When does it get hot?

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bound on event arrivals

- period: 120 ms
- jitter: 240 ms
- interarrival: 30 ms

workload model

- execution time: 30 ms
bound on event arrivals

- period: 120 ms
- jitter: 240 ms
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workload model

- execution time: 30 ms

peak temperatures

- average workload of tasks (25%): 342.5K
- random trace (500 s): 362.2 K
- reasonable heuristic: 363.5 K
- worst case: 366.9 K
heuristic worst case
Given

- a bound on workload arrivals (arrival curves)
- a computation model (from workload to task executions)
- a power model (from task executions to power)
- a temperature model (from power to temperature)

What is the worst case peak temperature?
The Models
**Workload Arrival Model**

- **Cumulative workload:** In time interval \([s, t)\), tasks with an accumulated workload of \(R(s,t)\) arrive.
- **Arrival curve:** The cumulative workload is upper bounded by the arrival curve:

\[
R(s, t) \leq \alpha(t - s) \quad \forall s < t
\]

- **Multiple inputs:**

\[
\alpha(\Delta) = \sum_{\text{inputs}} \alpha_i(\Delta)
\]
Event Stream

$R(0, 2.5)$: total workload in $t = [0 .. 2.5]$ ms

Arrival Curve $\alpha$

maximum workload in *any interval* of length 2.5 ms
**Computation Model**

- Arriving workload is buffered in FIFO
- *Work conserving schedule* (EDF, FP, GPS, …)

\[
C(s, t) = \inf_{s \leq u \leq t} \{(t - u) + R(s, u)\}
\]
$C(0, t)$
**Computation Model**

- Bound on the computing time

\[ C(t - \Delta, t) \leq \gamma(\Delta) = \inf_{0 \leq \lambda \leq \Delta} \left\{ (\Delta - \lambda) + \alpha(\lambda) \right\} \]
Given bound on task arrivals \( \alpha \) :
all feasible accumulated computing times are bounded by \( \gamma \)
Power Model

- active and idle modes

\[ P^a = \alpha^a T + \beta^a \]
\[ P^i = \alpha^i T + \beta^i \]

Temperature Model

\[ C \frac{dT}{dt} = -(G - \alpha)T + (\beta + GT^0) \]

- temperature-dependent leakage
- environment temperature
- thermal conductance
- thermal capacity
What is the worst-case task arrival sequence that leads to maximal peak temperature?
The Results
Suppose that the accumulated computing time function

\[ C^*(0, \Delta) = \gamma(\tau) - \gamma(\tau - \Delta) \]

for all \( 0 \leq \Delta \leq \tau \) leads to temperature \( T^*(\tau) \) at time \( \tau \). Then \( T^*(\tau) \) is an upper bound on the highest temperature

\[ T^*(\tau) \geq T(t) \]

for all \( 0 \leq t \leq \tau \) for all feasible workload traces that are bounded by the service curve \( \alpha \).
Does there exist a feasible input trace that leads to the peak temperature?

\[ R^*(s, t) \rightarrow C^*(s, t) \]

w.c. cumulative workload

w. c. accumulated computing time
The worst-case workload function

\[ R^*(0, \Delta) = C^*(0, \Delta) \]

for \( 0 \leq \Delta \leq \tau \) leads to the accumulated computing time
\( C^*(0, \Delta) \) and complies to the arrival curve \( \alpha \).

If the step size of \( \alpha(\Delta) \) is an integer multiple of \( c \) then

\[ \hat{R}^*(0, \Delta) = c \cdot \left[ \frac{1}{c} R^*(0, \Delta) \right] \]

has stepsize \( c \) and is a feasible worst case trace as well.
$C^*(0, t) = R^*(0, t)$

maximal temperature
How large should $\tau$ be?

All observation times $\tau$ that satisfy the following relation guarantee a peak-temperature precision of $T^a(\tau) - T^i(\tau)$

$$
\tau \geq \frac{1}{a^i} \cdot \log \left( \frac{(T^\infty)^a - (T^\infty)^i}{T^a(\tau) - T^i(\tau)} \right)
$$

$(T^\infty)^i$ and $(T^\infty)^a$ denote the steady state temperatures in idle and active mode, respectively.

$T^i(\tau)$ and $T^a(\tau)$ denote the temperatures at time $\tau$ with initial temperatures $(T^\infty)^i$ and $(T^\infty)^a$, respectively.
Some Simulations
**bound on event arrivals**

- period: 120 ms
- jitter: 240 ms
- interarrival: 30 ms

**workload model**

- execution time: 30 ms

**peak temperatures**

- average workload of tasks (25%): 342.5K
- random trace (500 s): 362.2 K
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- worst case: 366.9 K
\[
\alpha(\Delta) \quad \tau(\Delta) \\
C^*(0, \Delta) = R^*(0, \Delta)
\]

\[
T \ [K] \\
\Delta \ [s]
\]
<table>
<thead>
<tr>
<th></th>
<th>Video</th>
<th>Audio</th>
<th>Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>period</td>
<td>[20, 90]ms</td>
<td>30ms</td>
<td>30ms</td>
</tr>
<tr>
<td>jitter</td>
<td>[20, 90]ms</td>
<td>10ms</td>
<td>10ms</td>
</tr>
<tr>
<td>min. interarrival</td>
<td>1ms</td>
<td>1ms</td>
<td>1ms</td>
</tr>
<tr>
<td>execution demand</td>
<td>6ms</td>
<td>3ms</td>
<td>2ms</td>
</tr>
<tr>
<td>deadline</td>
<td>[20, 90]ms</td>
<td>30ms</td>
<td>30ms</td>
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</table>

Table I
PARAMETERS OF THE VIDEO CONFERENCING APPLICATION.

<table>
<thead>
<tr>
<th>$G$</th>
<th>$C$</th>
<th>$\alpha_i = \alpha^a$</th>
<th>$\beta_i$</th>
<th>$\beta^a$</th>
<th>$T^o$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.3 \frac{W}{K}$</td>
<td>0.03 $\frac{J}{K}$</td>
<td>$0.1 \frac{W}{K}$</td>
<td>$-25W$</td>
<td>$-11W$</td>
<td>$300K$</td>
<td>$1.5s$</td>
</tr>
</tbody>
</table>

Table II
PARAMETERS OF THE CONSIDERED EMBEDDED SYSTEM ARCHITECTURE.
worst case peak temperature

100 random simulations
not schedulable under EDF

schedulable under EDF

change task arrival of video
Conclusion
Critical instance for real-time analysis
Critical instance for temperature analysis

\[ \gamma = \Delta \otimes \alpha \]

\[ R^*(0, t) \]

maximal temperature

\( t \)

\( \Delta \)