Games that "SYNCHRONOUS" People Play

Michael Mendler & Joaquín Aguado
Faculty of Information Systems and Applied Computer Sciences,
University of Bamberg, Germany

SYNCHRON'07 29-Nov-2007

Overview A la Memento

1. Conclusions.

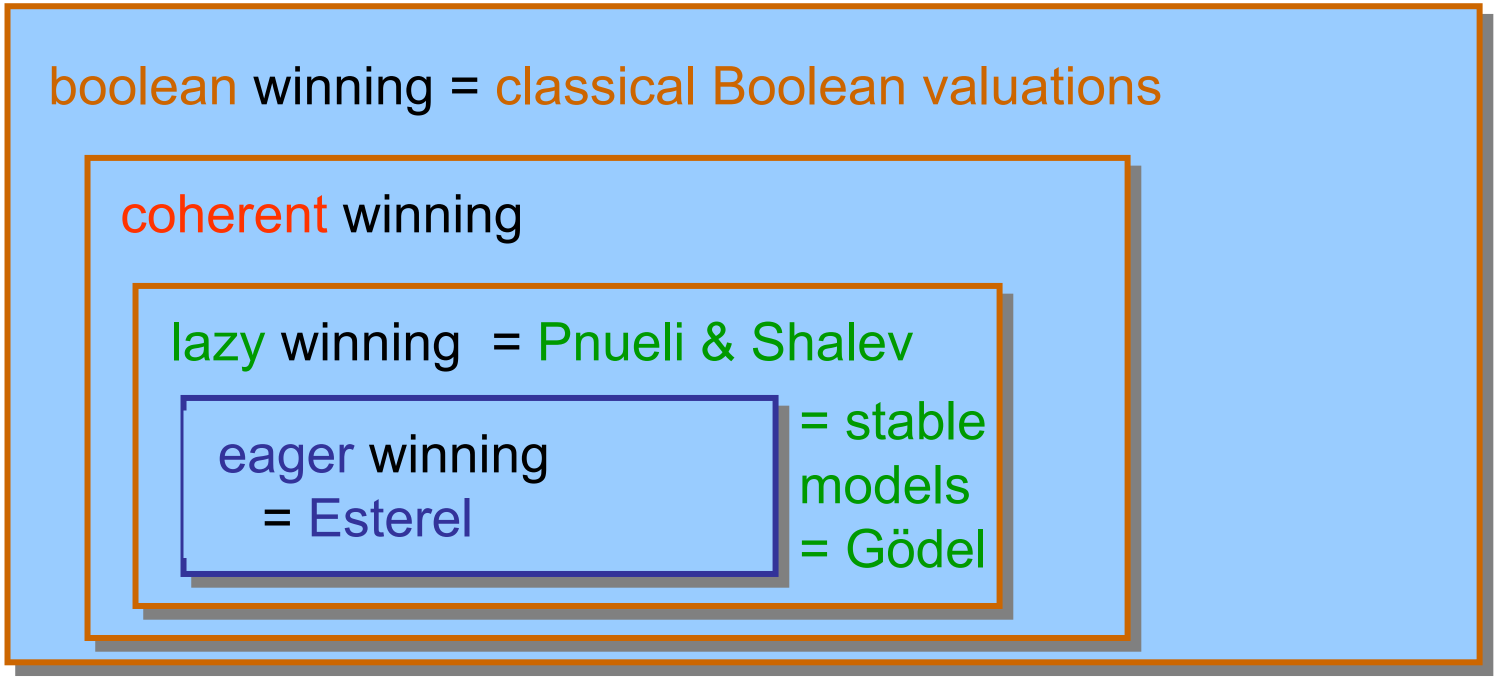
2. Boolean winning.
3. Coherent winning.
4. Lazy winning.
5. Eager winning.

6. Hierarchical State Machines.
7. Games that People Play.

8. Introduction.

1. Conclusions

We have identified four (natural) levels of semantics for synchronous (instantaneous) response in a game theoretic setting according to increasing restrictions on winning conditions.

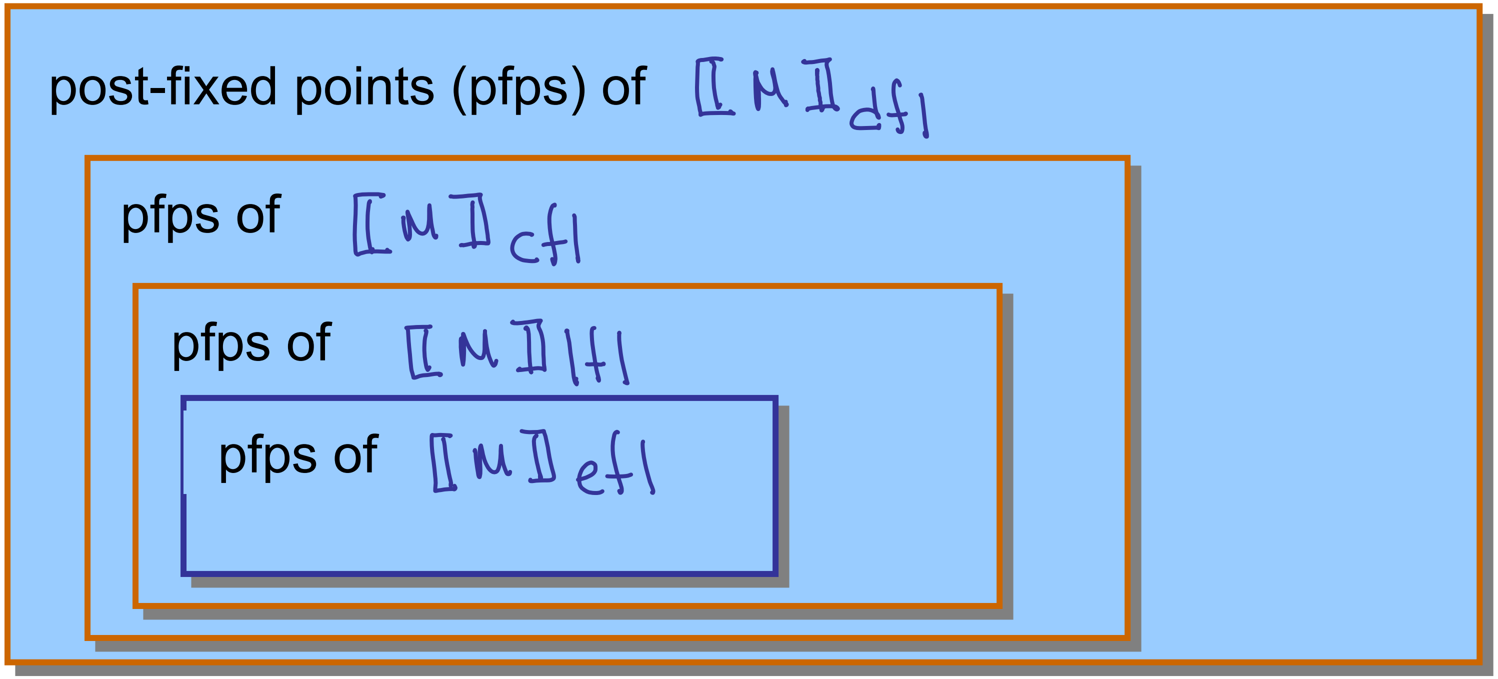


1. Conclusions

M induces monotone response functions on front-lines

$$[\![M]\!]_{\mathsf{efl}} \preceq [\![M]\!]_{\mathsf{lfl}} \preceq [\![M]\!]_{\mathsf{cfl}} \preceq [\![M]\!]_{\mathsf{dfl}} : \mathsf{FL} \rightarrow \mathsf{FL}$$

implementing the game-theoretic winning conditions:



1. Conclusions

The constructive refinament of the system function:

$$pe^+(P,O) := \langle t \rangle P \vee \langle i \rangle O \qquad pe^-(P,O) := [\tau]O \wedge [i]P$$

is reasembled as follows

$$[\![M]\!]_{dfl}(P,O) := (P \vee pe^+(P,O),\ O \wedge pe^-(P,O))$$

$$[\![M]\!]_{cfl}(P,O) := (pe^+(P,O),\ pe^-(P,O))$$

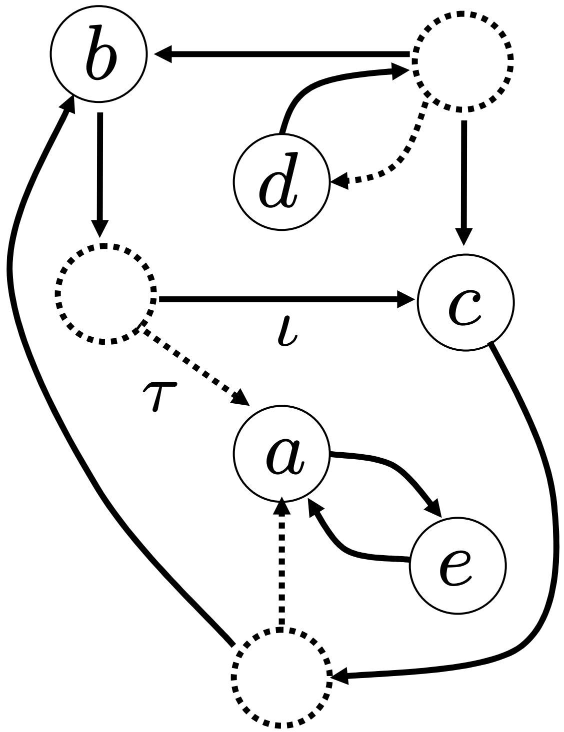
$$[\![M]\!]_{lfl}(P,O) := (\mu X.pe^+(P \wedge X, O),\ pe^-(P,O))$$

$$[\![M]\!]_{efl}(P,O) := \mu(X,Y).(pe^+(P \wedge X, O \wedge Y),\ pe^-(P \wedge X, O \wedge Y))$$

2. Boolean Winning

Two players $\mathbb{P} = \{A, B\}$ move a token through M taking turns.

- visible move = turn changes
- invisible move = player continues
- winning condition (example):
  last player loses



A maze is a transition system $M = (\mathbb{R}, \longrightarrow)$ with

rooms (states) $\mathbb{R} = \mathbb{R}_\iota \uplus \mathbb{R}_\tau$ $\mathbb{R}_\iota$ visible $\mathbb{R}_\tau$ secret

corridors (transition relation) $\longrightarrow \subseteq \mathbb{R} \times \{\iota, \tau\} \times \mathbb{R}$

$x \xrightarrow{\iota} y$ visible corridor $x \xrightarrow{\tau} y$ secret corridor

2. Boolean Winning

A strategy for a player U is a subset of plays in which U's moves are uniquely determined at each stage when U holds the turn, while keeping unconstrained the decisions of the opponent.

We are interested only in positional and consistent strategies.

For a strategy Σ,

$$P_\Sigma = \{ m \mid \exists \pi, \sigma.\ \pi \cdot (m, A) \cdot \sigma \in \Sigma \}$$

$$O_\Sigma = \{ m \mid \exists \pi, \sigma.\ \pi \cdot (m, B) \cdot \sigma \in \Sigma \}$$

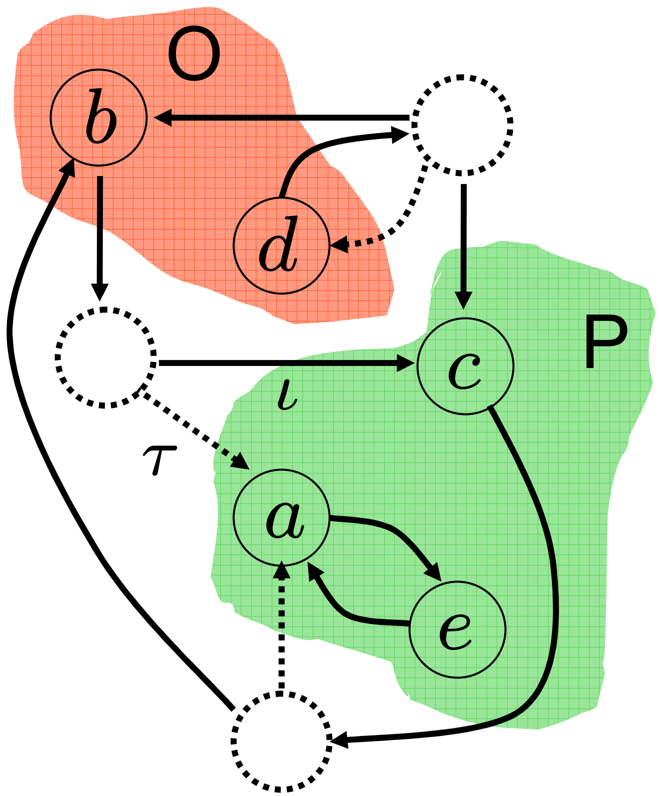
That is, $P_\Sigma \cap O_\Sigma = \emptyset$

2. Boolean Winning

(P,O) front-line.

Player A starts in P
Player B starts from O

For (P,O) to be a truth valuation
it must be „defensible“ by A



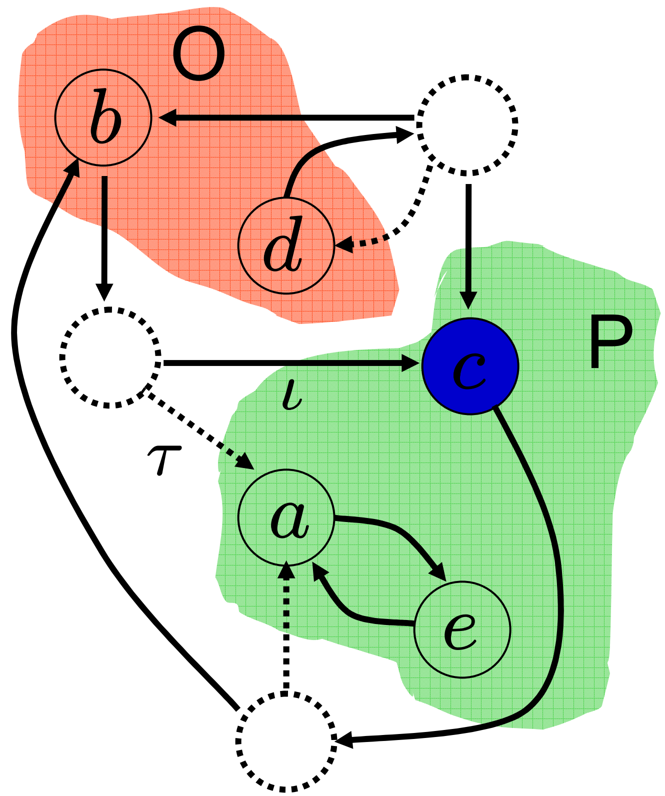
We assume M is a fixed, finitely branching maze.

2. Boolean Winning

(P,O) front-line.

Player A starts in P
Player B starts from O

For (P,O) to be a truth valuation
it must be „defensible“ by A



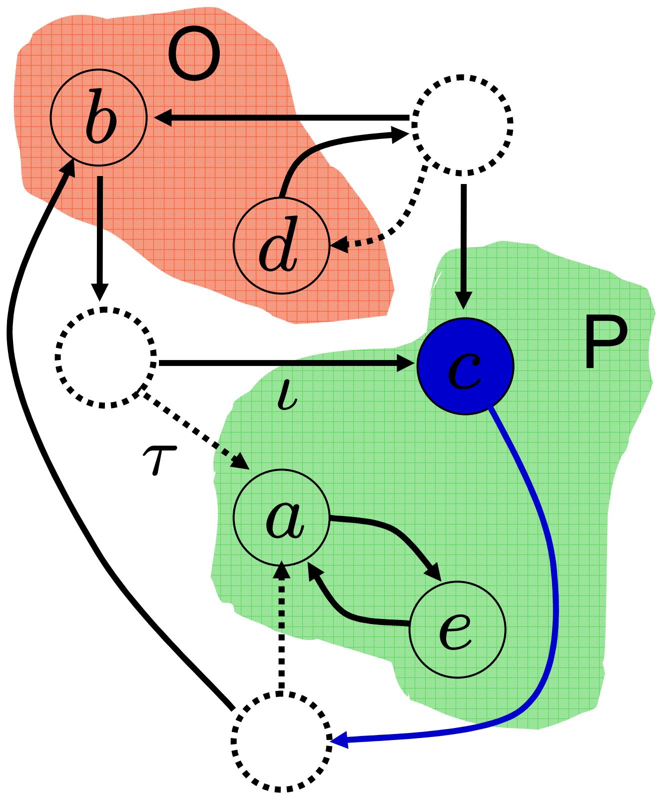
We assume M is a fixed, finitely branching maze.

2. Boolean Winning

(P,O) front-line.

Player A starts in P
Player B starts from O

For (P,O) to be a truth valuation
it must be „defensible“ by A



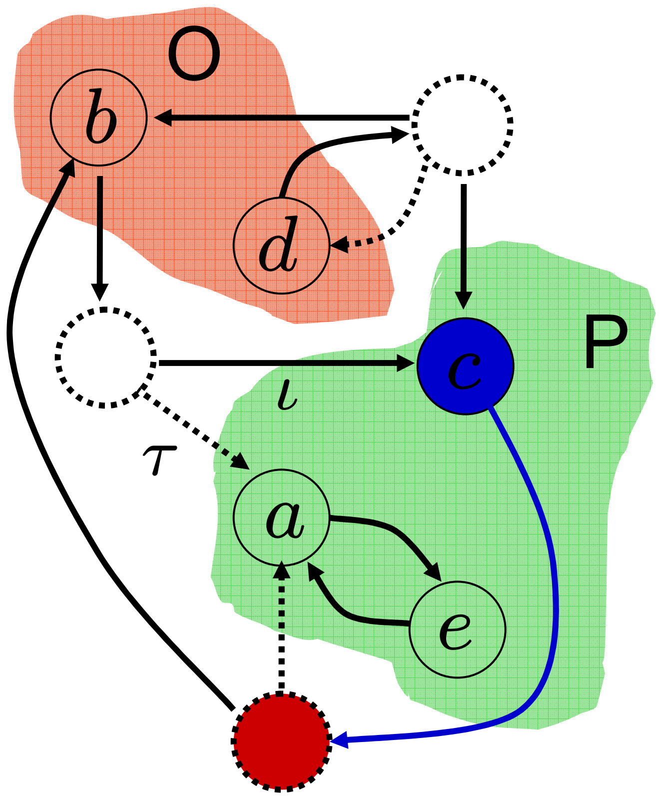
We assume M is a fixed, finitely branching maze.

2. Boolean Winning

(P,O) front-line.

Player A starts in P
Player B starts from O

For (P,O) to be a truth valuation
it must be „defensible" by A



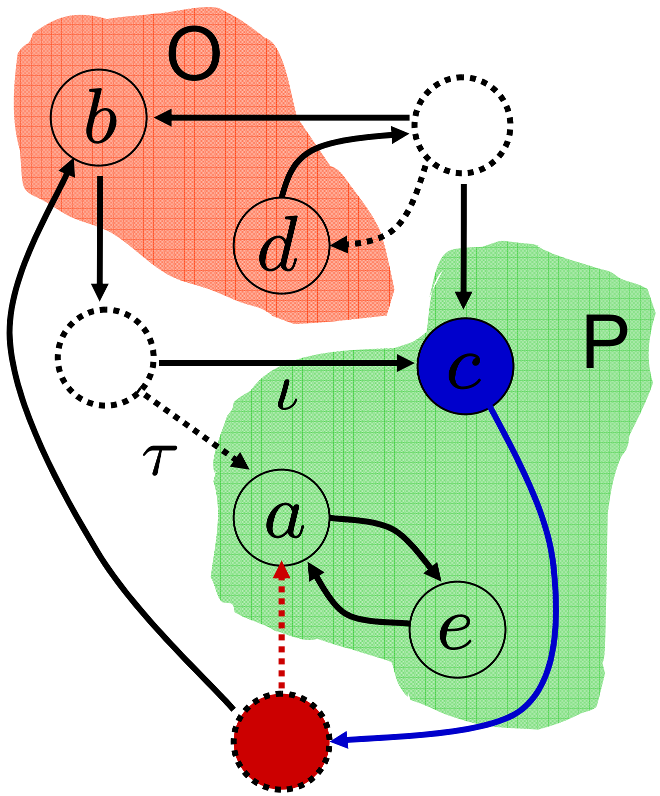
We assume M is a fixed, finitely branching maze.

2. Boolean Winning

(P,O) front-line.

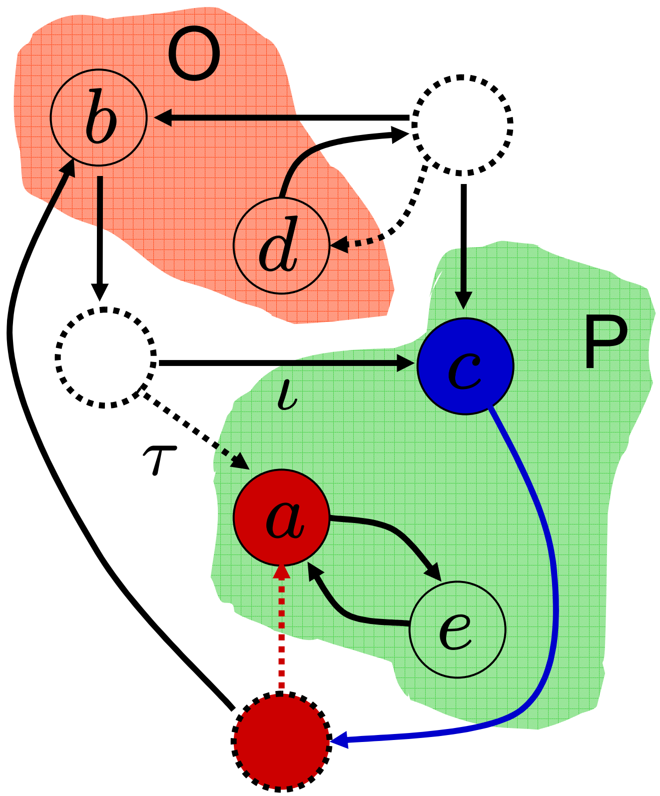
Player A starts in P
Player B starts from O

For (P,O) to be a truth valuation
it must be „defensible“ by A



We assume M is a fixed, finitely branching maze.

2. Boolean Winning



(P,O) front-line.

Player A starts in P
Player B starts from O

For (P,O) to be a truth valuation
it must be „defensible“ by A

Opponent B captures P-room a. So,
front line is not defensible

We assume M is a fixed, finitely branching maze.

2. Boolean Winning

We call pairs (P,O) of regions with the property $P \cap O = \emptyset$, front lines, and (P_Σ, O_Σ) the front line defended by Σ. We are interested in maximal front lines that are defensible using particular types of strategies and characterise them in terms of post-fixed points.

To understand the connection it is useful to view the maze as a Kripke transition structure in which regions may be specified as formulas of propositional modal μ-calculus.

$$\langle \gamma \rangle \, [\gamma] : 2^S \longrightarrow 2^S$$

$$\langle \gamma \rangle (R) := \{ m \mid \exists m' \in R.\ m \xrightarrow{\gamma} m' \}$$

$$[\gamma] (R) := \{ m \mid \forall m' \in R.\ m \xrightarrow{\gamma} m' \}$$

2. Boolean Winning

The logical connectives ‘and’, ‘or’ correspond to union and intersection on 2^S. Negation is complementation.

Further, there are least fixed point μX.φ and greatest fixed point νX.φ operators with the usual interpretation.

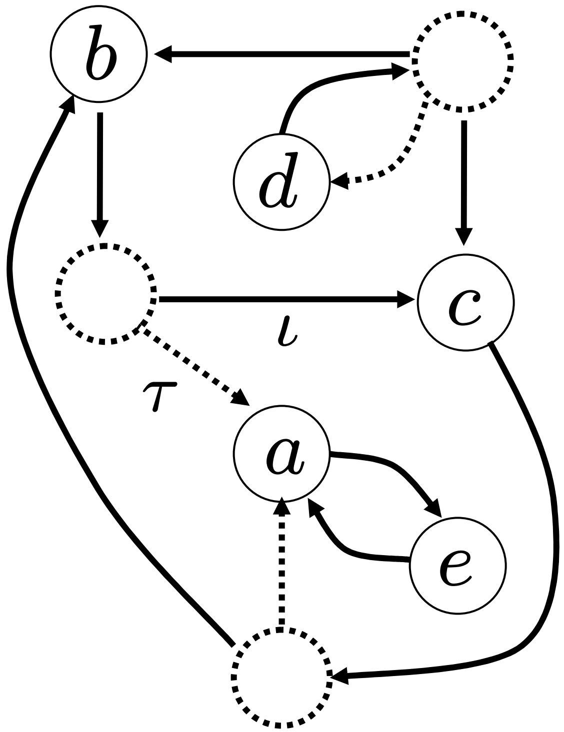
Assuming that the formula scheme φ is monotonic in the recursion variable X.

$$\nu X.\phi := \bigcup \{ X \mid X \subseteq \phi(X) \} \quad \text{greatest}$$

$$\mu X.\phi := \bigcap \{ X \mid \phi(X) \subseteq X \} \quad \text{least}$$

2. Boolean Winning

Win = Π



Observation:

If (P,O) defensible then it is defensible by α = Ø.

2. Boolean Winning

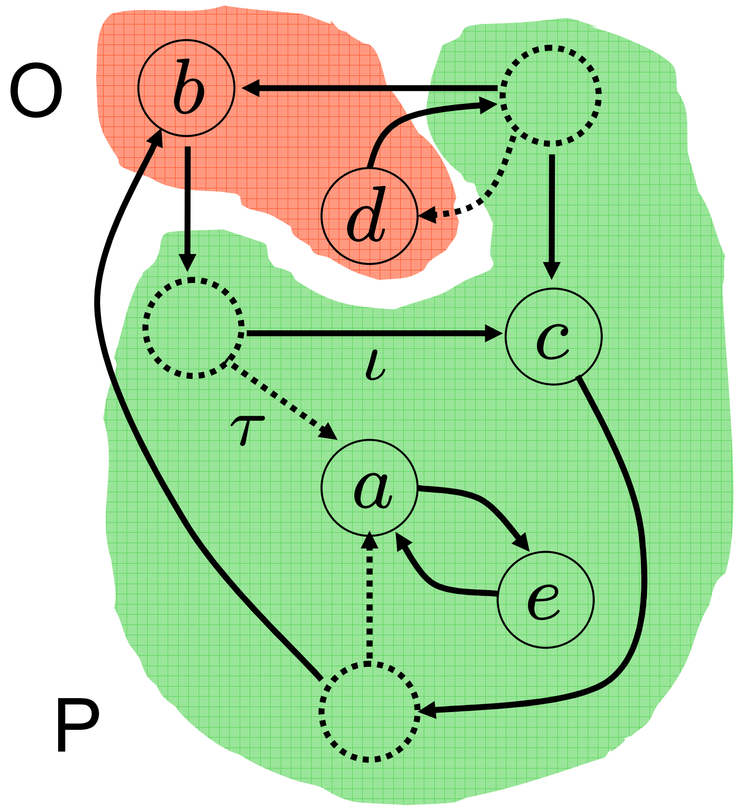
Win = Π

(P,O) is defensible

P = { a,c,e,... }
O = { b, d... }

Observation:

If (P,O) defensible then it is defensible by α = Ø.



$O \subseteq [\tau]O \wedge [\iota]P$

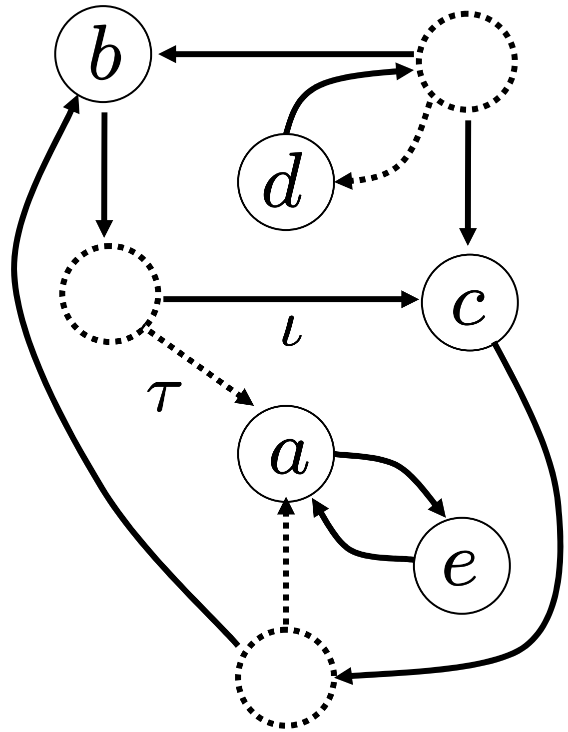
maximal since

$P = \neg O$

2. Boolean Winning

Win = Π

The maximal defensible (P,O) are binary and coincide with the classical models of φ.

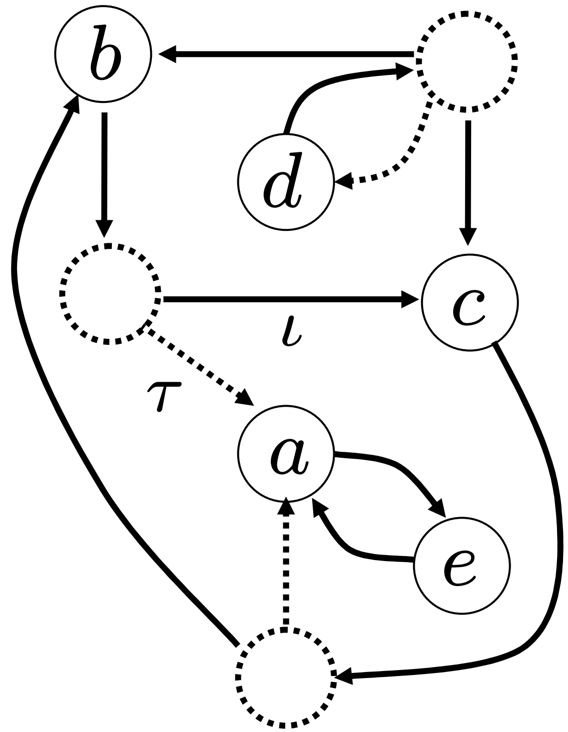


$$
\begin{aligned}
\phi = \; & (c \wedge \neg a) \rightarrow b \; \wedge \\
& (b \wedge c \wedge \neg d) \rightarrow d \; \wedge \\
& (b \wedge \neg a) \rightarrow c \; \wedge \\
& \neg a \rightarrow e \; \wedge \\
& \neg e \rightarrow a \; \wedge \\
& \neg a \rightarrow c
\end{aligned}
$$

3. Coherent Winning

(P, O) is coherent if it is defensible by a live strategy α,

A always makes a move when he gets the turn.

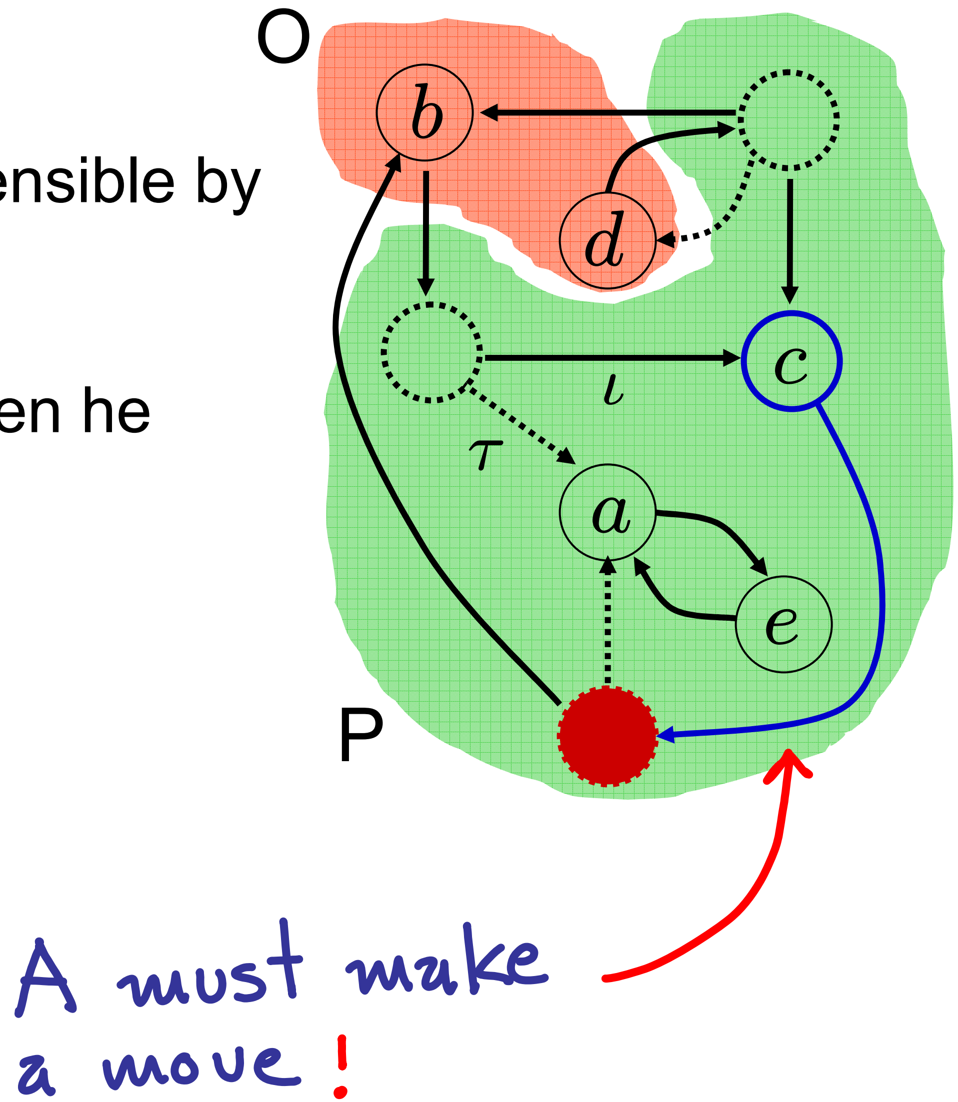


3. Coherent Winning

(P, O) is coherent if it is defensible by a live strategy α,

A always makes a move when he gets the turn.

(P,O) is not coherent !

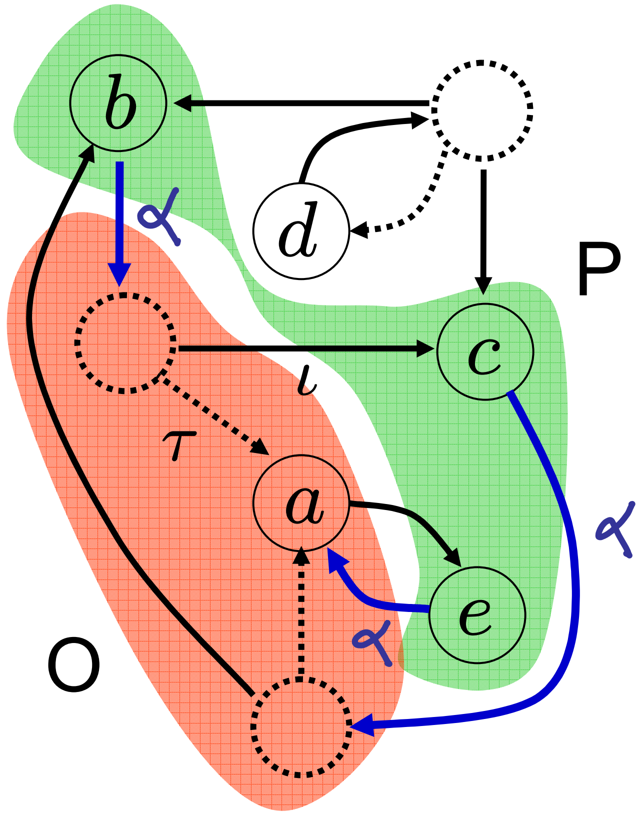


3. Coherent Winning

(P, O) is coherent if it is defensible by a live strategy α,

A always makes a move when he gets the turn.

(P,O) is maximal coherent under α.



$$O \subseteq [\tau]O \wedge [\iota]P$$

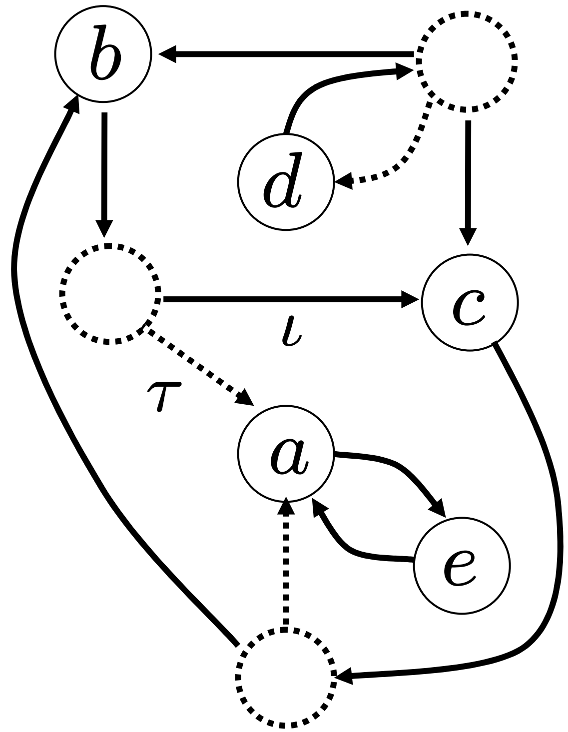
$$P \subseteq \langle\tau\rangle P \vee \langle\iota\rangle O$$

$$(P, \neg P) \text{ iff } P = \langle\tau\rangle P \vee \langle\iota\rangle \neg P$$

4. Lazy Winning

(P, O) is lazy if it is defensible by a reactive strategy α,

A always eventually hands over to B in a *visible* room.

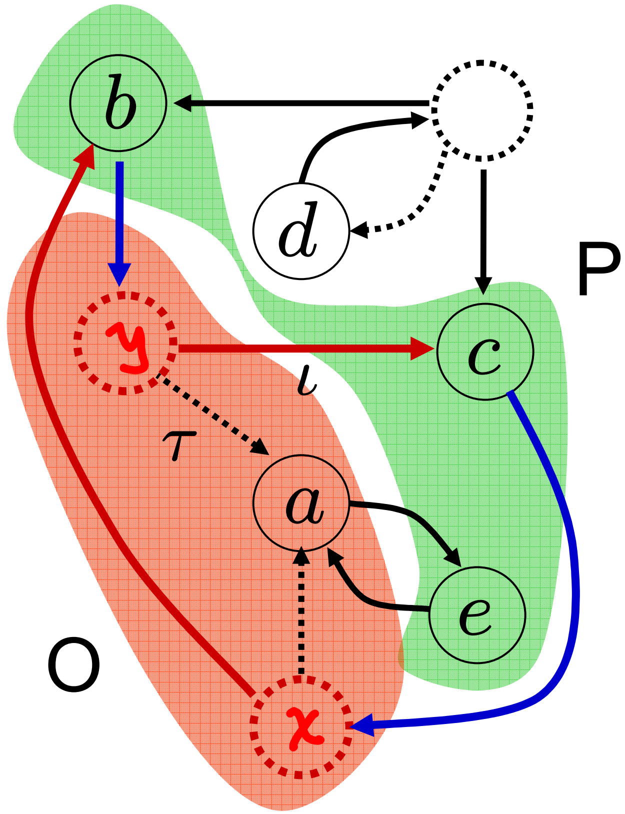


4. Lazy Winning

(P, O) is lazy if it is defensible by a reactive strategy α,

A always eventually hands over to B in a *visible* room.

(P,O) is not lazy !



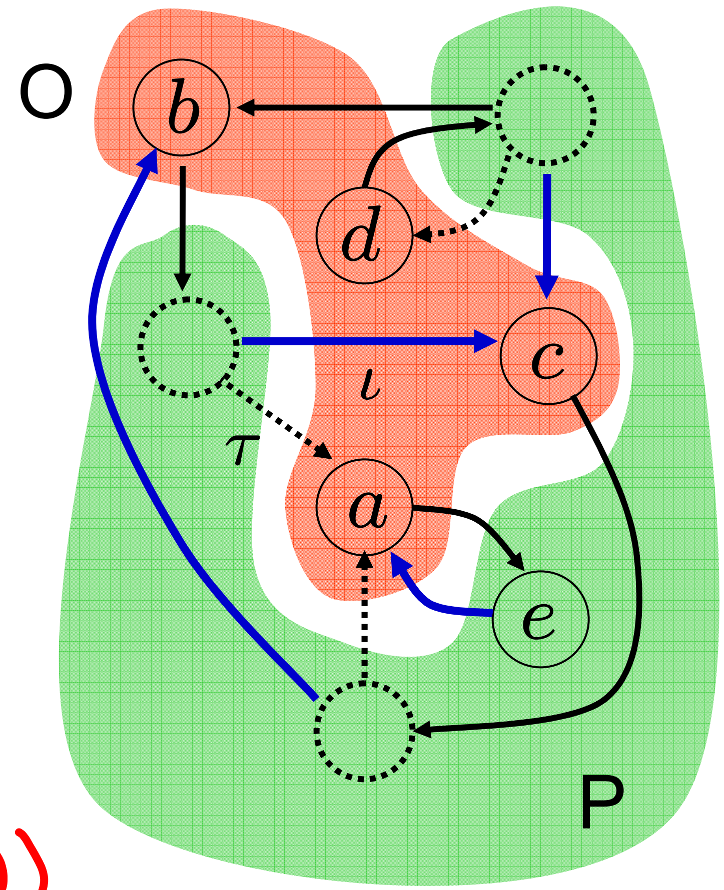
In the cyclic play
b → y → c → x → b
B only plays from secret rooms !

4. Lazy Winning

(P, O) is lazy if it is defensible by a reactive strategy α,

A always eventually hands over to B in a *visible* room.

(P,O) is maximal lazy under α.



$O \subseteq [\tau]O \wedge [\iota]P$

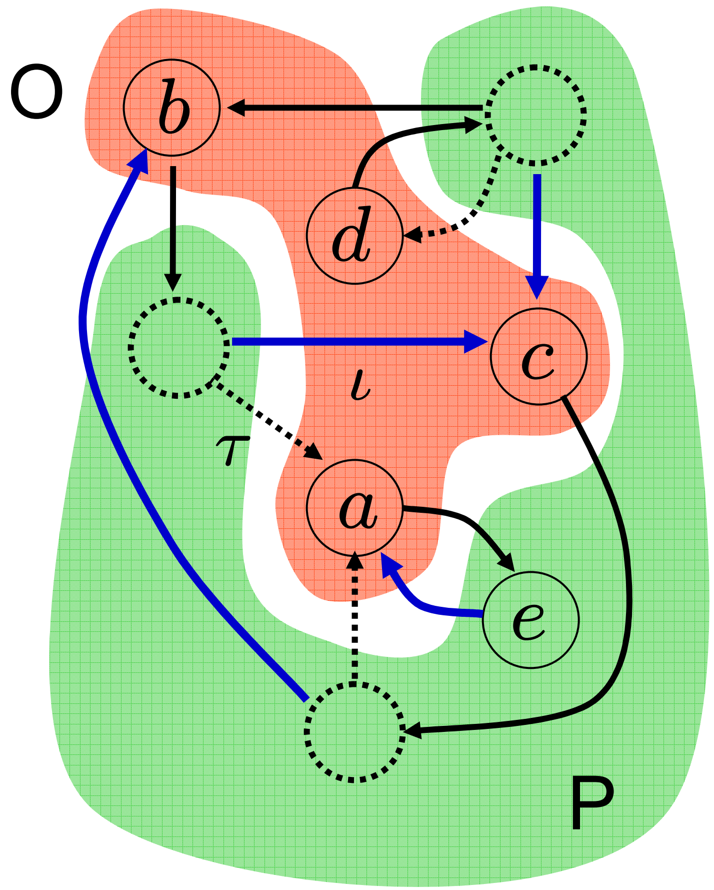
$P \subseteq \mu X.(\langle\tau\rangle(P \wedge X) \vee \langle\iota\rangle O)$

(P, ¬P) is maximal lazy iff

$P = \mu X.(\langle\tau\rangle(P \wedge X) \vee \langle\iota\rangle \neg P).$

4. Lazy Winning

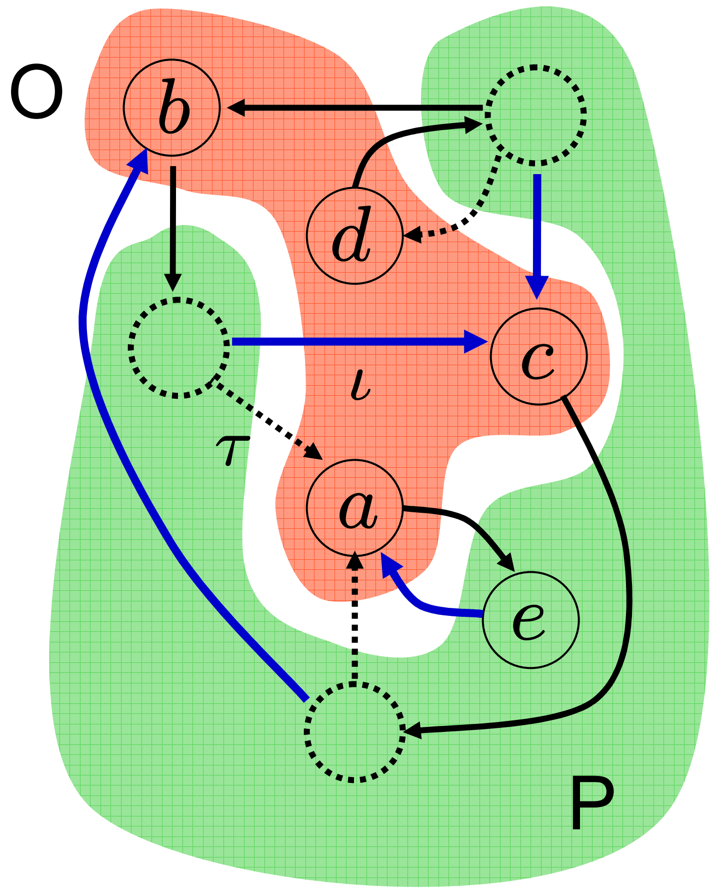
The *binary* lazy (P,O) coincide with the minimal intuitionistic models of φ in Gödel‘s three-valued logic (0 ≤ ½ ≤ 1).



4. Lazy Winning

The *binary* lazy (P,O) coincide with the minimal intuitionistic models of φ in Gödel's three-valued logic (0 ≤ ½ ≤ 1).

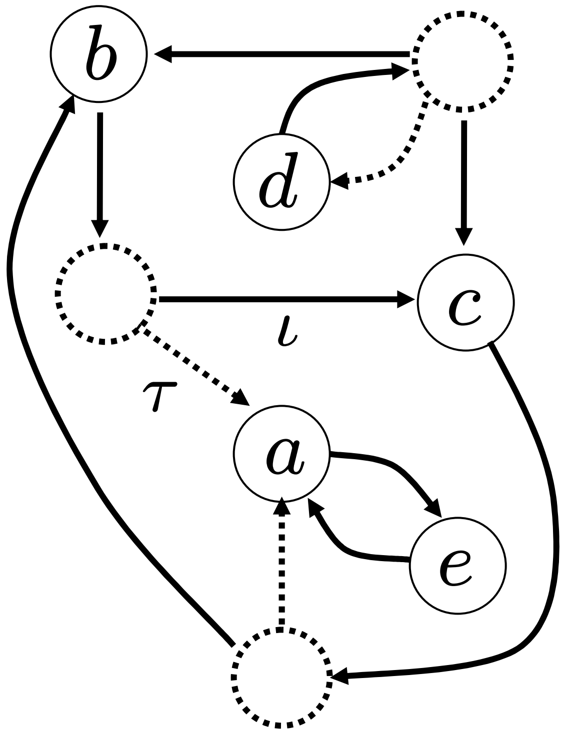
The maximal lazy (P,O) coincide with the stable models (*Prolog*) or Statecharts step responses (*Pnueli&Shalev*) of φ.



5. Eager Winning

(P, O) is eager if it is defensible by a terminating strategy α,

All plays are finite making B get stuck in a *visible* room.

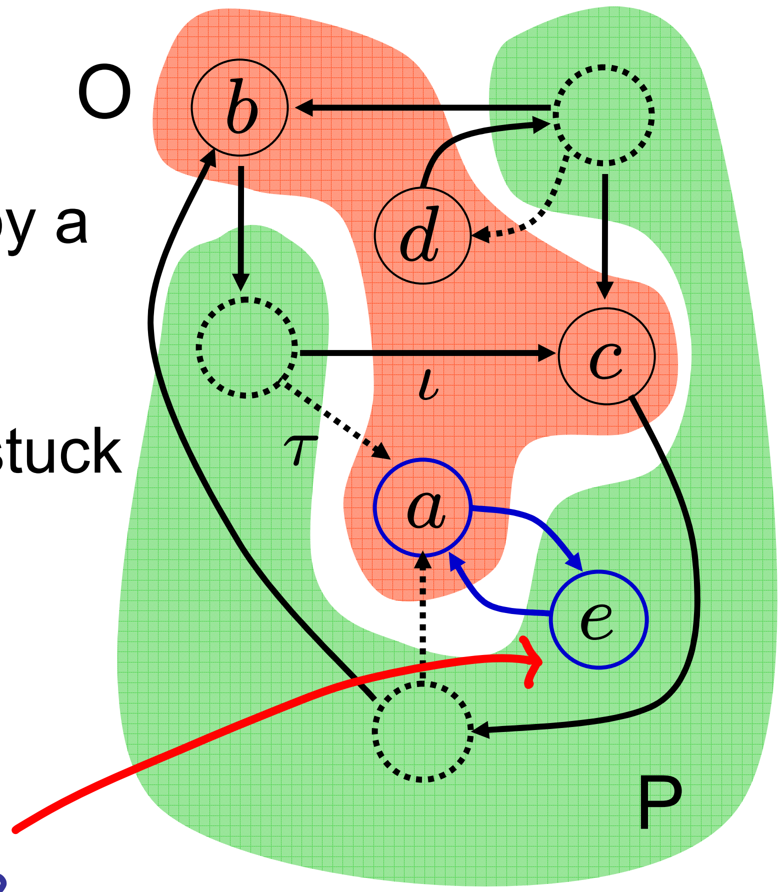


5. Eager Winning

(P, O) is eager if it is defensible by a terminating strategy α,

All plays are finite making B get stuck in a *visible* room.

(P,O) is not eager !



Every play from e has an infinite cycle

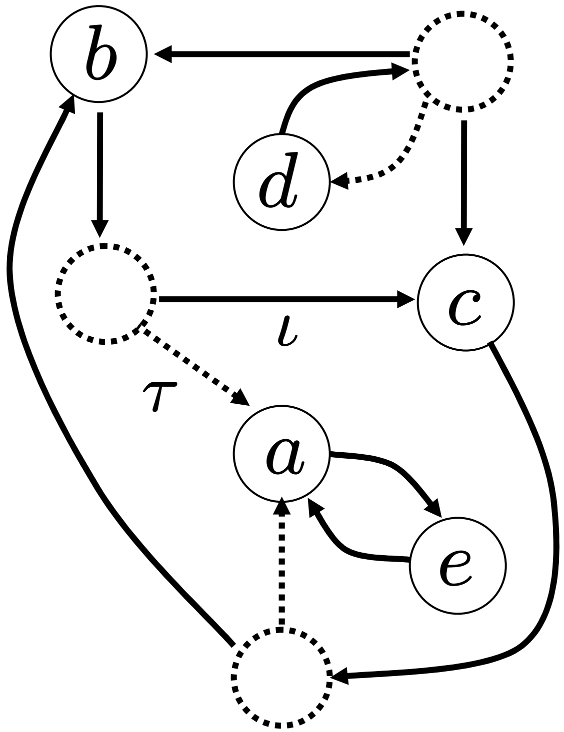
5. Eager Winning

(P, O) is eager if it is defensible by a terminating strategy α,

All plays are finite making B get stuck in a *visible* room.

Indeed ….

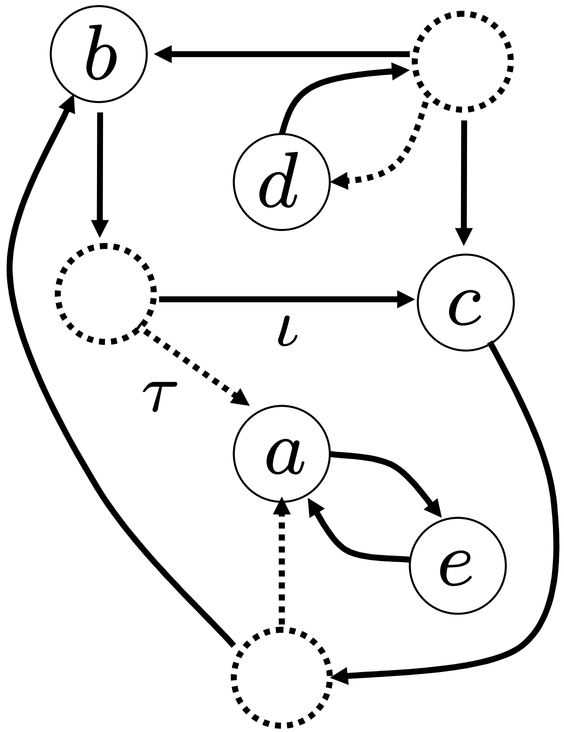
this maze has no eager front-line since the opponent B can always force infinite plays



5. Eager Winning

We need some “dungeons”

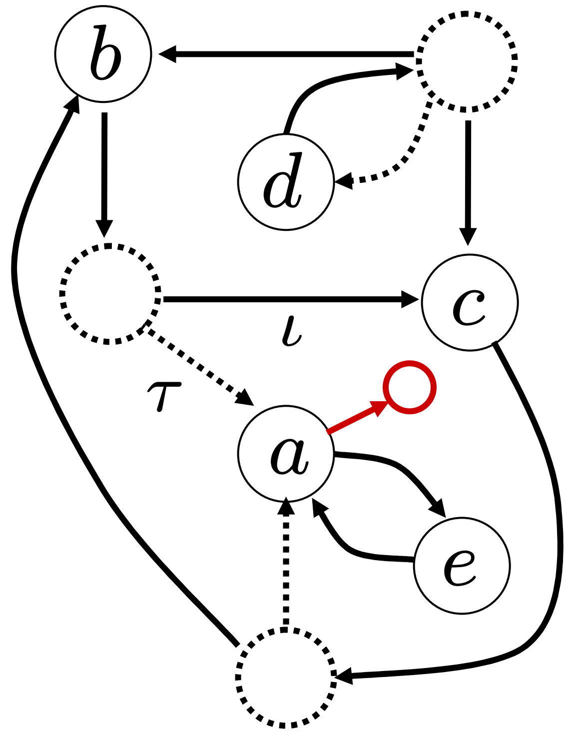
$$
\begin{aligned}
\phi = \; & (c \wedge \neg a) \rightarrow b \;\wedge \\
& (b \wedge c \wedge \neg d) \rightarrow d \;\wedge \\
& (b \wedge \neg a) \rightarrow c \;\wedge \\
& \neg a \rightarrow e \;\wedge \\
& \neg e \rightarrow a \;\wedge \\
& \neg a \rightarrow c
\end{aligned}
$$



5. Eager Winning

We need some "dungeons"

$$\phi = (c \wedge \neg a) \rightarrow b \ \wedge$$
$$(b \wedge c \wedge \neg d) \rightarrow d \ \wedge$$
$$(b \wedge \neg a) \rightarrow c \ \wedge$$
$$\neg a \rightarrow e \ \wedge$$
$$\neg e \rightarrow a \ \wedge$$
$$\neg a \rightarrow c \ \wedge$$
$$\top \rightarrow a$$

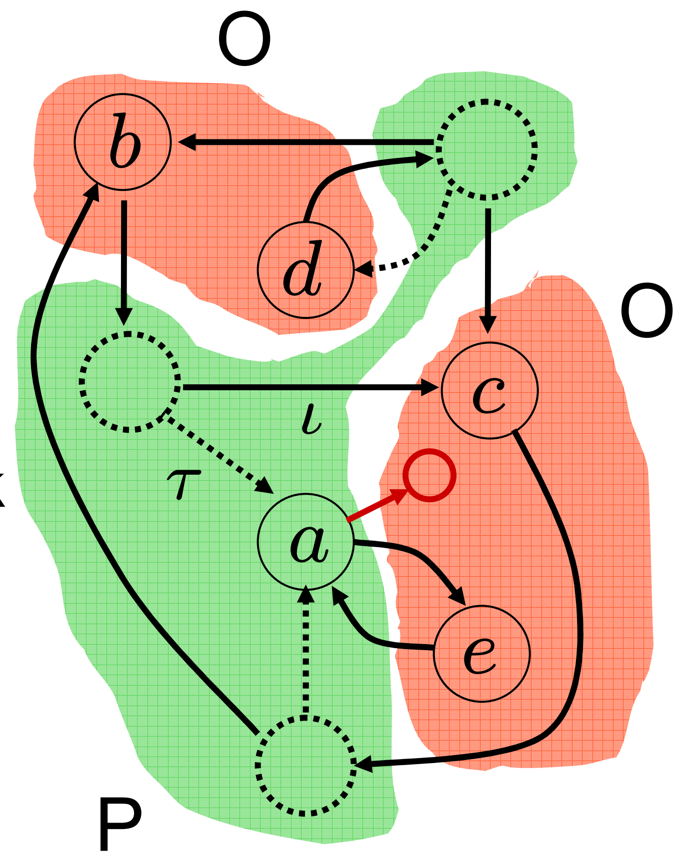


5. Eager Winning

(P, O) is eager if it is defensible by a terminating strategy α,

All plays are finite making B get stuck in a *visible* room.

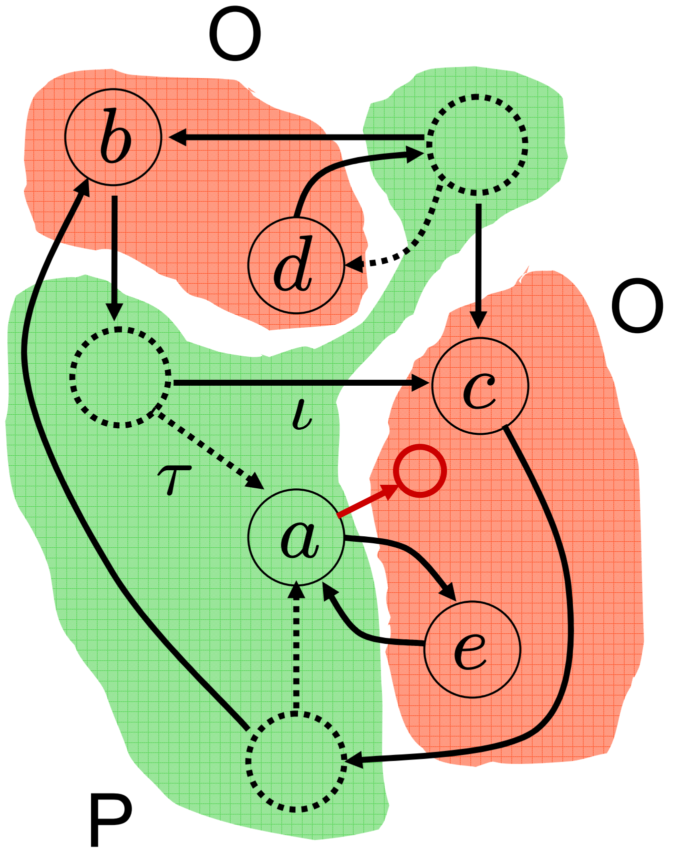
(P,O) is maximal eager under α.



5. Eager Winning

The maximal eager (P,O) coincide with Esterel step responses (*Berry*) of φ.

There is also a characterisation of eager front lines. It is known that (Win, Lose) can be obtained as:



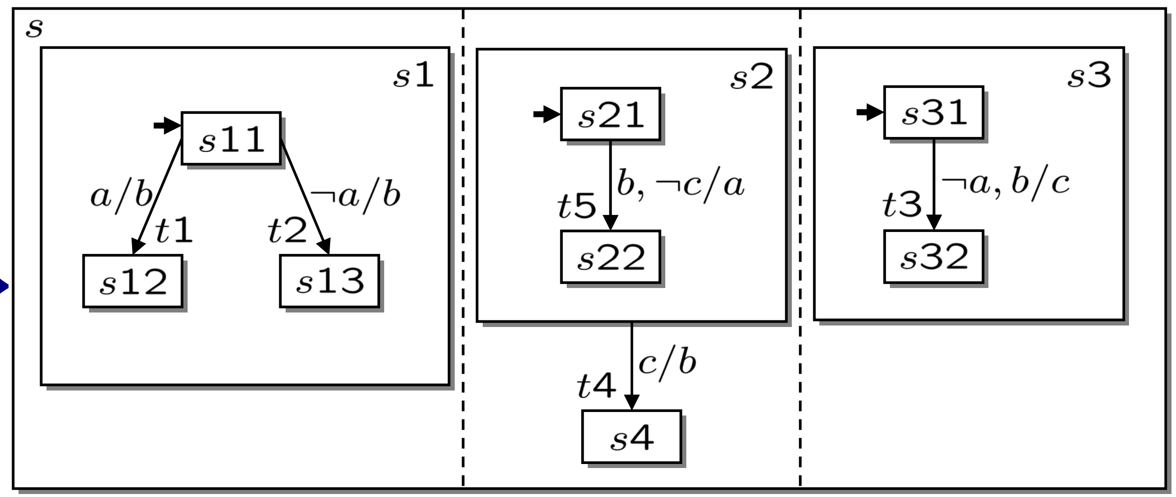
$$(Win, Lose) = \mu(X,Y).\,(\langle\tau\rangle X \vee \langle\iota\rangle Y,\ [\tau]Y \wedge [\iota]X)$$

6. Hierarchical State Machines

$C = \{s, s_1, s_2, \ldots\}$, $T = \{t_1, t_2, \ldots\}$, $E = \{a, b, c\}$

STATES TRANSITIONS EVENTS

Input Stimuli ⇒

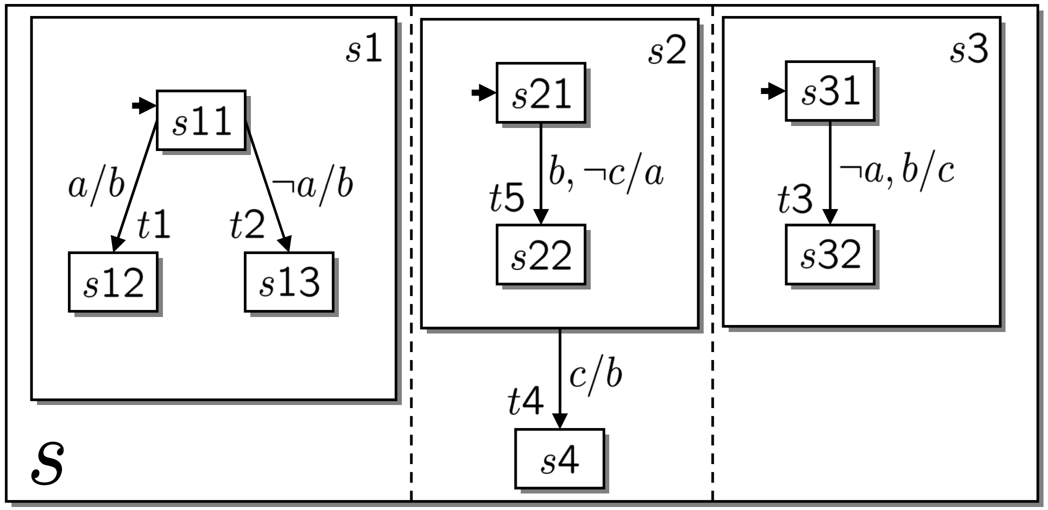


⇒ Synchronous Reaction = Macro Step

start configuration
transition schedule
environment events

next states
fired transitions
produced events

6. Hierarchical State Machines



$$\Phi = t_1 \Rightarrow b \;\wedge$$
$$t_2 \Rightarrow b \;\wedge$$
$$t_3 \Rightarrow c \;\wedge \;\ldots$$
$$(s_2 \wedge c) \Rightarrow t_4 \;\wedge$$
$$\ldots$$

- flat conjunction of transitions
- obtained from visual syntax, structurally and incrementally
- negations code non-determinism, priorities and hierarchy

7. Games that People Play

- ban negations [Modecharts 1994]
- only accept conflict-free, deterministic programs [Argos, Normal Logic Programming]
- give up global consistency [Huizing&al. 1988]
- add consistency as implicit trigger [Maggiolo-Schettini&al. 1996, Lüttgen&al. 1999]
- speculate on absence, if necessary backtrack [Pnueli&Shalev 1991]
- only schedule causally independent transitions, give up synchrony hypothesis [Statemate]
- negation as “positive“ absence [Berry 2000]
- negation with implicit delay [Saraswat‘s TCCP 1994]

8. Introduction

This talk is about a new game-theoretic approach offering a uniform understanding of several constructive solutions.

In particular reactions specified in

Esterel/SynchCharts (Berry)
Statecharts (Pnueli & Shalev)

that are long known but whose fundamental nature is only recently becoming clearer.

8. Introduction

Synchronous programming involves non-monotonic step functions.

Non-monotonic step function can be refined into 2-player games.

Different winning conditions can model different degrees of constructive truth values (defensible front-lines).

Truth-values are characterised as maximal post-fixed points of monotone functionals on cpo of front-lines.