Issues in Design Space Exploration Using Randomized Search Algorithms

Eckart Zitzler

Computer Engineering (TIK), ETH Zurich, Switzerland

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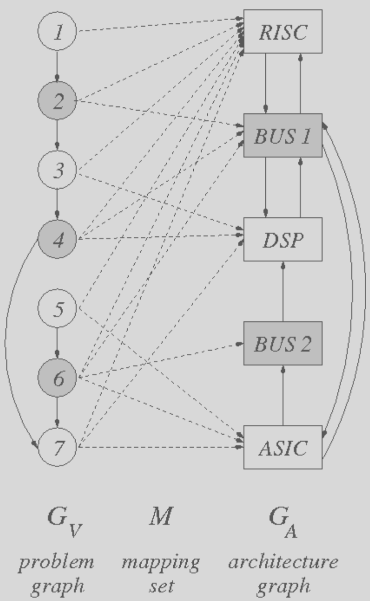
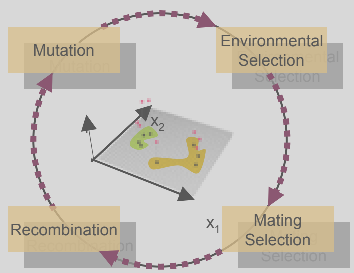
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

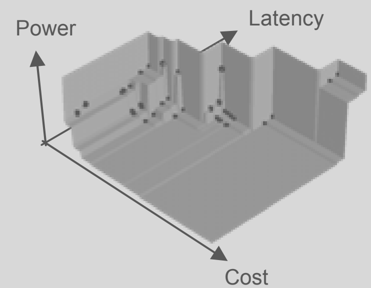
TIK
Computer Engineering
and Networks Laboratory

Design Space Exploration: Setting

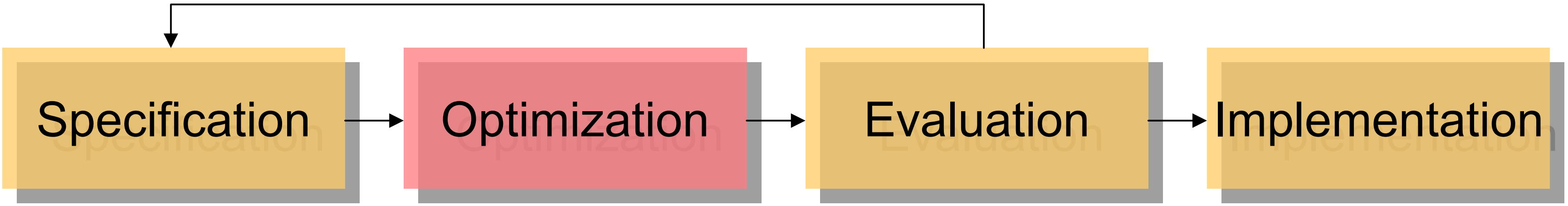
Specification → Optimization → Evaluation → Implementation

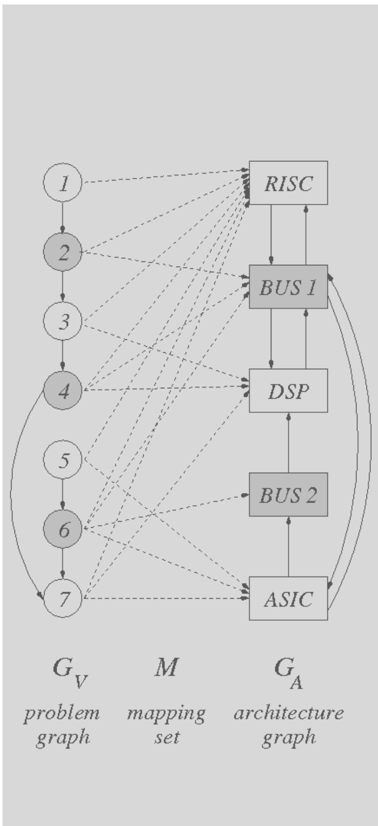


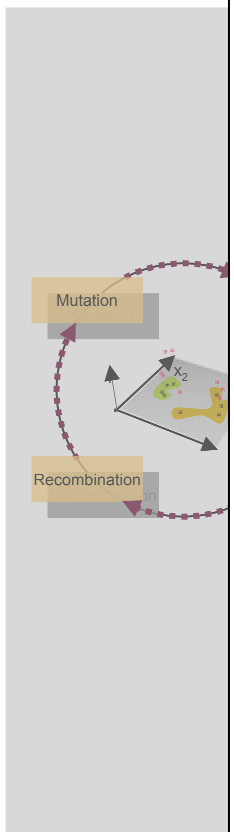


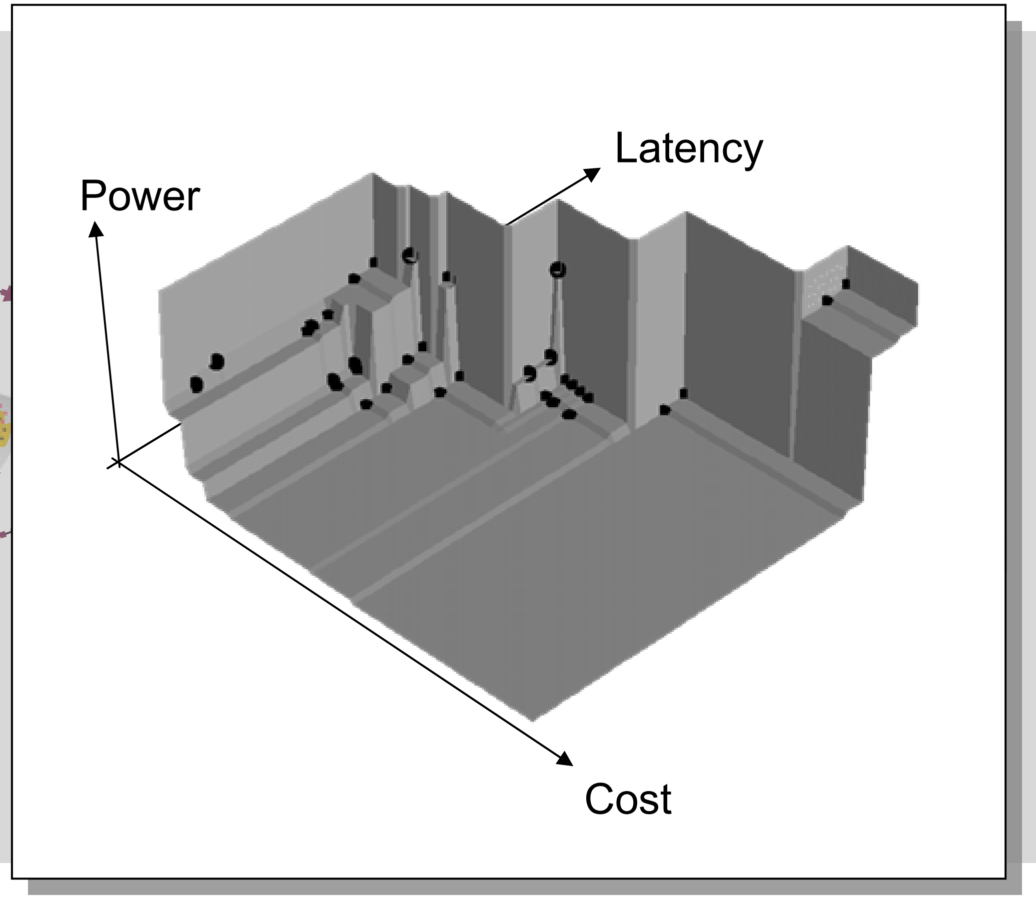


Design Space Exploration: Setting



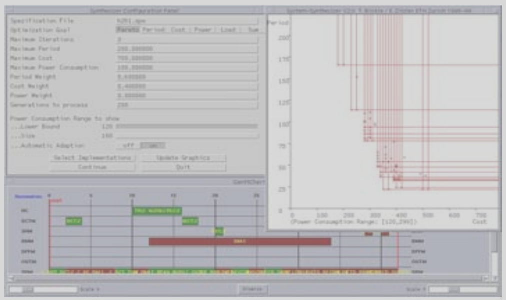






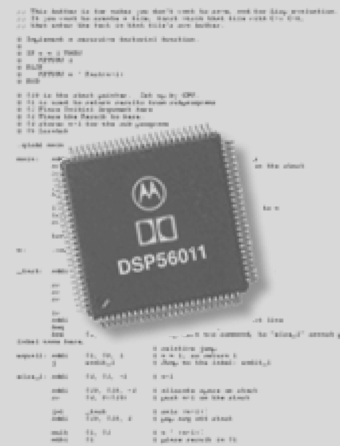
Example Applications

Architecture Synthesis for Embedded Systems



Network Processor Design

Software Synthesis for DSPs



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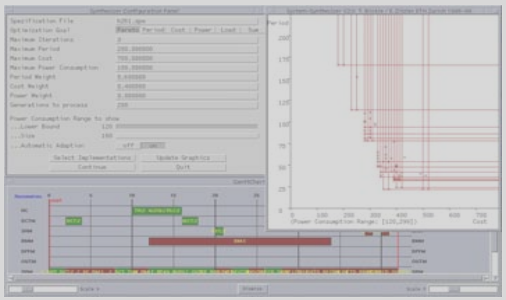
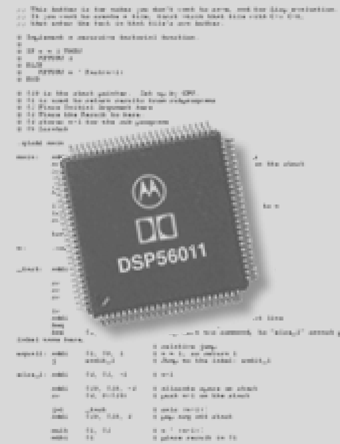
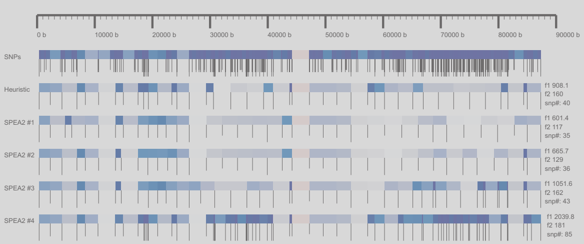


Network Processor Design

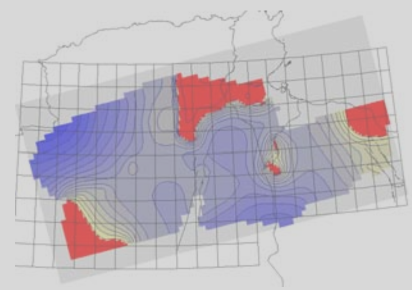
Software Synthesis for DSPs



Experiment Design for Genotyping

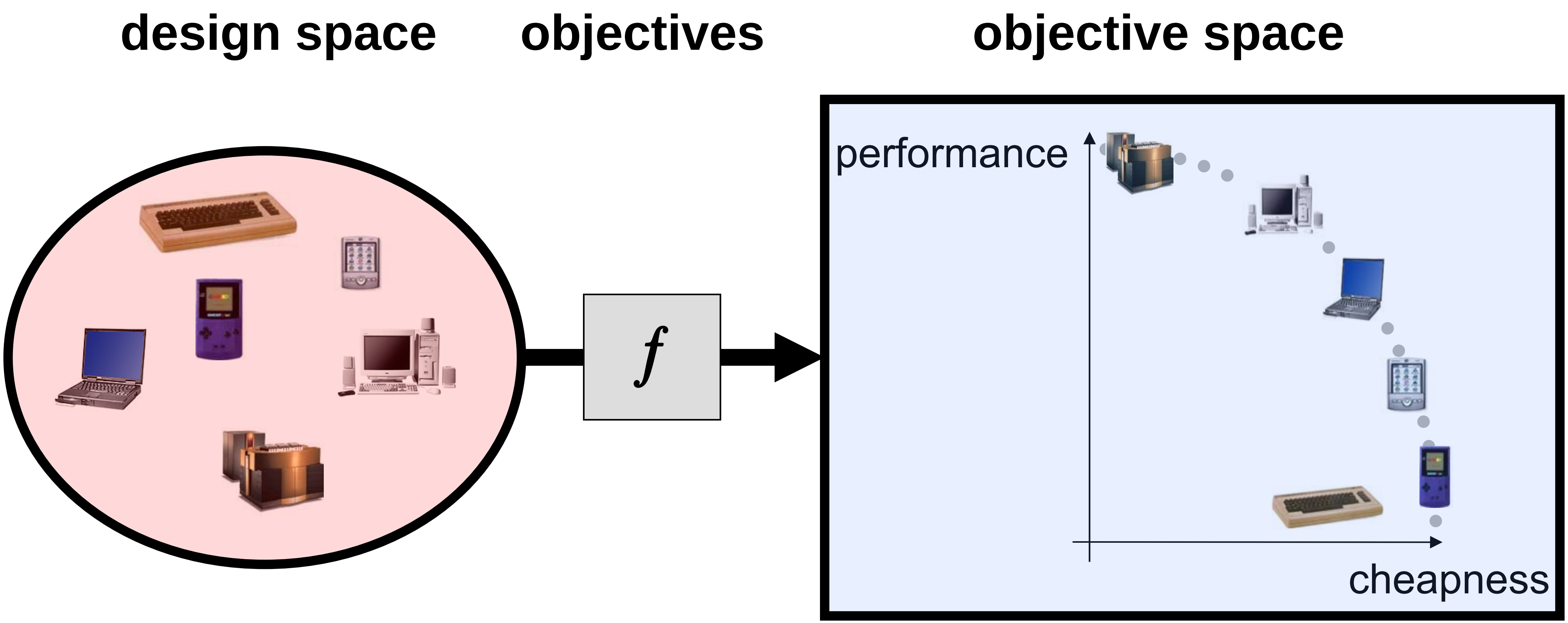


Optimal Usage of Non-renewable Groundwater Resources in the Sahara



Tobias Siegfried
Prof. Dr. Wolfgang Kinzelbach
Institut für Hydromechanik und Wasserwirtschaft (IHW)

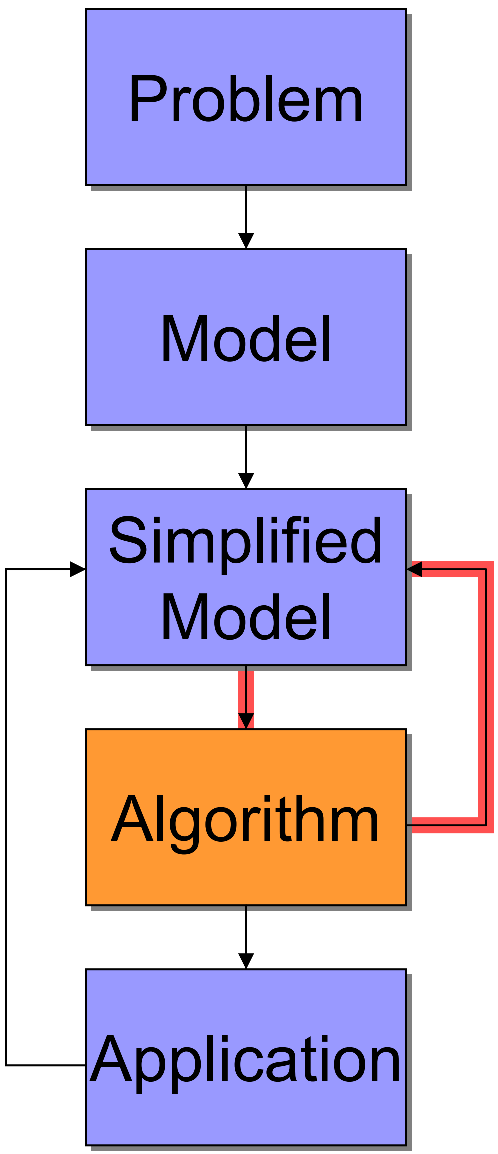
Global Optimization



Challenges:

- complexity of the solution space
- conflicting optimization criteria
- uncertain information
- computationally expensive objective functions

Bottom-Up Approach



Focus: a good algorithm…

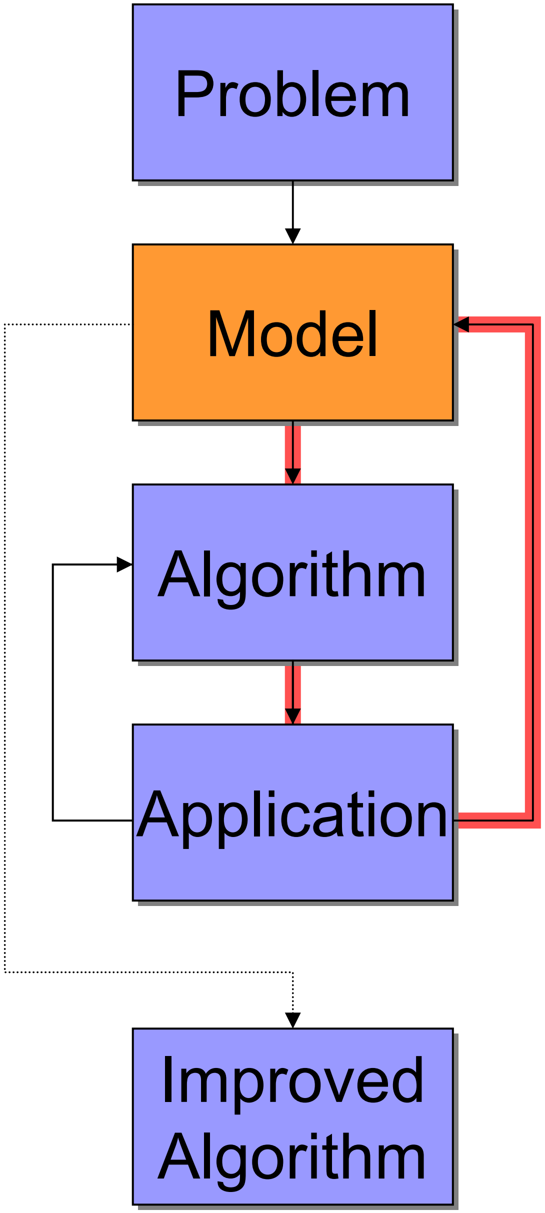
❶ Design of a problem-specific algorithm
using known concepts:

- Dynamic programming
- Branch and bound
- Divide and conquer

❷ Usage of a general search algorithm with model constraints:

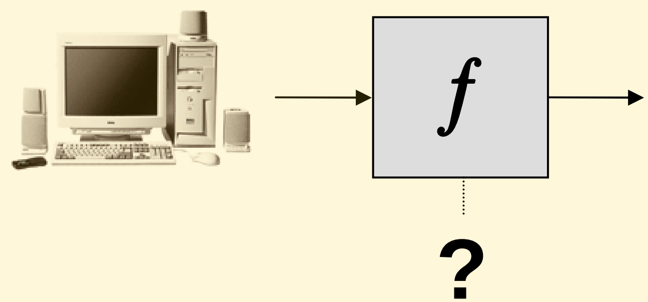
- Integer linear programming
- Gradient methods

Top-Down Approach



Focus: a good model…

Usage of black-box optimization methods to approximate the optima:



Performance: 1000
Cheapness: 50

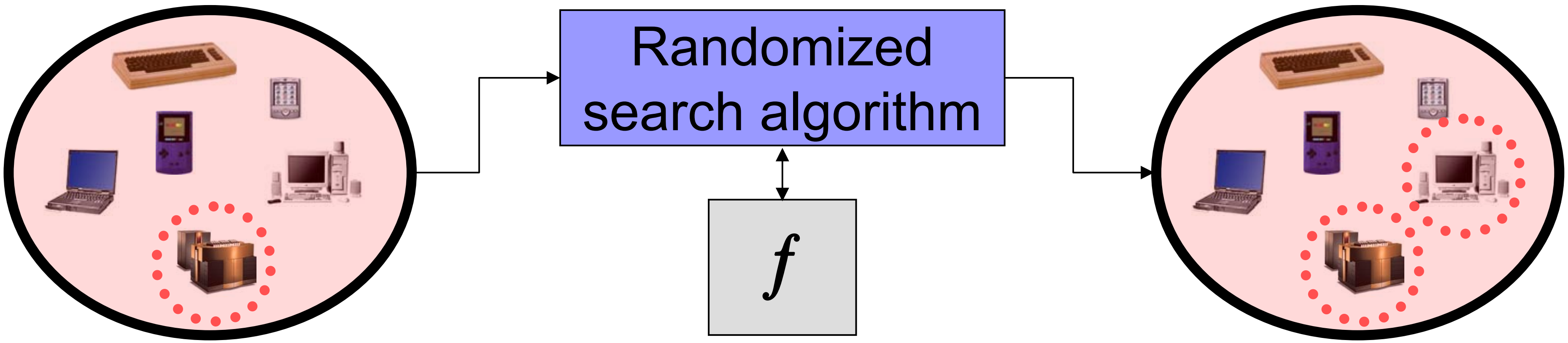
nonlinear function
simulation
experiment

- Evolutionary algorithms
- Simulated annealing

Randomized Search Algorithms (RSAs)

Idea: find good solutions without investigating all solutions

Assumption: better solutions can be found in the neighborhood of good solutions



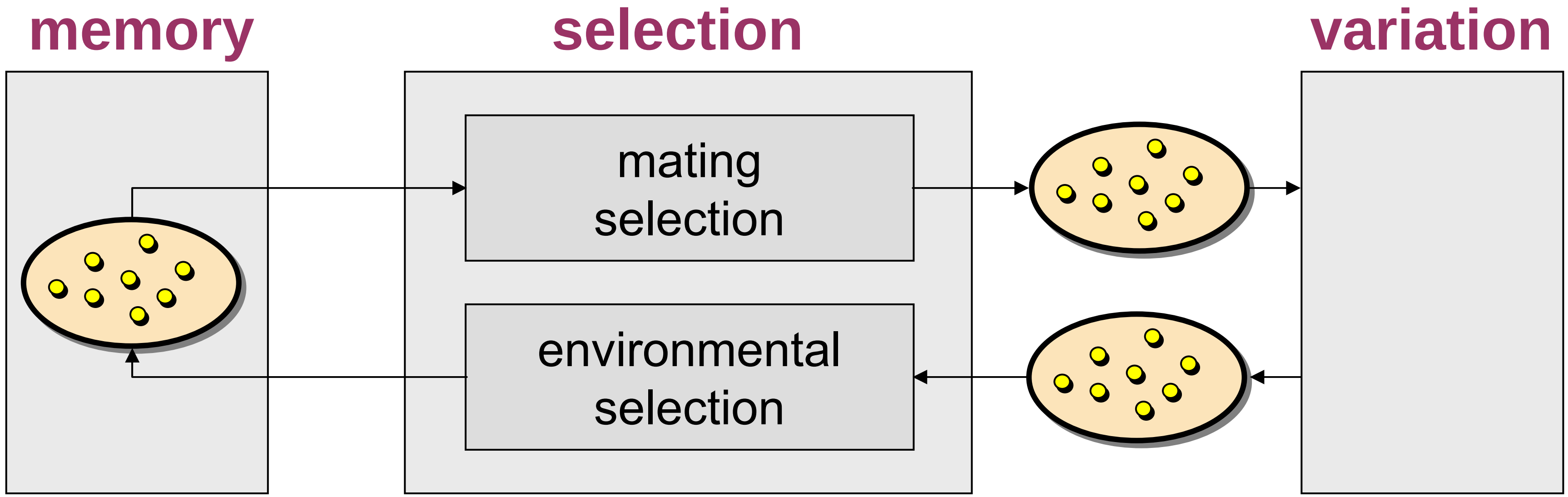
t = 1:

(randomly) choose a solution x_1 to start with

t → t+1:

(randomly) choose a solution x_{t+1} using solutions $x_1, \ldots, x_t$

Types of Randomized Search Algorithms



EA	≥ 1	both	≥ 1	N : M
evolutionary algorithm			≥ 1	randomized
TS	1	no mating selection	1	1 : M
tabu search			≥ 1	deterministic
SA	1	no mating selection	1	1 : M
simulated annealing			≥ 1	randomized
ACO	1	neither	1	1 : 1
ant colony optimization			1	randomized

Key Issues in Design Space Exploration Using RSA

How to...

- handle multiple objectives
- account for uncertainty
- optimize for robustness
- incorporate user preferences
- achieve diversity in the parameter space
- reduce the design space complexity
- statistically assess the quality of the outcome
- simplify implementation and maximize reusability
- deal with limited memory resources
- postprocessing of tradeoff surface

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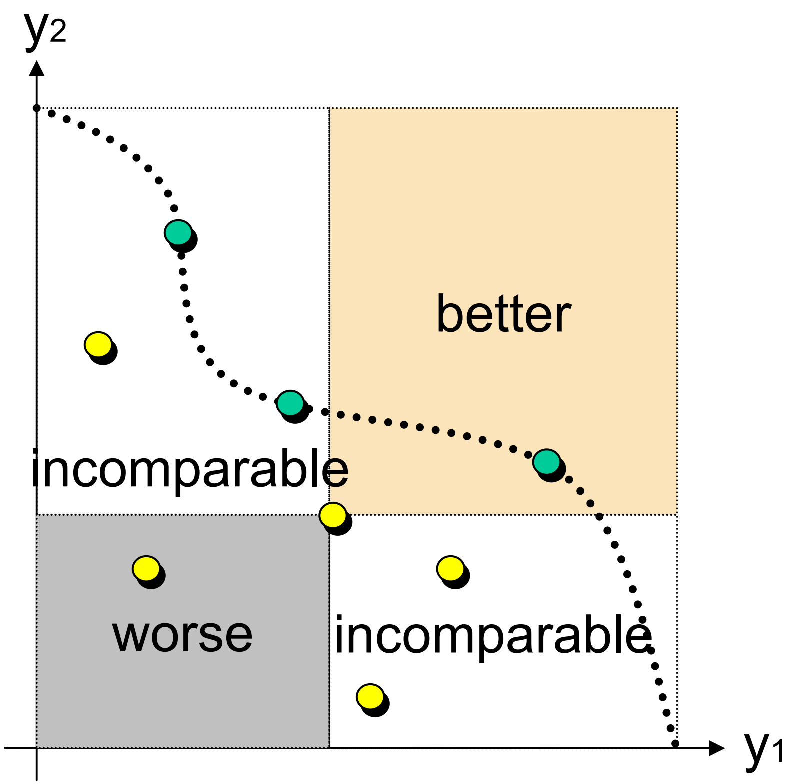
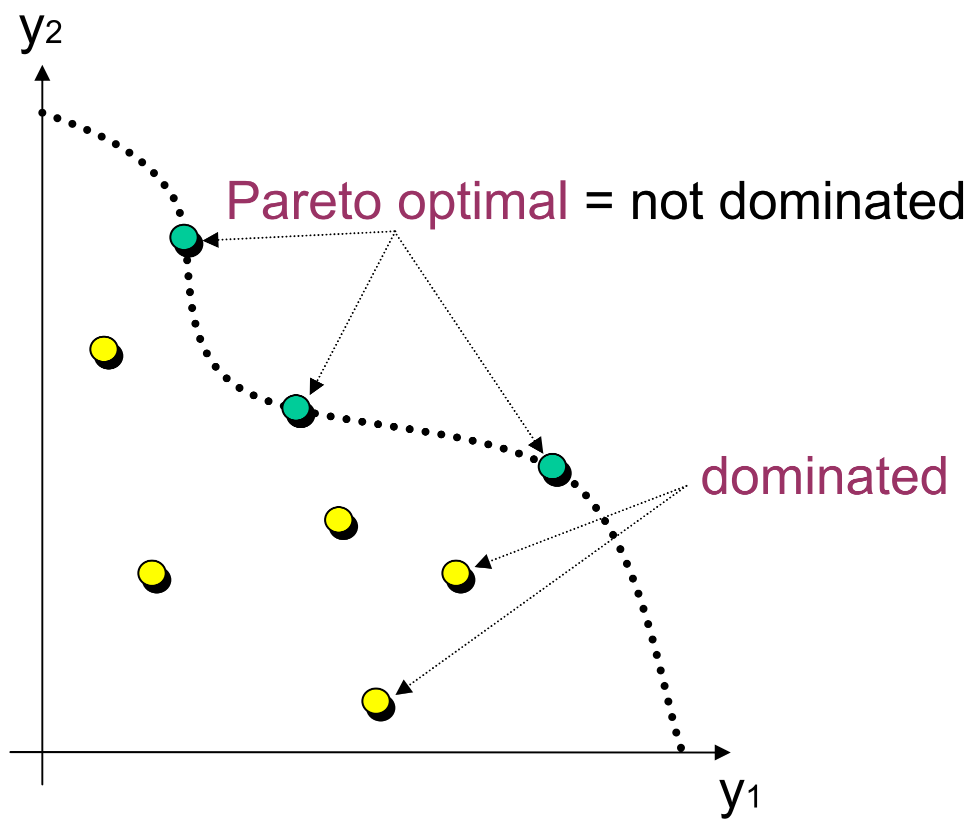
The Multiobjective Scenario

Maximize $(y_1, y_2, \ldots, y_n) = (f_1(x_1, x_2, \ldots, x_k), \ldots, f_n(x_1, x_2, \ldots, x_k))$





Problem is underdetermined...

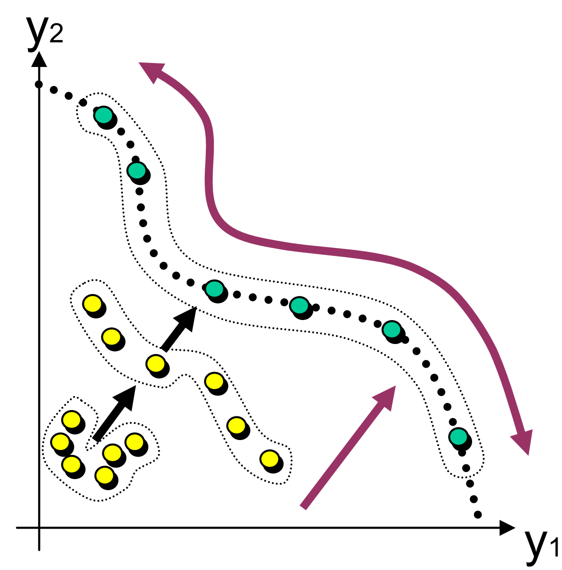
What Is the Optimization Goal?

- Find all Pareto-optimal solutions?
  - Impossible in continuous search spaces
  - How should the decision maker handle 10000 solutions?
- Find a representative subset of the Pareto set?
  - Many problems are NP-hard
  - What does representative actually mean?
- Find a good approximation of the Pareto set?
  - What is a good approximation?
  - How to formalize intuitive understanding:

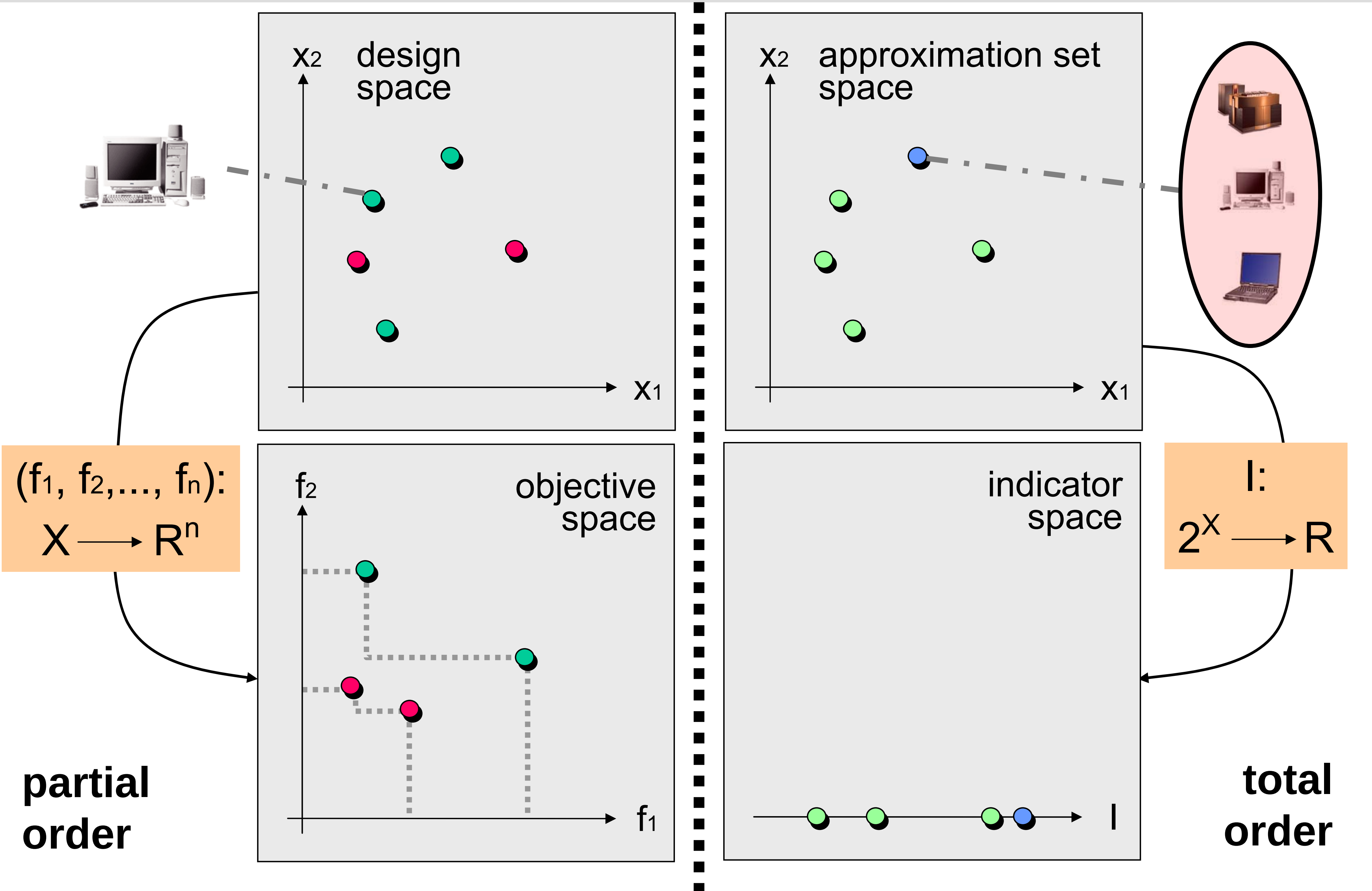
    ❶ close to the Pareto front

    ❷ well distributed

    [Deb:01]



The Actual Problem

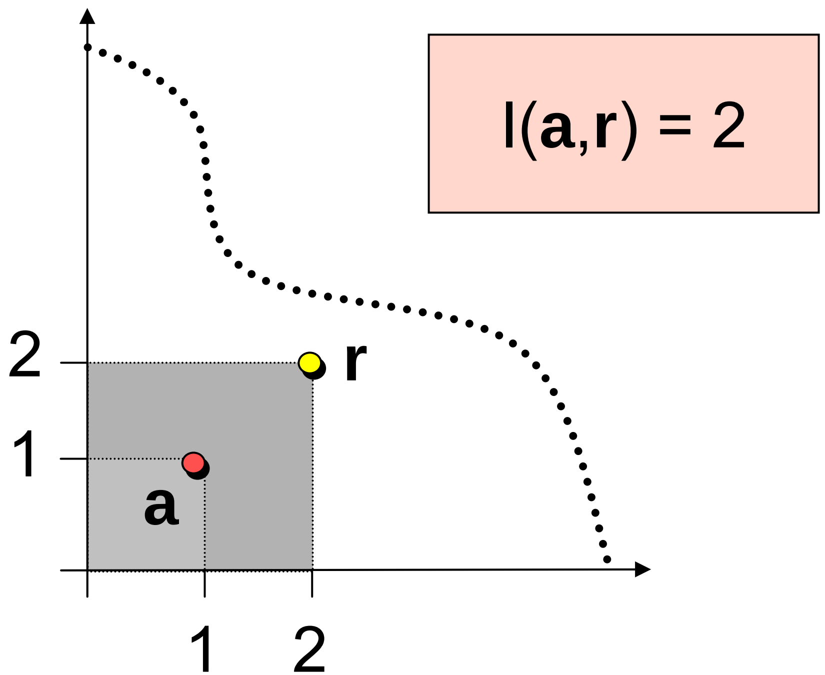


The ε-Quality Indicator

Two solutions:

$I(\mathbf{a},\mathbf{r}) =$

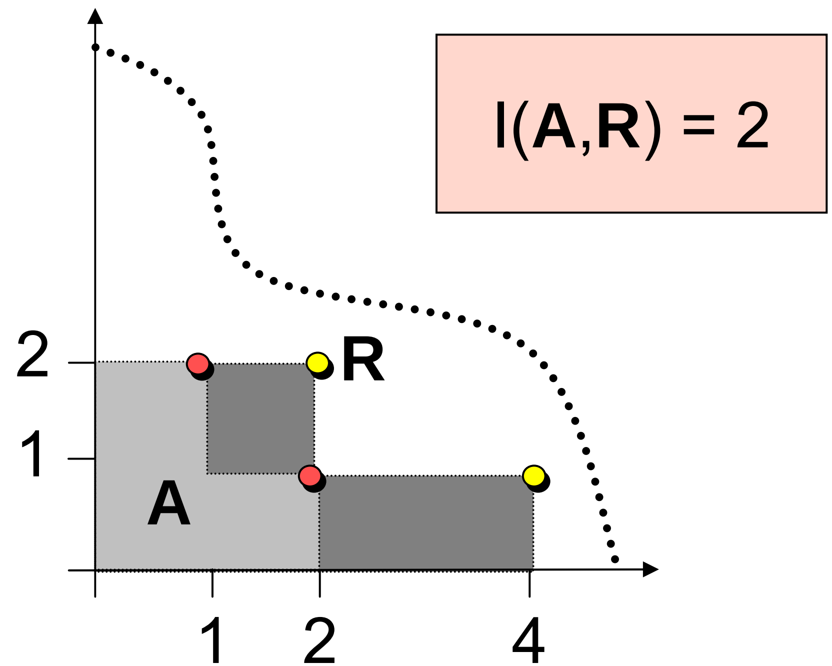
$\max_{1 \le i \le n} \min_{\varepsilon} \; \varepsilon \cdot f_i(\mathbf{a}) \ge f_i(\mathbf{r})$



Two approximations:

$I(\mathbf{A},\mathbf{R}) =$

$\max_{\mathbf{r} \in \mathbf{R}} \min_{\mathbf{a} \in \mathbf{A}} \; I(\mathbf{a},\mathbf{r})$

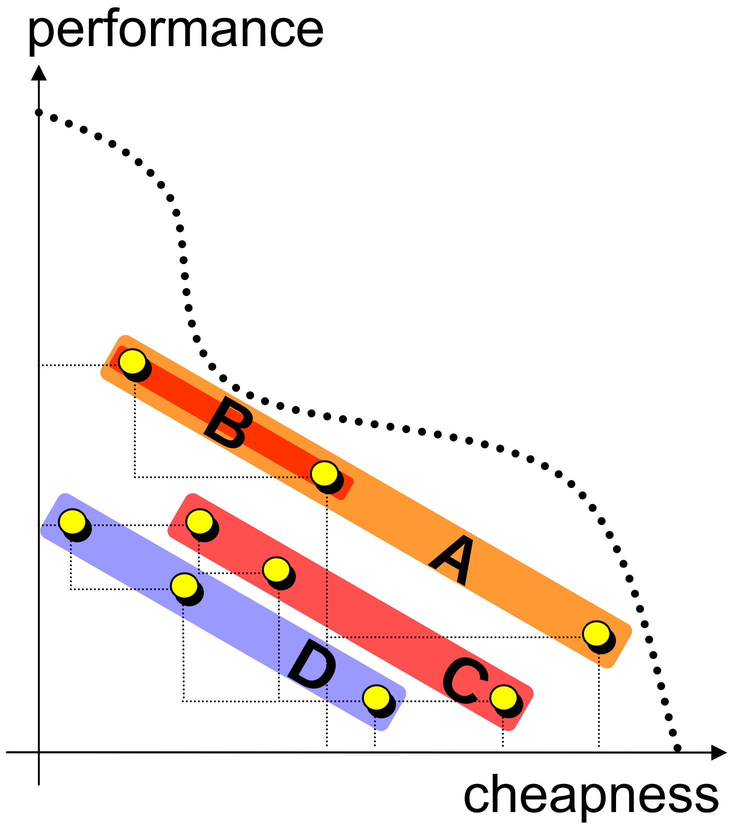


$I(\mathbf{A},\mathbf{R}) =$ minimum factor by which **A** needs to be "improved" such the (fixed) reference set **R** is entirely covered

[Zitzler et al. : 03]

Pareto Compliance

Indicators should be **Pareto compliant** =
the order induced by I should be an extension of the order induced by $f_1, \ldots, f_n$



Pareto dominance:

A better than B

B better than C

C better than D

Indicator ranking:

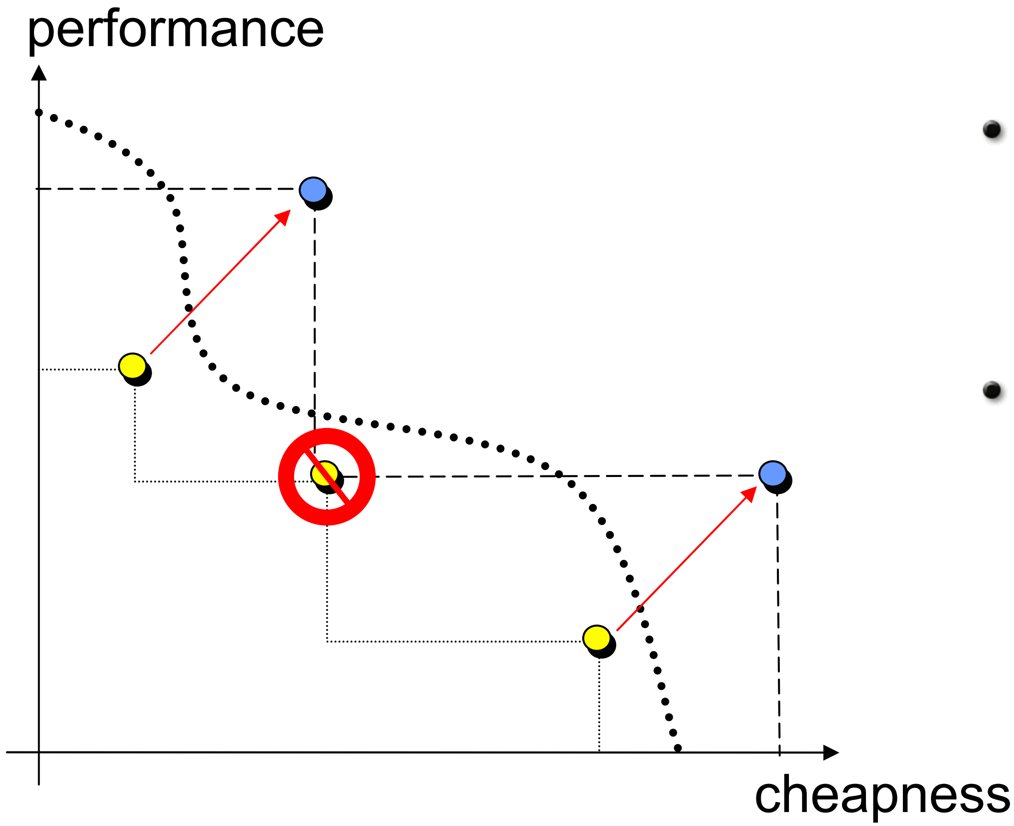
$I(A) \geq I(B) \geq I(C) \geq I(D)$

Indicator-Based Selection: Main Idea

Fitness x = loss in indicator value if x is removed

= I(**A**-{**x**}, **A**)

= I(**A**-{**x**}, {**x**})



- “optimal” in steady state (one solution per iteration)
- needs to be extended to break ties

[Zitzler,Künzli:04]

Implementation for the ε-Quality Indicator

Fitness x = loss in indicator value if x is removed

= I(**A**-{**x**}, {**x**})

= $\min_{\mathbf{a} \in A-\{x\}} \{ I(\{\mathbf{a}\}, \{\mathbf{x}\}) \}$

More precisely:

Fitness vector = sorted pairwise indicator values

Fast approximation:

$$F(\boldsymbol{x}^1) = \sum_{\boldsymbol{x}^2 \in P \setminus \{\boldsymbol{x}^1\}} -e^{-I(\{\boldsymbol{x}^2\},\{\boldsymbol{x}^1\})/\kappa}$$

Empirical Validation

		SPEA2		NSGA-II		$SPEA2_{adap}$		$IBEA_{\epsilon,adap}$	
		P value	T	P value	T	P value	T	P value	T
ZDT6	NSGA-II	$5.6073 \cdot 10^{-4}$	↑						
	$SPEA2_{adap}$	$> 5\%$	⇌	$8.1975 \cdot 10^{-6}$	↓				
	$IBEA_{\epsilon,adap}$	$8.1014 \cdot 10^{-9}$	↑	$2.0023 \cdot 10^{-5}$	↑	$1.9568 \cdot 10^{-9}$	↑		
	$IBEA_{HD,adap}$	0.0095	↑	$> 5\%$	⇌	$5.4620 \cdot 10^{-5}$	↑	$1.3853 \cdot 10^{-5}$	↓
DTLZ2	NSGA-II	$3.0199 \cdot 10^{-10}$	↓						
	$SPEA2_{adap}$	$> 5\%$	⇌	$3.0199 \cdot 10^{-10}$	↑				
	$IBEA_{\epsilon,adap}$	$3.0199 \cdot 10^{-10}$	↑	$3.0199 \cdot 10^{-10}$	↑	$3.0199 \cdot 10^{-10}$	↑		
	$IBEA_{HD,adap}$	$3.0199 \cdot 10^{-10}$	↑	$3.0199 \cdot 10^{-10}$	↑	$3.0199 \cdot 10^{-10}$	↑	$5.5329 \cdot 10^{-7}$	↓
DTLZ6	NSGA-II	$8.1014 \cdot 10^{-9}$	↓						
	$SPEA2_{adap}$	$> 5\%$	⇌	$6.1210 \cdot 10^{-9}$	↑				
	$IBEA_{\epsilon,adap}$	$3.0199 \cdot 10^{-10}$	↑	$3.0199 \cdot 10^{-10}$	↑	$3.0199 \cdot 10^{-10}$	↑		
	$IBEA_{HD,adap}$	$3.0199 \cdot 10^{-10}$	↑	$3.0199 \cdot 10^{-10}$	↑	$3.0199 \cdot 10^{-10}$	↑	$3.5923 \cdot 10^{-4}$	↓
KUR	NSGA-II	$> 5\%$	⇌						
	$SPEA2_{adap}$	$> 5\%$	⇌	$> 5\%$	⇌				
	$IBEA_{\epsilon,adap}$	$3.0199 \cdot 10^{-10}$	↓	$3.0199 \cdot 10^{-10}$	↓	$6.6955 \cdot 10^{-10}$	↓		
	$IBEA_{HD,adap}$	$3.0199 \cdot 10^{-10}$	↓	$3.0199 \cdot 10^{-10}$	↓	$4.9752 \cdot 10^{-10}$	↓	$> 5\%$	⇌
Knap.	NSGA-II	$> 5\%$	⇌						
	$SPEA2_{adap}$	$> 5\%$	⇌	$> 5\%$	⇌				
	$IBEA_{\epsilon,adap}$	$> 5\%$	⇌	$> 5\%$	⇌	$> 5\%$	⇌		
	$IBEA_{HD,adap}$	$> 5\%$	⇌	$> 5\%$	⇌	$> 5\%$	⇌	$> 5\%$	⇌
EXPO2	NSGA-II	$> 5\%$	⇌						
	$SPEA2_{adap}$	$> 5\%$	⇌	0.0189	↑				
	$IBEA_{\epsilon,adap}$	$1.0837 \cdot 10^{-8}$	↑	$2.6753 \cdot 10^{-9}$	↑	$6.4048 \cdot 10^{-8}$	↑		
	$IBEA_{HD,adap}$	$1.9638 \cdot 10^{-7}$	↑	$1.2260 \cdot 10^{-8}$	↑	$6.6261 \cdot 10^{-7}$	↑	$> 5\%$	⇌
EXPO3	NSGA-II	$> 5\%$	⇌						
	$SPEA2_{adap}$	$> 5\%$	⇌	$> 5\%$	⇌				
	$IBEA_{\epsilon,adap}$	$4.3165 \cdot 10^{-8}$	↑	$5.0801 \cdot 10^{-8}$	↑	$3.1159 \cdot 10^{-7}$	↑		
	$IBEA_{HD,adap}$	$2.4189 \cdot 10^{-7}$	↑	$1.5732 \cdot 10^{-7}$	↑	$1.1653 \cdot 10^{-6}$	↑	$> 5\%$	⇌
EXPO4	NSGA-II	$> 5\%$	⇌	-					
	$SPEA2_{adap}$	$> 5\%$	⇌	$9.4209 \cdot 10^{-4}$	↓				
	$IBEA_{\epsilon,adap}$	$1.8546 \cdot 10^{-10}$	↑	$6.9754 \cdot 10^{-10}$	↑	$1.8390 \cdot 10^{-10}$	↑		
	$IBEA_{HD,adap}$	$1.9883 \cdot 10^{-10}$	↑	$1.0221 \cdot 10^{-9}$	↑	$1.9716 \cdot 10^{-10}$	↑	$> 5\%$	⇌

significantly better than all other algorithms

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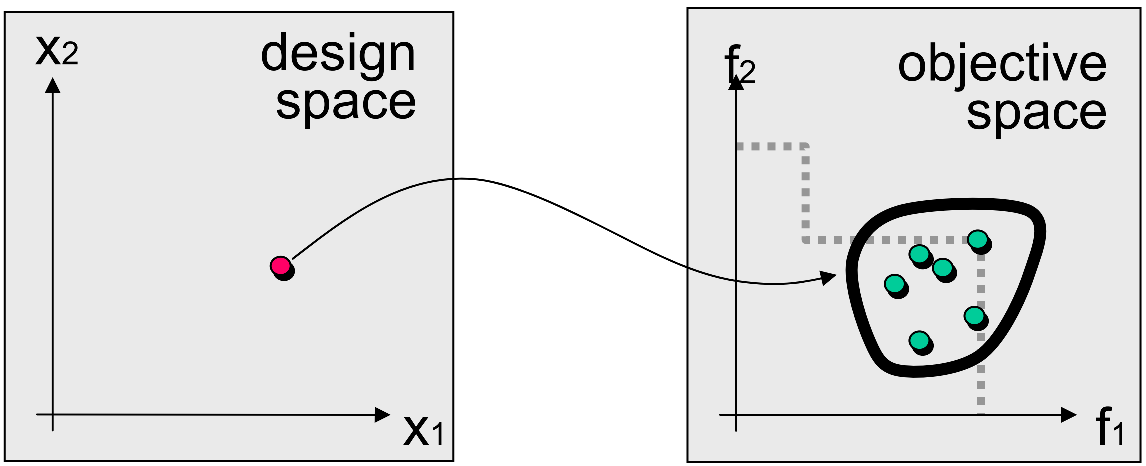
Uncertainty in Multiobjective Optimization

Uncertainty = each time a solution/design is evaluated, possibly different objective function values emerge due to

- stochastic system model (Monte Carlo simulation)
- model parameter variations (cost estimates)



Previous work: *[Hughes:01;Teich:01; Goldberg et al.:03,05]*

- assume a certain type of distribution (uniform,normal)
- mixed models (stochastic dominance, regular distance)

Indicators and Uncertainty

Deterministic model:

$$\mathrm{argmin}_{S \in \mathcal{M}(X)} \; I(f(S), f(R))$$

each solution is associated with one objective vector

Stochastic model:

$$\mathrm{argmin}_{S \in \mathcal{M}(X)} \; E(I(\mathcal{F}(S), \mathcal{F}(R)))$$

each solution is associated with a random variable

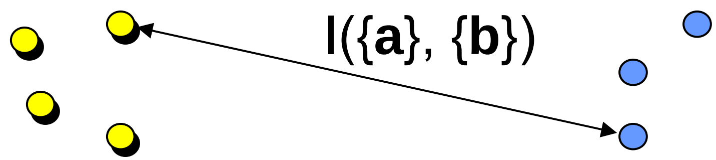
$$\begin{aligned} E(I(\mathcal{F}(S), \mathcal{F}(R))) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(\mathcal{F}(S) = A, \mathcal{F}(R) = B) \cdot I(A, B) \; \mathrm{d}A \mathrm{d}B \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(\mathcal{F}(S) = A) \cdot P(\mathcal{F}(R) = B) \cdot I(A, B) \; \mathrm{d}A \mathrm{d}B \end{aligned}$$

[Basseur, Zitzler:05]

Estimating the Expected ε-Value

$\mathcal{S}(\mathbf{x})$ = sample of objective vectors for solution **x**

Main idea: consider all combinations of objective vectors and compute expected (mean) indicator value



$$\hat{E}(I(\mathcal{F}(S), \{\mathbf{z}^*\}) = \sum_{\mathbf{z}_1 \in \mathcal{S}(\mathbf{x}_1)} \sum_{\mathbf{z}_2 \in \mathcal{S}(\mathbf{x}_2)} \cdots \sum_{\mathbf{z}_m \in \mathcal{S}(\mathbf{x}_m)} \frac{I(\{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_m\}, \{\mathbf{z}^*\})}{\prod_{1 \leq i \leq m} |\mathcal{S}(\mathbf{x}_i)|}$$

$$\hat{E}(I(\mathcal{F}(S), \mathcal{F}(\{\mathbf{x}^*\})) = \frac{1}{|\mathcal{S}(\mathbf{x}^*)|} \sum_{\mathbf{z}^* \in \mathcal{S}(\mathbf{x}^*)} \hat{E}(I(\mathcal{F}(S), \{\mathbf{z}^*\})$$

Alternative: mean value per objective function (looses distribution characteristics)

Preliminary Simulation Results

indicator-based average [Hug01]

		EIV		BCK		Exp		Avg		PDR	
		Z	X	Z	X	Z	X	Z	X	Z	X
n=1	EIV			> 5%	> 5%	> 5%	> 5%	> 5%	> 5%	> 5%	> 5%
	BCK	> 5%	> 5%			> 5%	> 5%	> 5%	> 5%	> 5%	> 5%
	Exp	> 5%	> 5%	> 5%	> 5%			> 5%	> 5%	> 5%	> 5%
	Avg	> 5%	> 5%	> 5%	> 5%	> 5%	> 5%			> 5%	> 5%
	PDR	> 5%	> 5%	> 5%	> 5%	> 5%	> 5%	> 5%	> 5%		
n=5	EIV			> 5%	> 5%	> 5%	1.56%	0.014%	0.014%	0.014%	0.014%
	BCK	> 5%	> 5%			1.89%	0.041%	0.014%	0.014%	0.014%	0.014%
	Exp	> 5%	> 5%	> 5%	> 5%			0.014%	0.014%	0.014%	0.014%
	Avg	> 5%	> 5%	> 5%	> 5%	> 5%	> 5%			2.28%	> 5%
	PDR	> 5%	> 5%	> 5%	> 5%	> 5%	> 5%	> 5%	0.019%		
n=10	EIV			> 5%	> 5%	> 5%	> 5%	0.014%	0.014%	0.014%	0.014%
	BCK	> 5%	> 5%			> 5%	0.45%	0.014%	0.014%	0.014%	0.014%
	Exp	> 5%	> 5%	> 5%	> 5%			0.014%	0.014%	0.014%	0.014%
	Avg	> 5%	> 5%	> 5%	> 5%	> 5%	> 5%			> 5%	> 5%
	PDR	> 5%	> 5%	> 5%	> 5%	> 5%	> 5%	1.28%	> 5%		

- no significant differences in <3 objectives
- highly significant differences in higher dimensions

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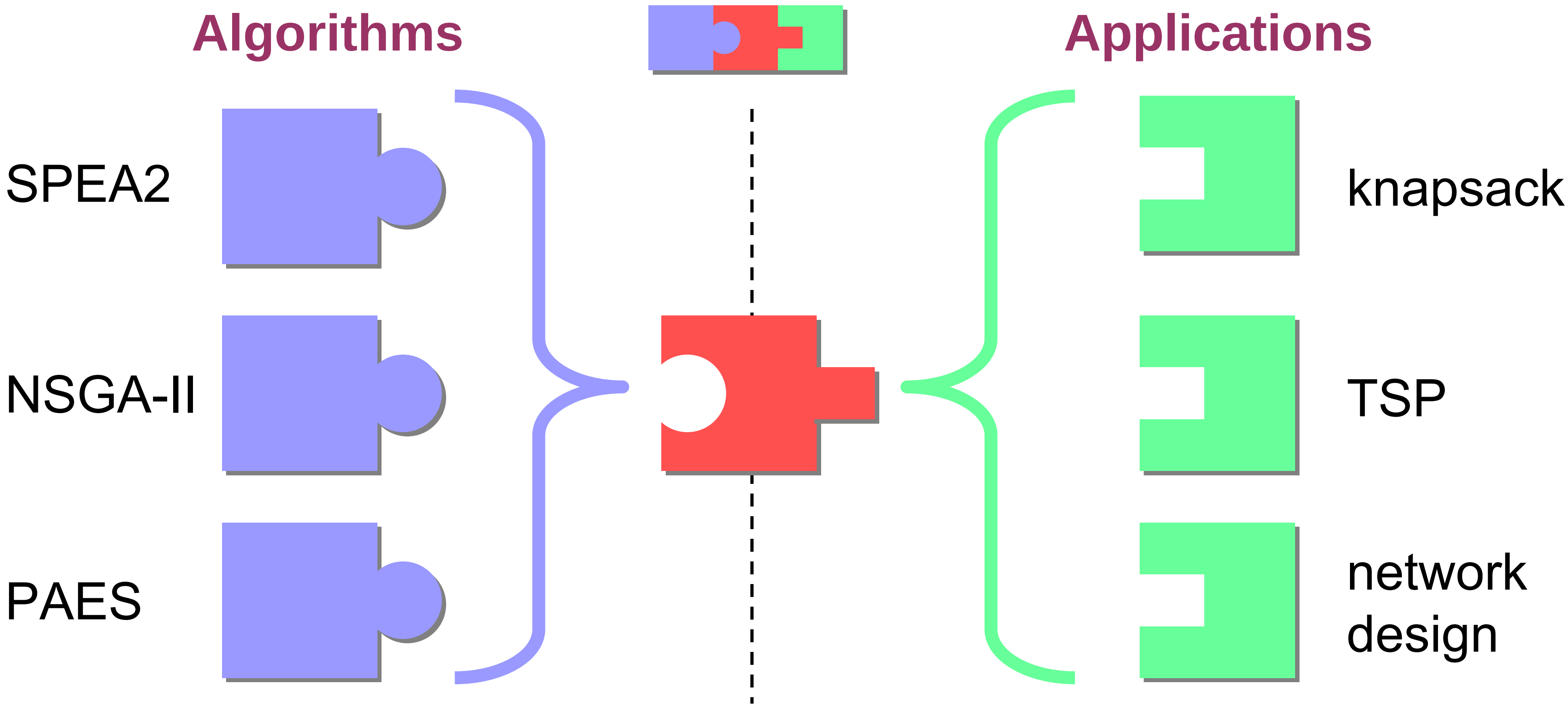
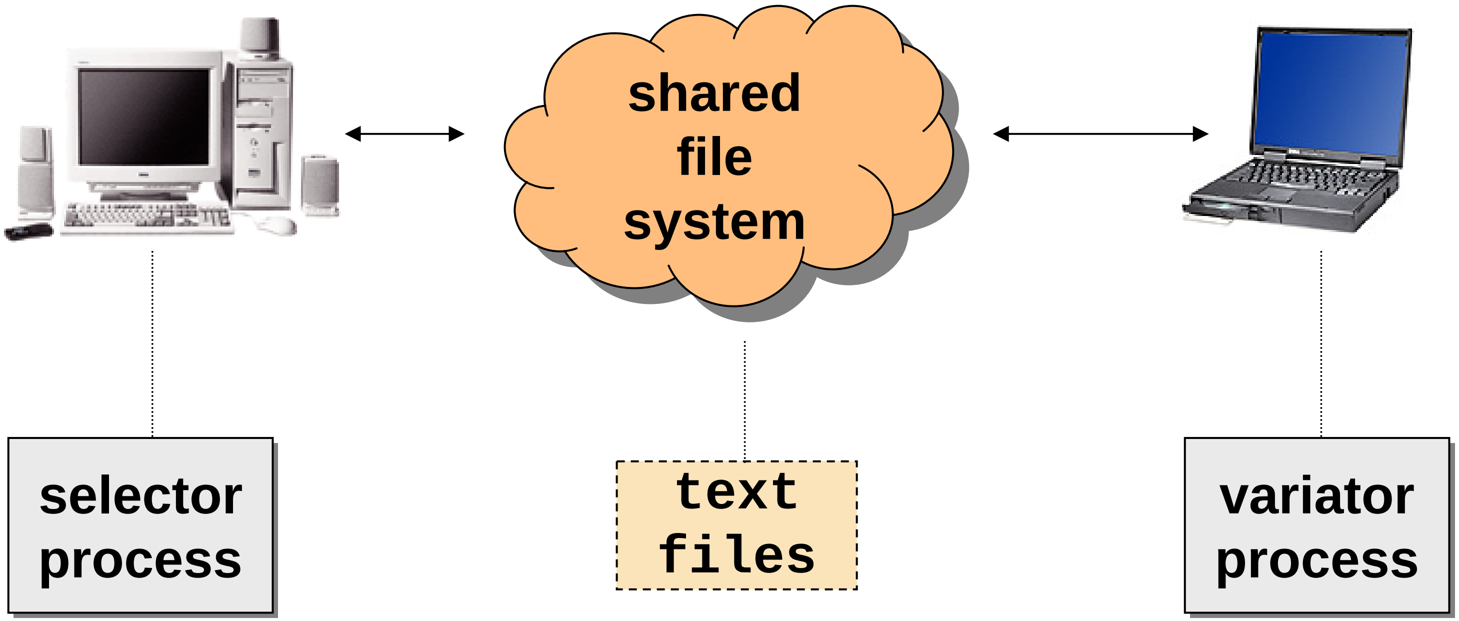
The Concept of PISA



text-based

Platform and programming language independent **I**nterface for **S**earch **A**lgorithms *[Bleuler et al.:03; Künzli et al. 05]*

PISA: Implementation



application independent:

- mating / environmental selection
- individuals are described by IDs and objective vectors

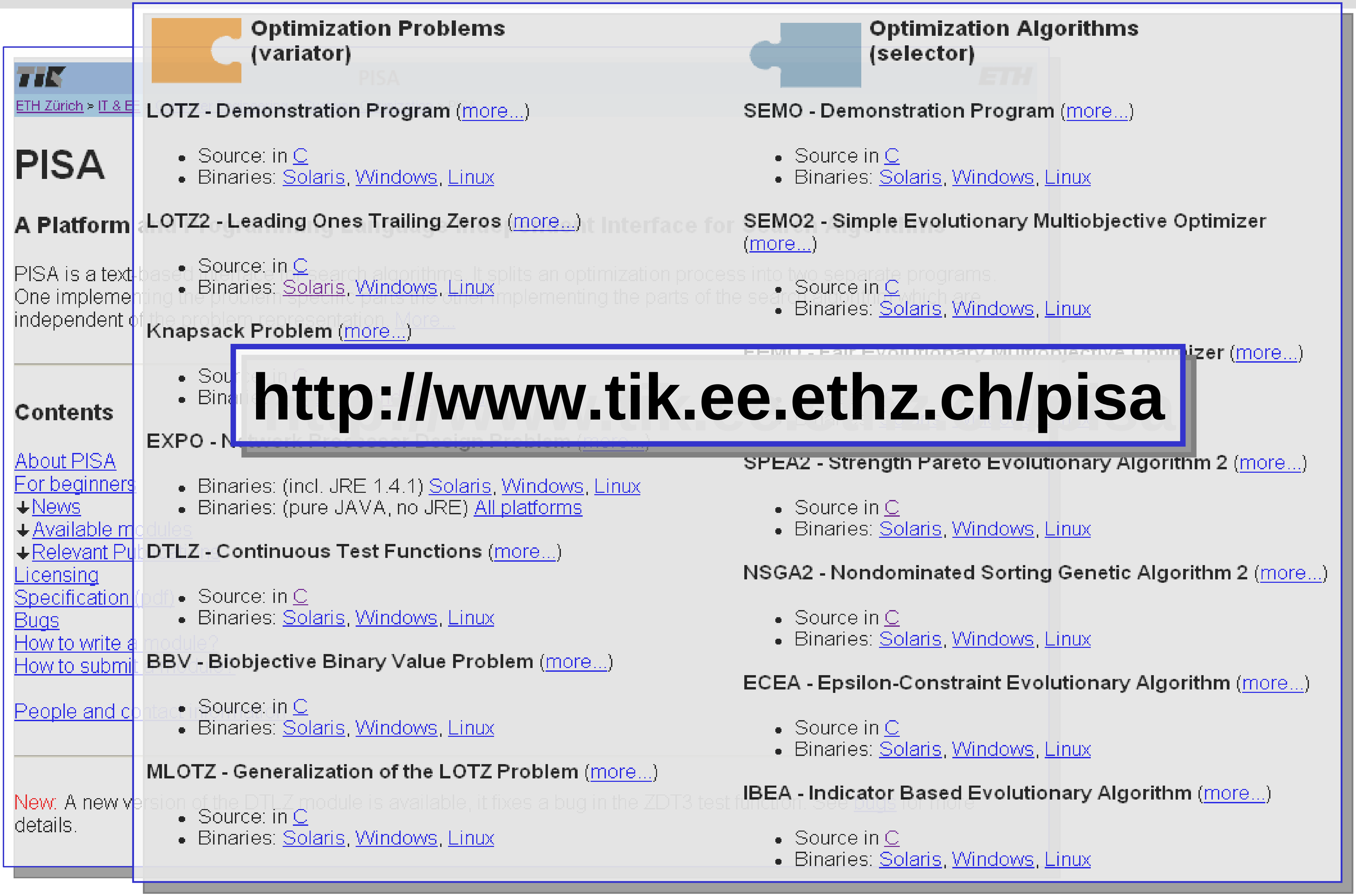
handshake protocol:

- state / action
- individual IDs
- objective vectors
- parameters

application dependent:

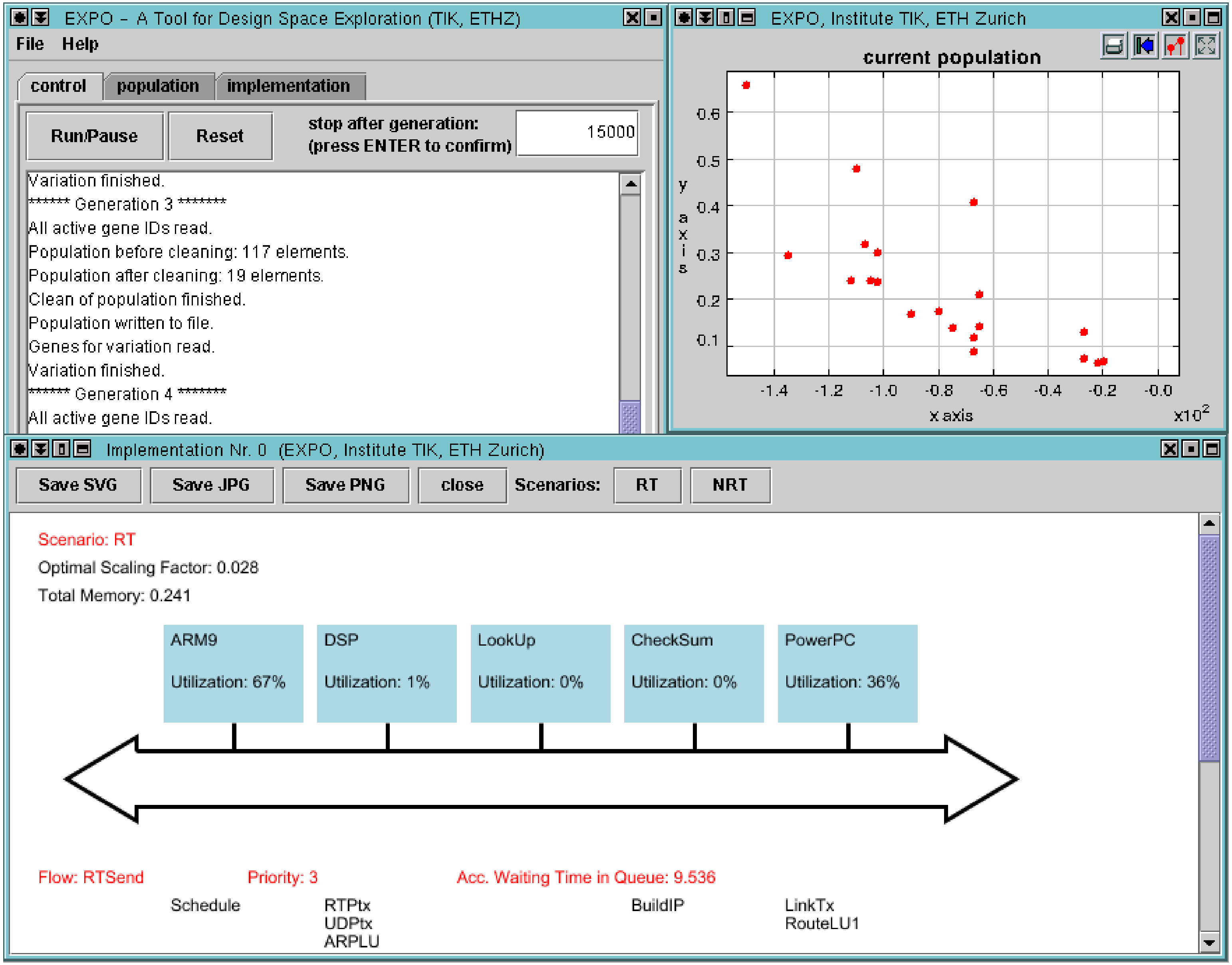
- variation operators
- stores and manages individuals

PISA Website



http://www.tik.ee.ethz.ch/pisa

Network Processor Design Application (EXPO)



[Thiele,Künzli et al.:03,04,05]

Open Issues in Design Space Exploration Using RSA

How to...

- handle multiple objectives
- account for uncertainty
- optimize for robustness
- incorporate user preferences
- achieve diversity in the parameter space
- reduce the design space complexity
- statistically assess the quality of the outcome
- simplify implementation and maximize reusability
- deal with limited memory resources
- postprocessing of tradeoff surface

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