Semantics of Dynamic Structure Hybrid Simulation Models

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November 27th, 2007

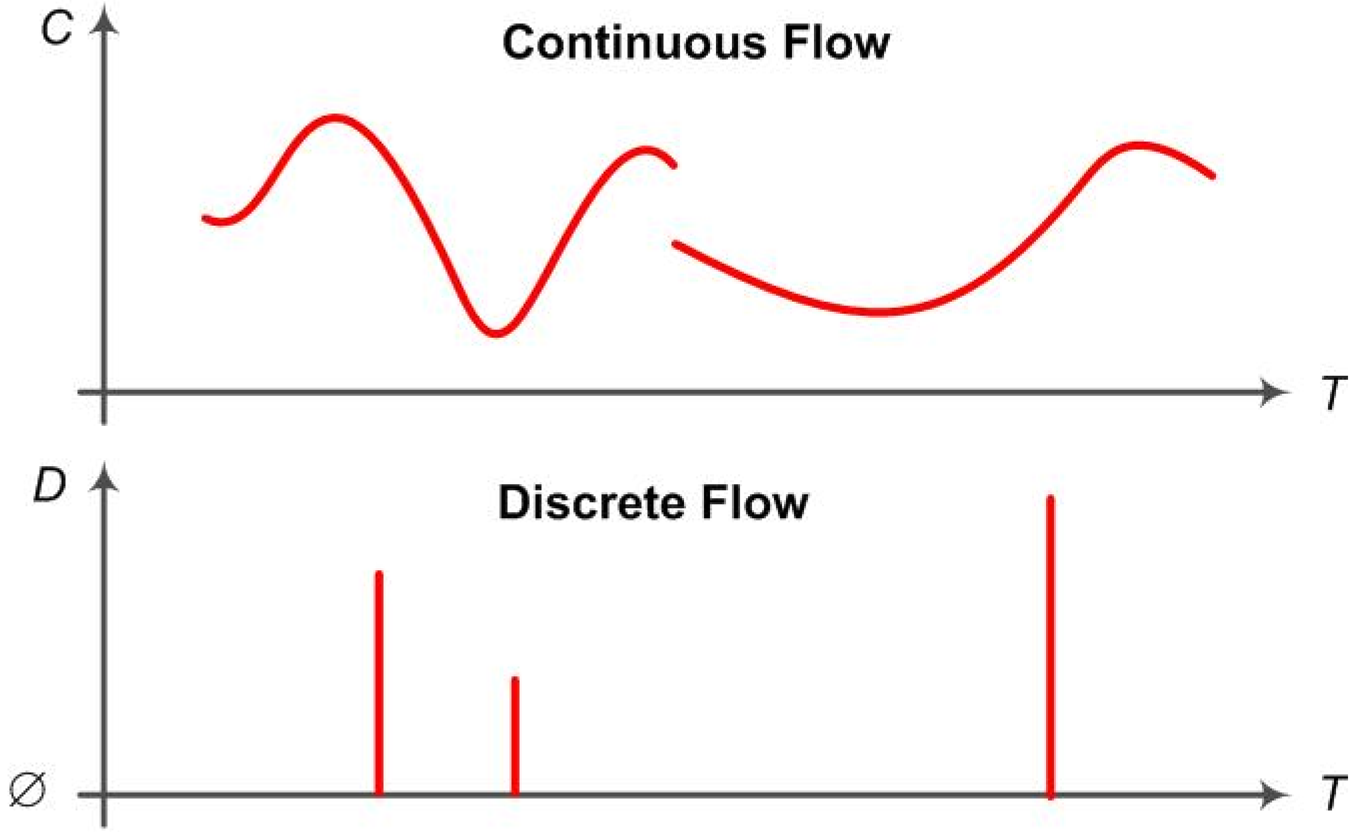
Introduction

- Modeling and simulation of hybrid systems has been subjected to intense research.
- The traditional trend in modeling and simulation of continuous and hybrid systems rely on ideal continuous machines.
- The ODE solvers are usually hidden from the user and are not explicitly represented.
- This approach does not encourage model interoperability.
- Semantics are commonly not well defined.

HFSS Formalism

- The *Heterogeneous Flow System Specification* (HFSS) is a modular formalism able to describe hybrid hierarchical models with a time-varying structure.
- HFSS uses a traditional representation of discrete event systems but it introduces the new concept of **generalized sampling** to achieve the description of continuous signals.
- HFSS treats sampling as a first order concept.
- Both time and component varying sampling are supported.
- The long accepted discrete time machines are limited to represent single and time-invariant sampling rate.
- Discrete-Time machines were superseded by HFSS machines.

HFSS Flows

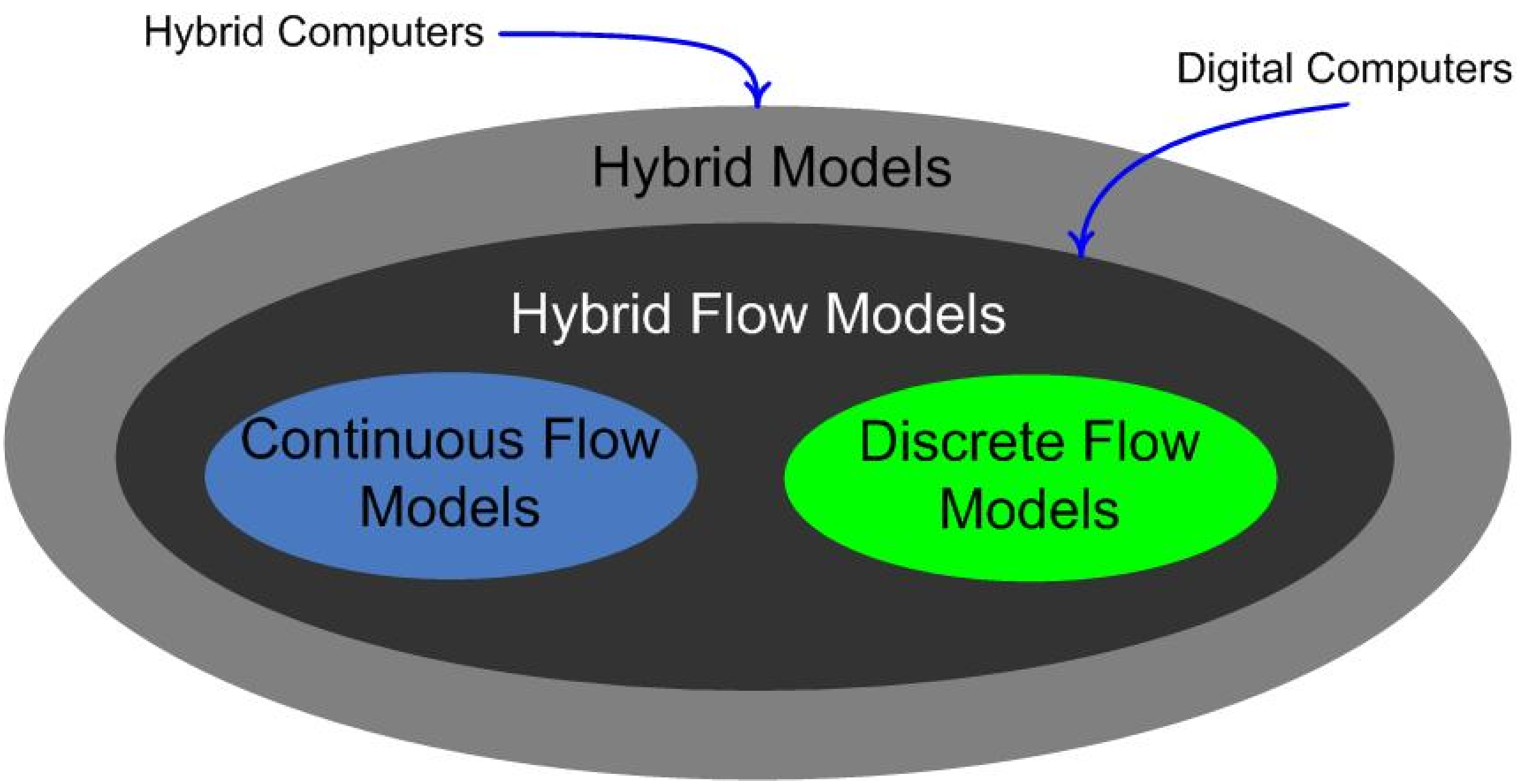


$\varnothing \equiv$ absence of value (null value)

Digital Computers

- A digital computer can only represent a set of states $S = \{(p,e)|p \in P, e \in \mathbb{R}_0^+\}$ where P is a set of piecewise constant partial states (p-states) and e the time elapsed in p-state p.
- All components that can be represented in a digital computer are discrete since the number of changes in a component p-state is finite during a finite time interval.
- Under this assumption the term **discrete** looses its discriminating expressiveness.
  - models can be better categorized by the type of flow they can handle.

Modeling Formalisms



HFSS Basic Model I

$$M_B = (X, Y, P, \rho, \omega, s_0, \delta, \bar{\Lambda}, \lambda), \text{ for } B \in \widehat{B}$$

$X = \bar{X} \times \check{X}$ set of input flow values

$\bar{X}$ set of continuous input flow values

$\check{X}$ set of discrete input flow values

$Y = \bar{Y} \times \check{Y}$ set of output flow values

$\bar{Y}$ set of continuous output flow values

$\check{Y}$ set of discrete output flow values

P set of partial states (p-states)

$\rho : P \to \mathbb{R}_0^+$ time-to-input function

$\omega : P \to \mathbb{R}_0^+$ time-to-output function

HFSS Basic Model II

$S = \{(p, e) | p \in P, 0 \leq e \leq \nu(p)\}$ state set

$\nu(p) = \min\{\rho(p), \omega(p)\}$, time to transition function

$s_0 = (p_0, e_0) \in S$ initial state

$\delta : S \times X^{\varnothing} \rightarrow P$ the transition function

$X^{\varnothing} = \bar{X} \times \check{X}^{\varnothing}$

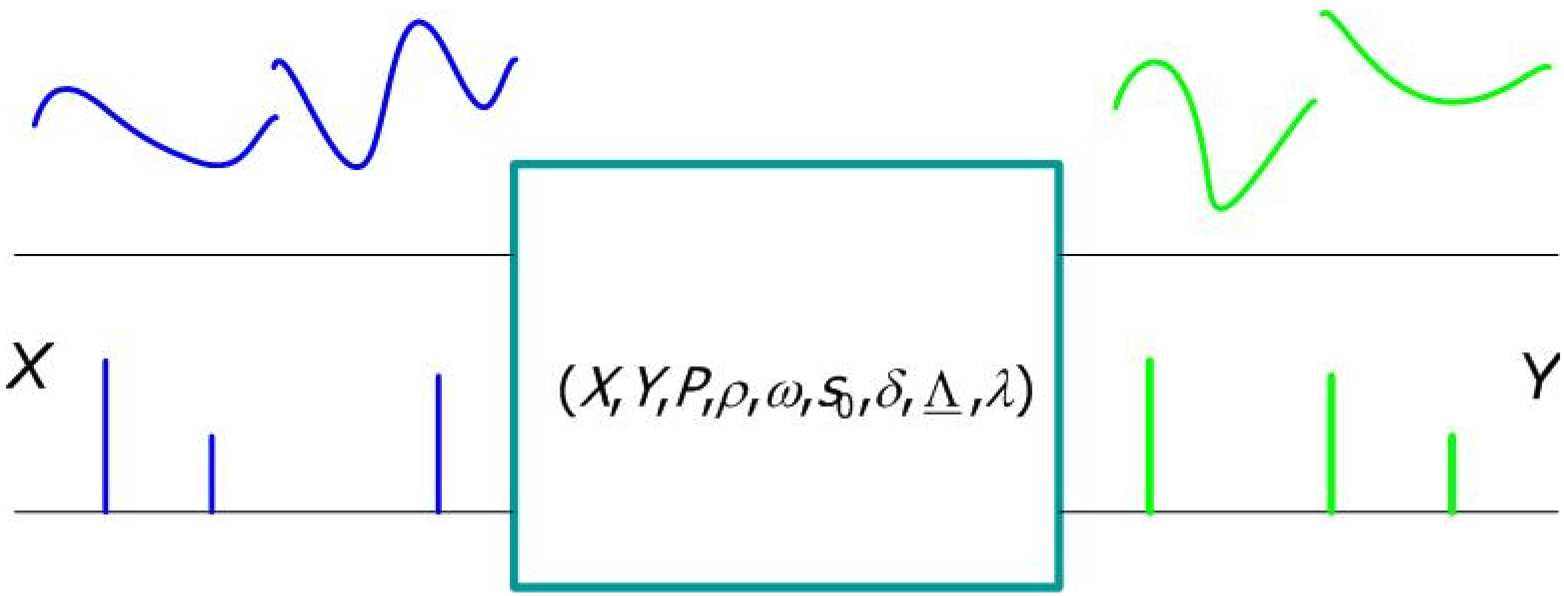
$\check{X}^{\varnothing} = \check{X} \cup \{\varnothing\}$

$\varnothing$ null value

$\bar{\Lambda} : S \rightarrow \bar{Y}$ continuous output function

$\lambda : P \rightarrow \check{Y}$ partial discrete output function

Atomic Component



Real-Time Intervals

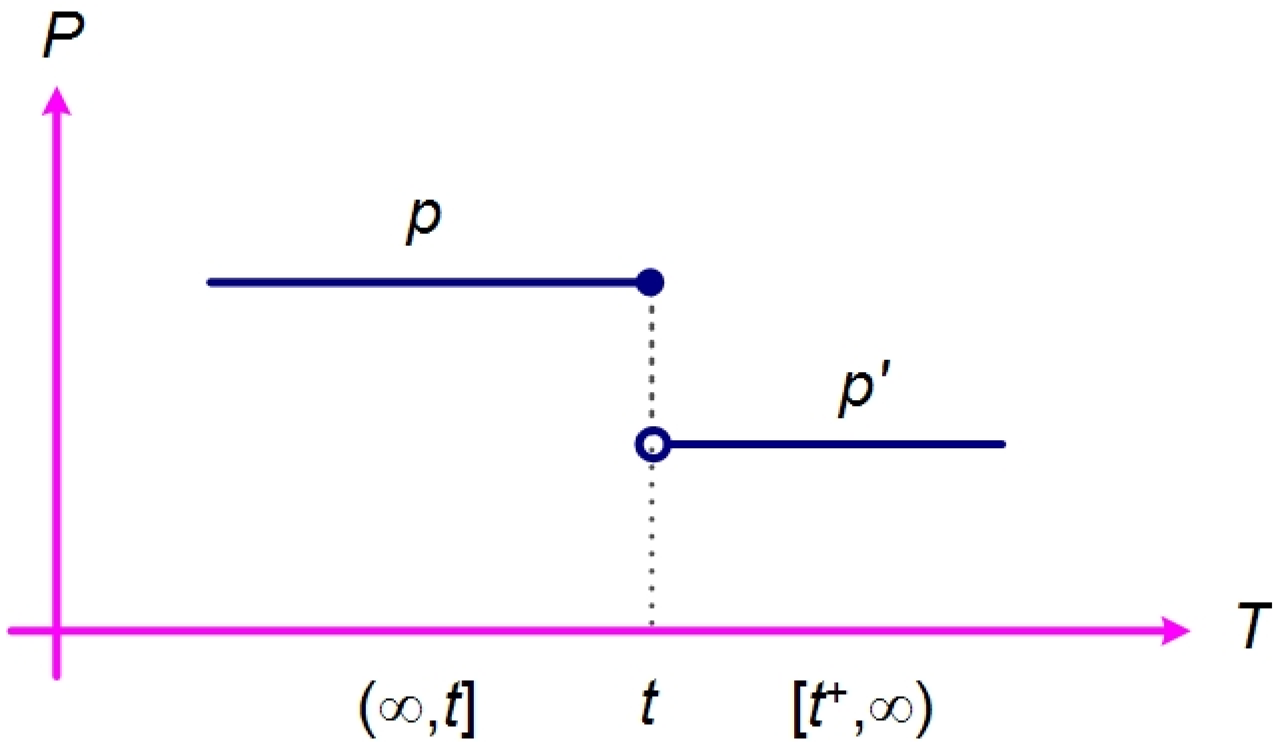
- $t \in \mathbb{R}, x \geqslant t \rightarrow [t, +\infty)$
- $t \in \mathbb{R}, x > t \rightarrow (t, +\infty)$.
- We denote the left extreme of the interval $(t, +\infty)$ by t^+.
- $(t, +\infty) \equiv [t^+, +\infty)$.
- $\varepsilon = t^+ - t$.

Non-instantaneous Propagation I

Assumption. *A HFSS component performing a transition at time t only changes its state at time t^+.*

- Gives an operational definition of causality by defining the time between the cause and its effect.
- Plays a key role for ensuring determinist simulations by allowing the existence of loops of zero-delay components.
- Enables determinism in time-warp simulations.

Non-Instantaneous Propagation II



Basic Component I

$$\Xi_B = (< s_m, s >, \mathsf{T}, \Delta, \Lambda), \text{ for } B \in \widehat{B}$$

$M_B = (X, Y, P, \rho, \omega, (p_0, e_0), \delta, \bar{\Lambda}, \lambda)$, component model

$S = \{(p,t) | p \in P, t \in \mathbb{R}\}$, component state set

$S^{\varnothing} = S \cup \{\varnothing\}$

$< s_m, s >$ with $s_m \in S^{\varnothing}$ and $s \in S^{\varnothing}$, component state

- s_m, component past state
- $s = (p,t)$, component current state
  - p, component current p-state
  - t, time of component last transition action

$\mathsf{T} : \{\varnothing\} \to \mathbb{R}$, time limit for the next transition

$\mathsf{T} = t + \nu(p)$, where $(p, t) \in s$ is the current state

$\Delta : \mathbb{R} \times X$, component transition action

$\Delta(t, x) \triangleq$

$s_m \leftarrow s$

$s \leftarrow (\delta((p, t - \tau), x), t + \varepsilon)$, assuming $s = (p, \tau)$

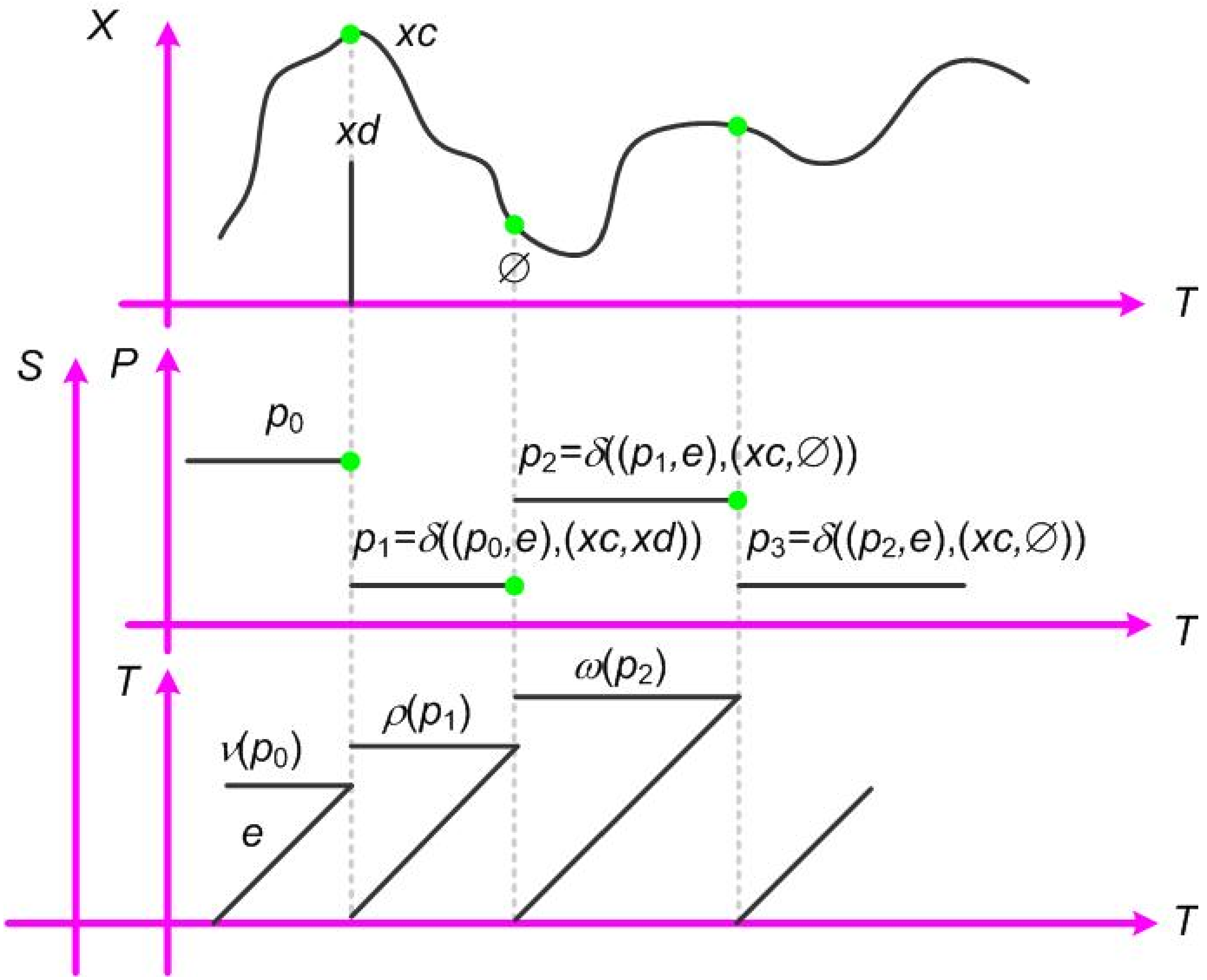
$\Lambda : \mathbb{R} \to Y^{\varnothing}$, with $Y^{\varnothing} = Y \cup \{\varnothing\}$, component output function

$$\Lambda(t) = \begin{cases} (\bar{\Lambda}(p, t - \tau), \lambda(p)) & \text{if} \quad t - \tau = \omega(p) \\ (\bar{\Lambda}(p, t - \tau), \varnothing) & \text{otherwise} \end{cases}$$

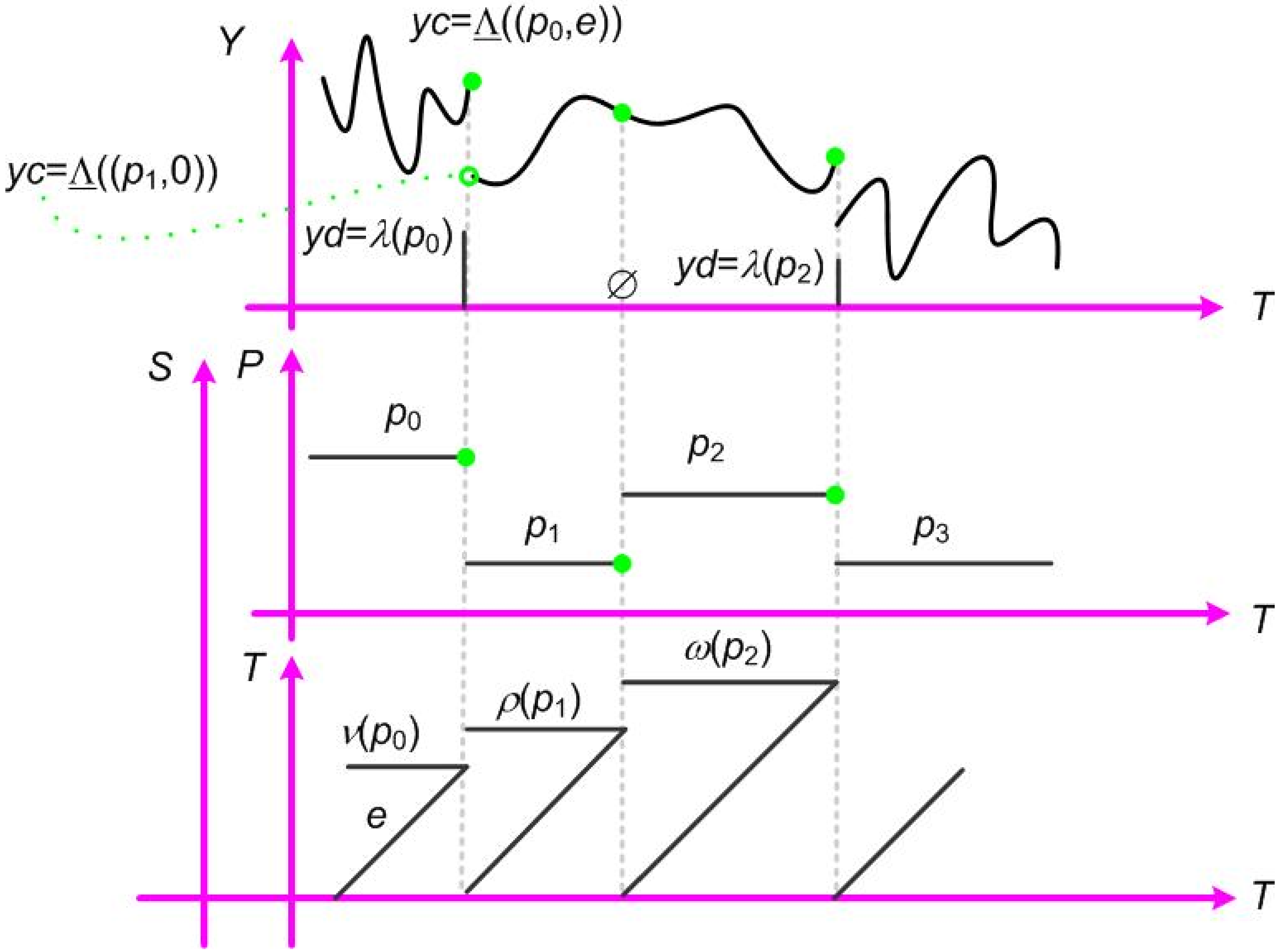
$$(p, \tau) = \begin{cases} (p_1, \tau_1) & \text{if} \quad t \geq \tau_1 \\ (p_2, \tau_2) & \text{if} \quad t < \tau_1 \end{cases}$$

with $(p_1, \tau_1) = s$ and $(p_2, \tau_2) = s_m$, restricted to $t \geq \tau_2$

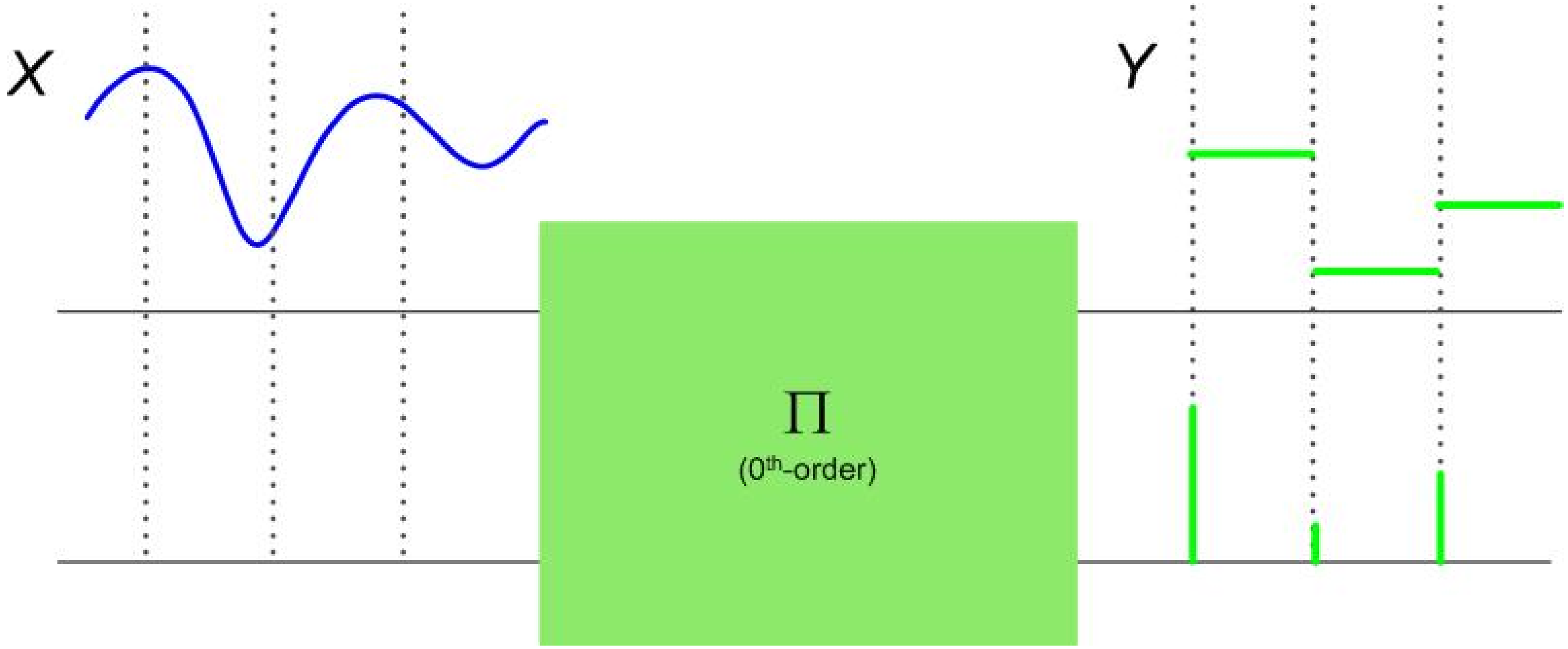
HFSS Behavior I



HFSS Behavior II



PID Controller I



PID Controller II

$$M_{\sqcap} = (X, Y, P, \rho, \omega, s_0, \delta, \bar{\Lambda}, \lambda)$$

$X = \mathbb{R} \times \{\text{stop}\}$

$Y = \mathbb{R} \times \mathbb{R}$

$P = \{\text{init,sample,out,stop}\} \times \mathbb{R}^5$

$\rho(phase, \alpha, \beta, int, der, \bar{x}_1) = \alpha$

$\omega(phase, \alpha, \beta, int, der, \bar{x}_1) = \beta$

$s_0 = ((\text{init}, 0, \infty, 0, 0, 0), 0)$

$\delta(((\text{init}, \alpha, \beta, int, der, \bar{x}_1), e), (\bar{x}, \check{x})) =$
$\quad (\text{sample}, 2, \infty, 0, 0, \bar{x})$

$$\delta(((\mathsf{sample}, \alpha, \beta, int, der, \bar{x}_1), e), (\bar{x}, \mathsf{stop})) =$$
$$((\mathsf{stop}, \infty, \infty, int + e \cdot (\bar{x} + \bar{x}_1)/2, (\bar{x} - \bar{x}_1)/e, \bar{x})$$

$$\delta(((\mathsf{stop}, \alpha, \beta, int, der, \bar{x}_1), e), (\bar{x}, \breve{x})) =$$
$$(\mathsf{stop}, \alpha, \beta, int, der, \bar{x}_1)$$

$$\delta(((\mathsf{sample}, \alpha, \beta, int, der, \bar{x}_1), e), (\bar{x}, \breve{x})) =$$
$$(\mathsf{out}, \infty, 0, int + e \cdot (\bar{x} + \bar{x}_1)/2, (\bar{x} - \bar{x}_1)/e, \bar{x})$$

$$\delta(((\mathsf{out}, \alpha, \beta, int, der, \bar{x}_1), e), (\bar{x}, \breve{x})) =$$
$$(\mathsf{sample}, 2, \infty, int, der, \bar{x}_1)$$

$$\bar{\Lambda}(((phase, \alpha, \beta, int, der, \bar{x}_1), e)) =$$
$$P \cdot \bar{x}_1 + I \cdot int - D \cdot der$$

$$\lambda((phase, \alpha, \beta, int, der, \bar{x}_1)) =$$
$$P \cdot \bar{x}_1 + I \cdot int - D \cdot der$$

Adaptive Step-Size Integrator I

$$M_{\int} = (X, Y, P, \rho, \omega, s_0, \delta, \bar{\Lambda}, \lambda)$$

$X = \mathbb{R} \times \mathbb{R}$

$Y = \mathbb{R} \times \{\text{detect}\}$

$P = \{(\alpha, \beta, x, y, par) | \alpha, \beta, x, y \in \mathbb{R}, par \in \mathbb{R}^4\}$

$\rho(\alpha, \beta, x, y, par) = \alpha$

$\omega(\alpha, \beta, x, y, par) = \beta$

$s_0 = ((0, \infty, x_0, y_0, (min, max, K, L)), 0)$

$\delta(((\alpha, \beta, x, y, (min, max, K, L)), e), (der, \breve{x})) =$
$\quad (\alpha', \beta', der, y', (min, max, K, L))$

$$\alpha' = \begin{cases} min & \text{if} \quad K \cdot |der|^{-0.5} \leqslant min \\ K \cdot |der|^{-0.5} & \text{if} \quad K \cdot |der|^{-0.5} \in (min, max) \\ max & \text{if} \quad K \cdot |der|^{-0.5} \geqslant max \end{cases}$$

$$\beta' = \begin{cases} (L - y')/der & \text{if} \quad (L - y')/der > 0 \\ \infty & \text{otherwise} \end{cases}$$

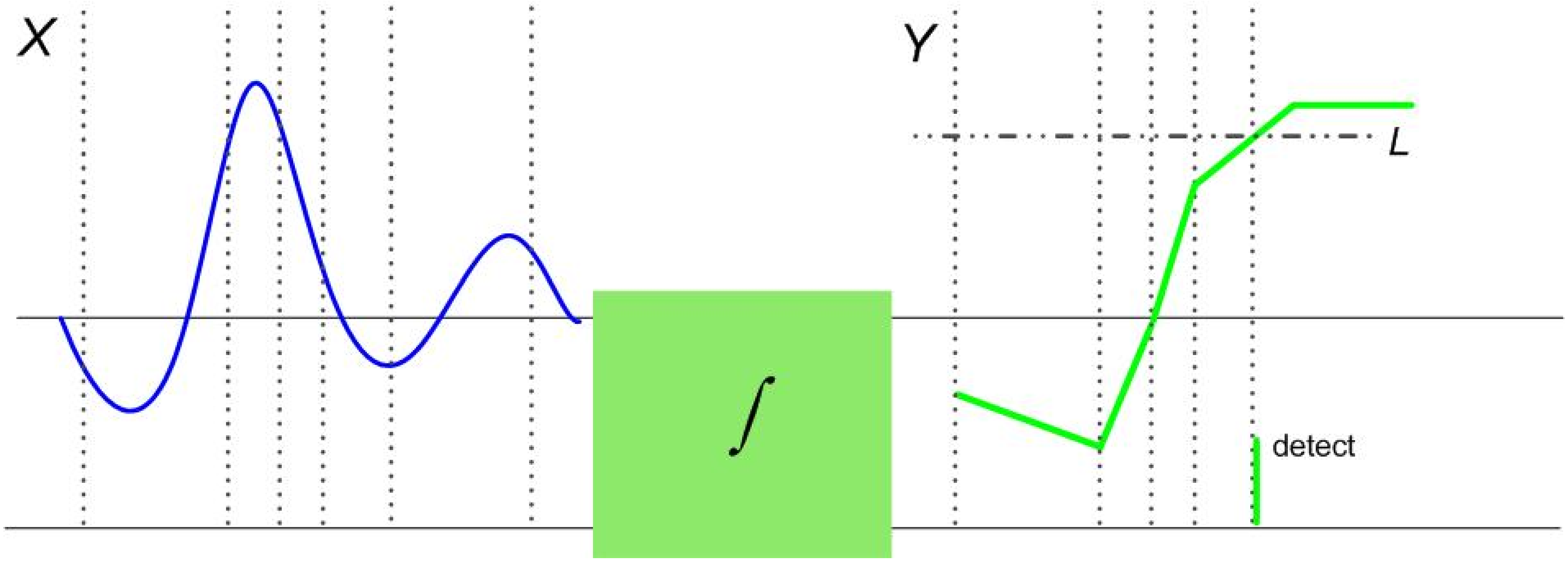
$$y' = y + e \cdot x$$

$$\bar{\Lambda}(((\alpha, \beta, x, y, par), e)) = y + e \cdot x$$

$$\lambda((\alpha, \beta, x, y, par)) = \text{detect}$$

The discrete flow value **detect** is produced whenever the continuous output flow reaches the threshold L.

Adaptive Step-Size Integrator II



Network Model

$$M_N = (X, Y, \eta), N \in \widehat{N}$$

N, network name

$X = \bar{X} \times \check{X}$, set of network input flows

$\bar{X}$, set of network continuous input flows

$\check{X}$, set of network discrete input flows

$Y = \bar{Y} \times \check{Y}$, set of network output flows

$\bar{Y}$, set of network continuous output flows

$\check{Y}$, set of network discrete output flows

$\eta \in \widehat{\eta}$, name of the dynamic structure network executive

Executive Model

$$M_\eta = (X_\eta, Y_\eta, P, \rho, \omega, s_0, \delta, \bar{\Lambda}, \lambda, \widehat{\Sigma}, \gamma), \eta \in \widehat{\eta}$$

$\widehat{\Sigma}$, set of network structures

$\gamma : P \rightarrow \widehat{\Sigma}$, structure function

The network structure $\Sigma_\alpha \in \widehat{\Sigma}$, corresponding to the p-state $p_\alpha \in P$ is given by:

$$\Sigma_\alpha = \gamma(p_\alpha) = (C_\alpha, \{I_{i,\alpha}\} \cup \{I_{\eta,\alpha}, I_{N,\alpha}\}, \{E_{i,\alpha}\} \cup \{E_{\eta,\alpha}, E_{N,\alpha}\}, F_{i,\alpha} \cup \{F_{\eta,\alpha}, F_{N,\alpha}$$

C_α, set of names associated with the executive state p_α

for all $i \in C_\alpha \cup \{\eta\}$

$I_{i,\alpha}$, sequence of **asynchronous** influencers of i

$E_{i,\alpha}$, set of the **synchronous** influencees of i

$F_{i,\alpha}$, input function of i

$I_{N,\alpha}$, sequence of network influencers

$E_{N,\alpha}$, set of synchronous network influencees

$F_{N,\alpha}$, network output function

For all $i \in C_\alpha$

$M_i = (X_i, Y_i, P_i, \rho_i, \omega_i, s_{0,i}, \delta_i, \bar{\Lambda}_i, \lambda_i)$ if $i \in \widehat{B}$

$M_i = (X_i, Y_i, \eta_i)$ if $i \in \widehat{N}$

Constraints

for every $p_\alpha \in P_\alpha$

$N \notin C_\alpha, N \notin I_{N,\alpha}, \eta \notin C_\alpha$

$N \notin E_{i,\alpha}$ for all $i \in C_\alpha \cup \{\eta, N\}$

$F_{N,\alpha} : \times_{k \in I_{N,\alpha}} Y_k \to Y^\varnothing$

$F_{i,\alpha} : \times_{k \in I_{i,\alpha}} V_k \to X_i^\varnothing$

$$V_k = \begin{cases} Y_k^\varnothing & \text{if} \quad k \neq N \\ X^\varnothing & \text{if} \quad k = N \end{cases}$$

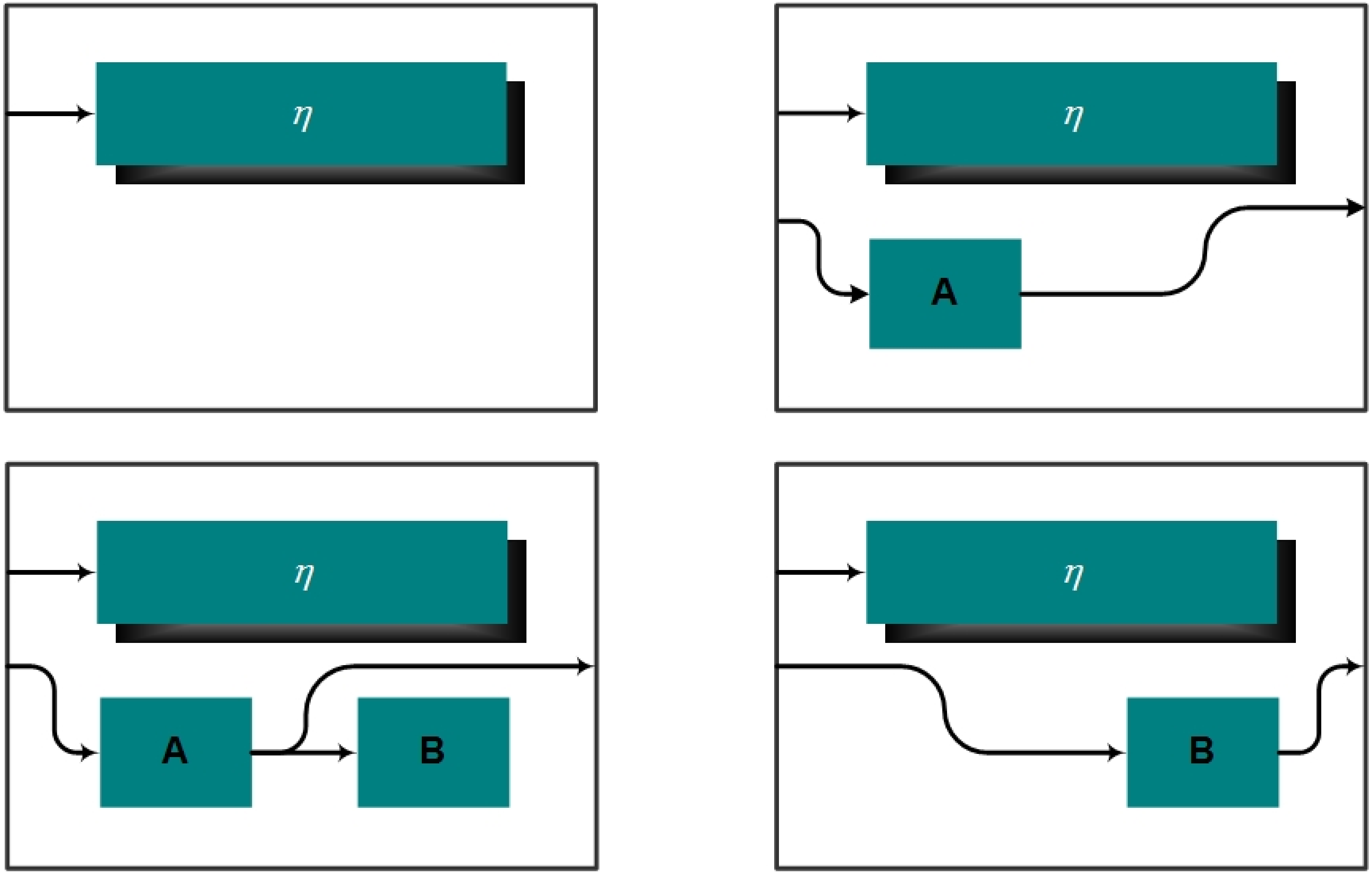
$F_{N,\alpha}((\bar{v}_{k_1}, \varnothing), (\bar{v}_{k_2}, \varnothing), \ldots) = (\bar{y}_N, \varnothing)$

$F_{i,\alpha}((\bar{v}_{k_1}, \varnothing), (\bar{v}_{k_2}, \varnothing), \ldots) = (\bar{x}_i, \varnothing)$

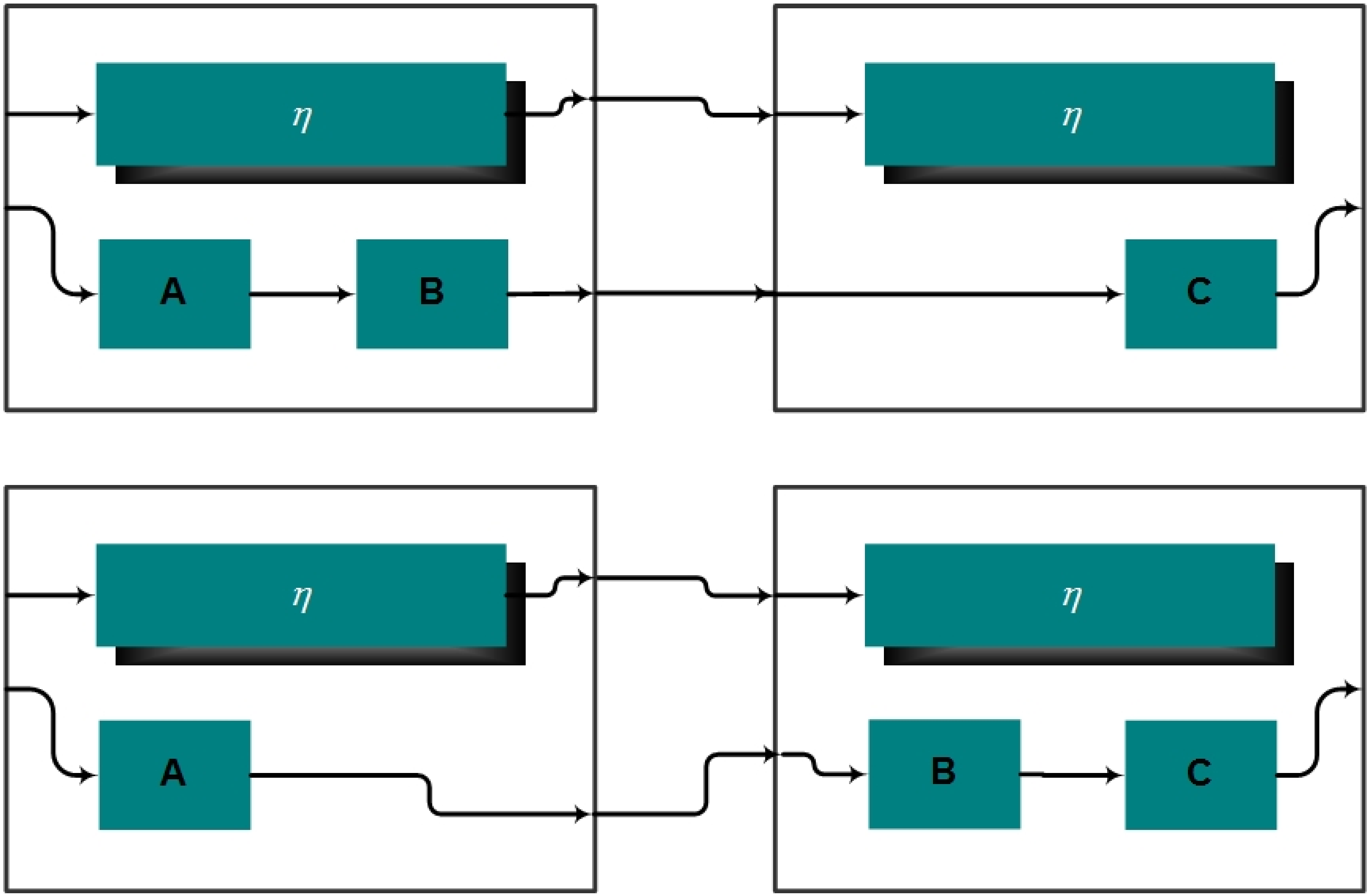
Dynamic Structure Networks I

- HFSS networks offer a general framework for defining structural changes:
  - components can be added/removed;
  - input functions, influencers and influencees can be modified during the lifetime of a component.

Dynamic Structure Networks II



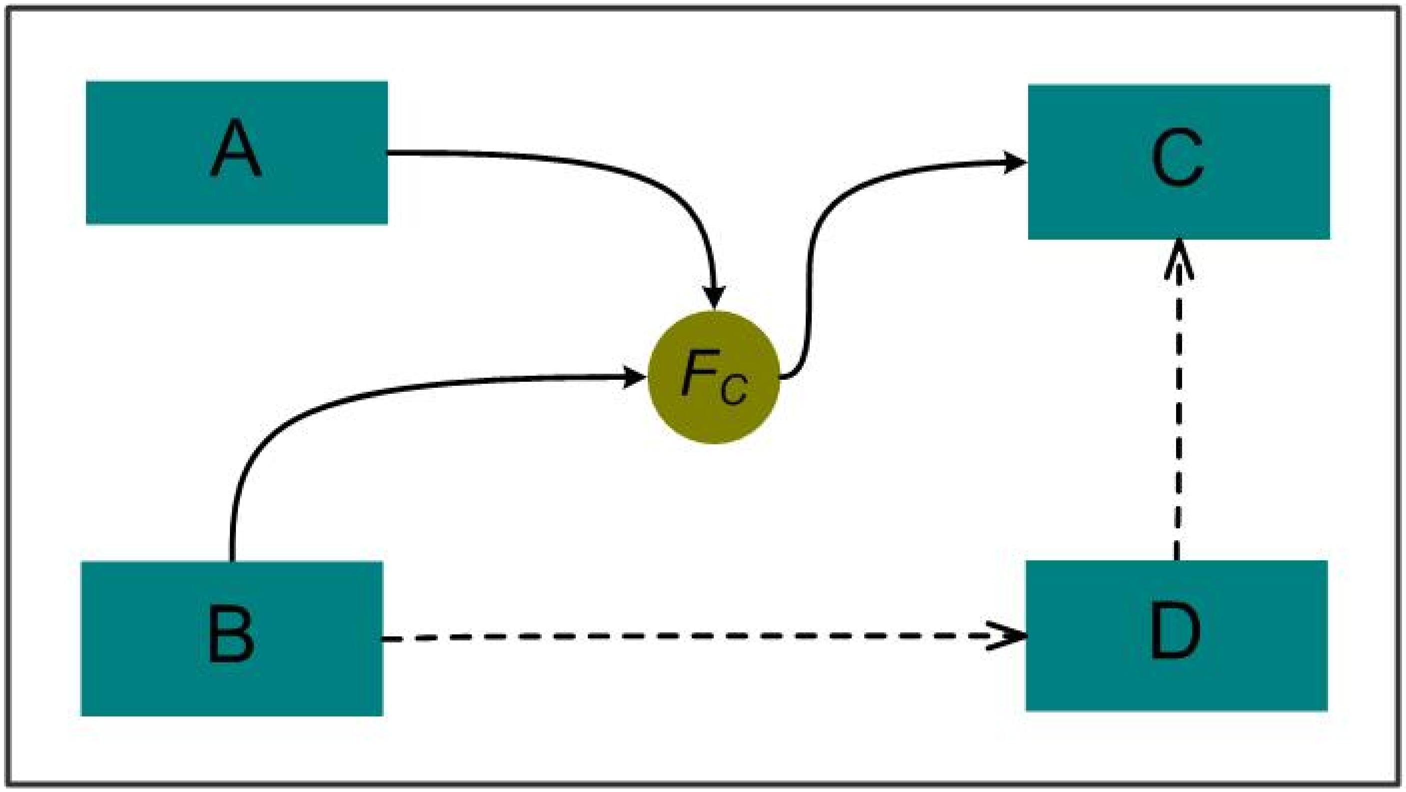
Mobility



Transitions I

- A transition is defined as the application of the transition function to a component.
- Transitions can be caused by three conditions:
  - a non-null discrete flow is presented at component input;
  - the component reaches the maximum time allowed in the current p-state;
  - a component belongs to the set of synchronous influencees of another component that is undergoing a transition;
- We use the term transition instead of event since the latter is ambiguous having usually different meanings.

Transitions II



$I_C = \{A, B\}$
$E_D = \{C\}$
$E_B = \{D\}$

Executive Component

$$\Xi_\eta = (< s_m, s >, \mathsf{T}, \Delta, \Lambda, \Gamma), \eta \in \widehat{\eta}$$

$M_\eta = (X_\eta, Y_\eta, P, \rho, \omega, s_0, \delta, \bar{\Lambda}, \lambda, \widehat{\Sigma}, \gamma)$, model of the executive

$\Gamma : \mathbb{R} \to \widehat{\Sigma}$, executive component structure function

$$\Gamma(\tau) = \gamma(p)$$

$$(p, t) = \begin{cases} (p_1, t_1) = s & \text{if} \quad \tau \geq t_1 \\ (p_2, t_2) = s_m & \text{if} \quad \tau < t_1 \end{cases}$$

restricted to $\tau \geq t_2$

Network Component

$$\Xi_N = (\mathsf{T}, \Delta, \Lambda), N \in \widehat{N}$$

with $M_N = (X, Y, \eta)$,$M_\eta = (X_\eta, Y_\eta, P, \rho, \omega, s_0, \delta, \bar{\Lambda}, \lambda, \widehat{\Sigma}, \gamma)$

$\mathsf{T} : \{\varnothing\} \to \mathbb{R}$, maximum time allowed in the current state

$\mathsf{T} = \min\{\Xi_k.\mathsf{T} | k \in C \cup \{\eta\}\}$, with $(C, \{I_i\}, \{E_i\}, \{F_i\}) = \Xi_\eta.\Gamma(t)$

$\Delta : \mathbb{R} \times X$, component transition action

$\Delta(t, x) \triangleq$

$H \leftarrow \{k | k \in C \cup \{\eta\}, \check{x}_k \neq \varnothing \vee t = \Xi_k.\mathsf{T}\}$

$L \leftarrow$transitive-closure$(H \cup E_N)$

for all $k \in L$ **do** $\Xi_k.\Delta(t, x_k)$

for all $c \in C' \setminus C$ **do** create-component$(c, t+\varepsilon)$

for all $c \in C \setminus C'$ **do** destroy-component(c)

$x_k = (\bar{x}_k, \check{x}_k) = F_k(\times_{i \in I_k} v_i)$

$$v_i = \begin{cases} \Lambda_i(t) & \text{if} \quad i \neq N \\ x & \text{if} \quad i = N \end{cases}$$

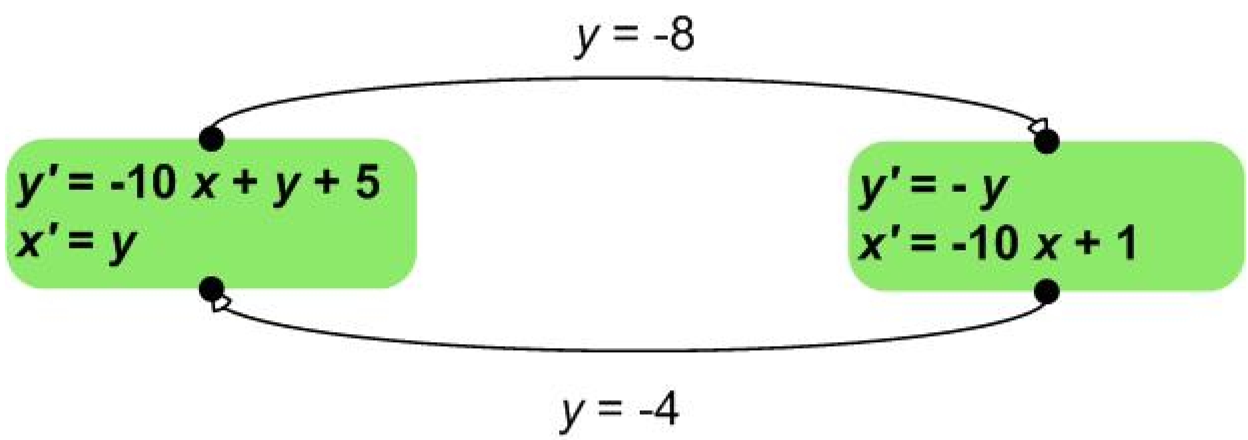
$(C, \{I_i\}, \{E_i\}, \{F_i\}) = \Xi_\eta.\Gamma(t)$ and

$(C', \{I'_i\}, \{E'_i\}, \{F'_i\}) = \Xi_\eta.\Gamma(t+\varepsilon)$

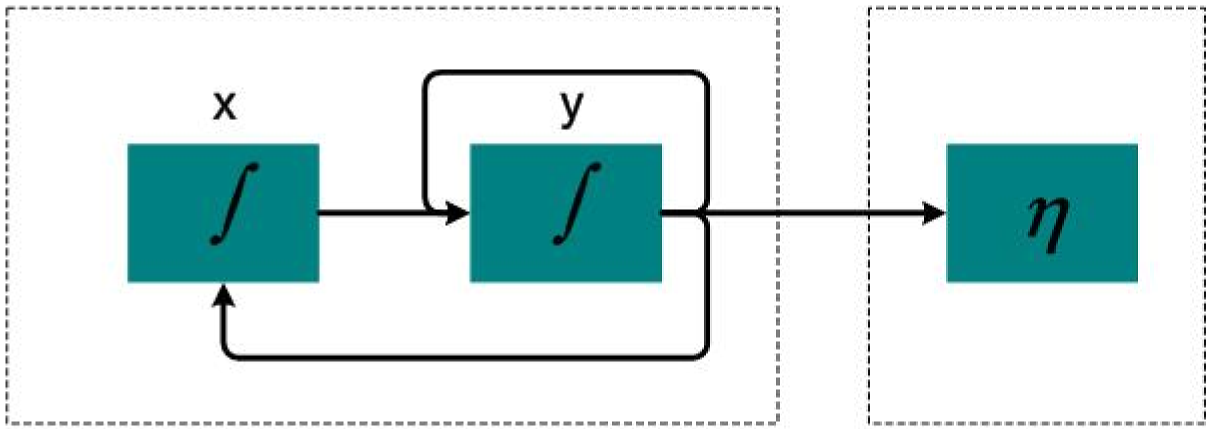
$\Lambda : \mathbb{R} \to Y$, network component output function

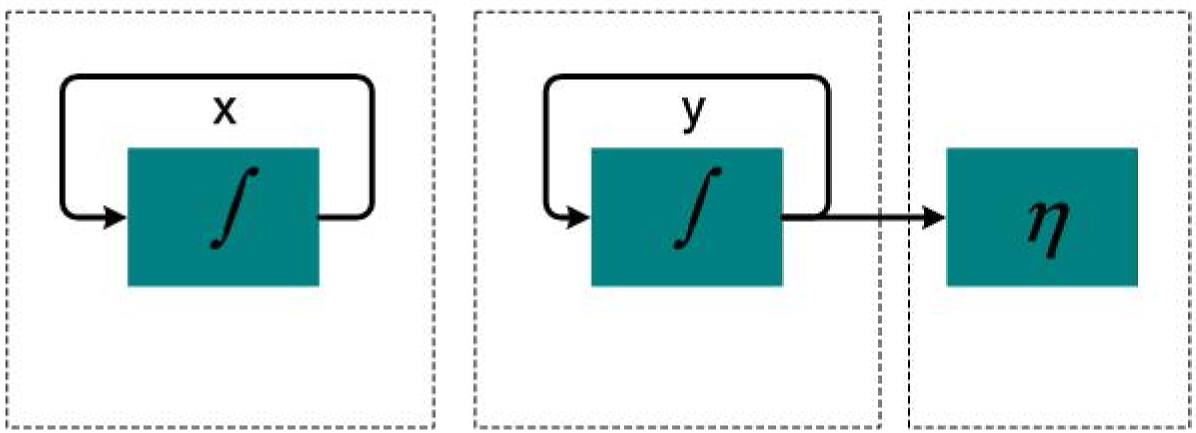
$\Lambda(t) = F_N(\times_{i \in I_N} \Lambda_i(t))$, with $(C, \{I_i\}, \{E_i\}, \{F_i\}) = \Xi_\eta.\Gamma(t)$

Switching System I

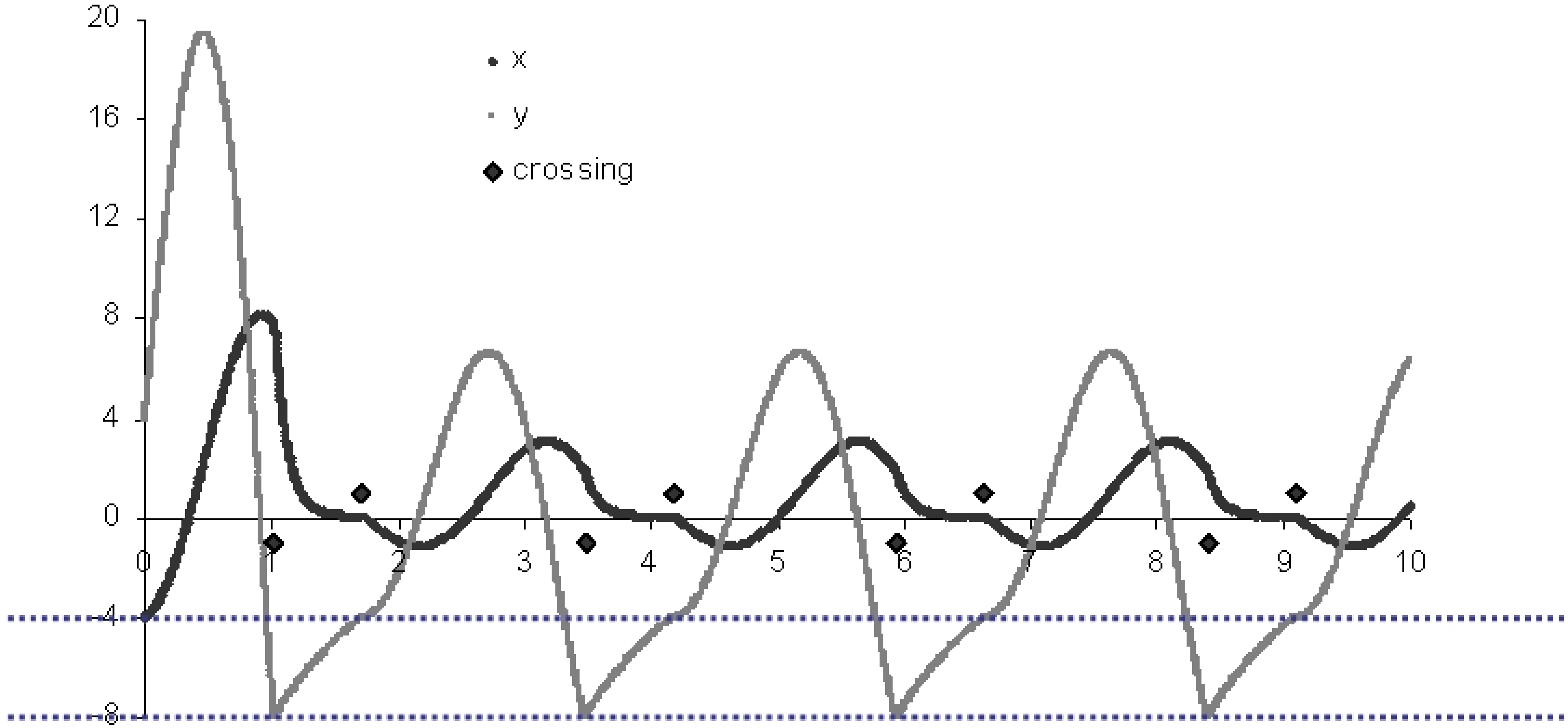


Switching System II

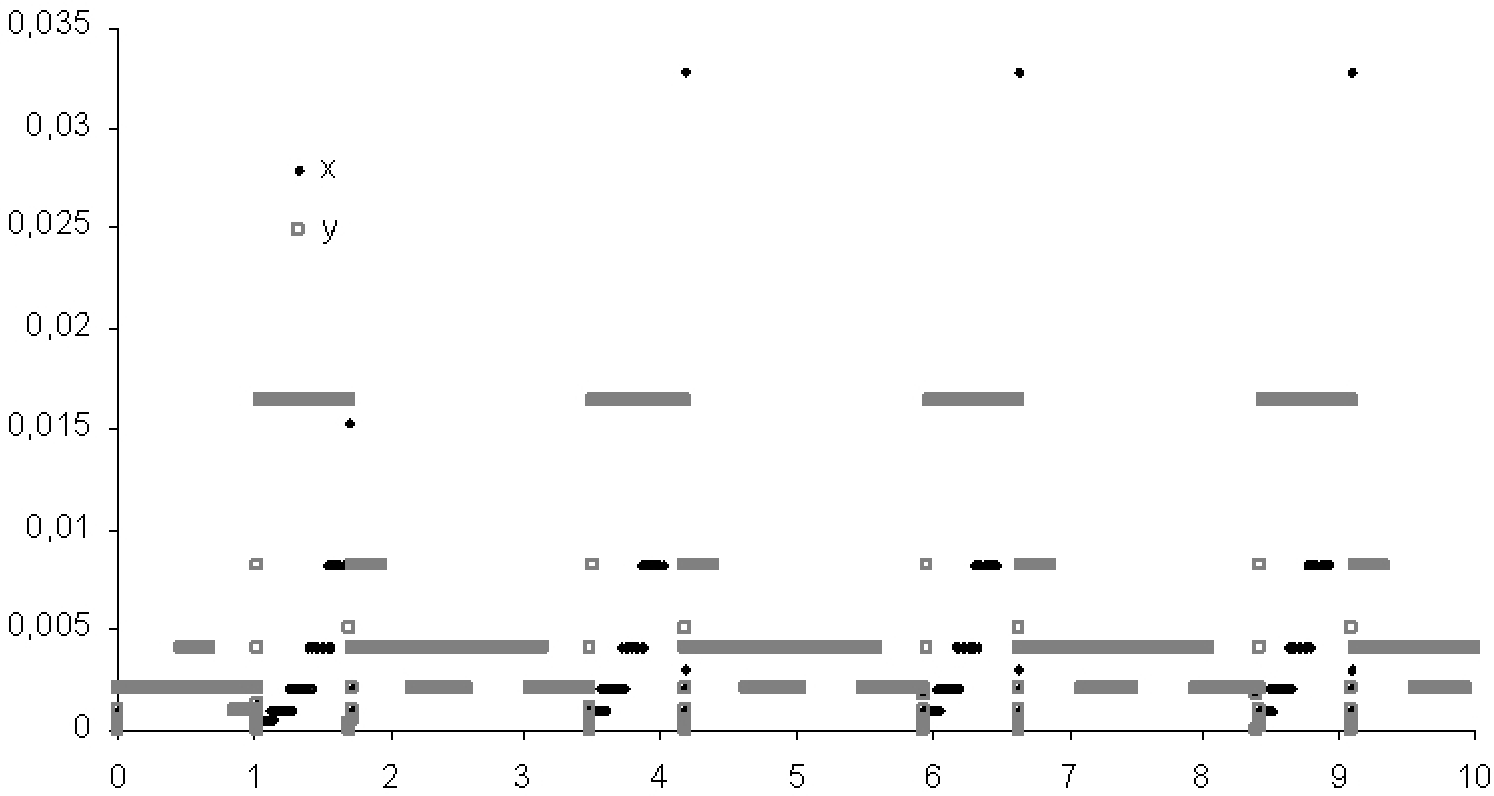




Switching System III



Step Sizes



Number of transitions: **#x** = 3589, **#y** = 2670.

(A)Synchronous Systems

- Partitions can be easily established in many systems
- In highway systems traffic is organized around platoons
  - in each platoon vehicles have a strong interaction and decisions made by a car of may influence the decisions of the other drivers (**synchronous**)
  - interactions may not exist between platoons (**asynchronous**)
- Many real systems, specially, when regarded at a global scale, like aircraft systems, have clusters of locally interacting entities with no relationship between clusters
- To effectively exploit the loose coupling it is necessary the ability to dynamically change the interaction relationships between integrators

Currently Supported Models

- Explicit adaptive-step Adams solvers
- Adaptive sampling PID controllers
- Adaptive sampling alpha-beta filters

Conclusions

- HFSS semantics provide an algorithm description of HFSS components enabling their unambiguous simulation.
- Semantics definition enables component interoperability.
- HFSS provides a base for multi-paradigm modeling.
- As a future work we plan to extend HFSS to support non-causal models.

For More Information

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