OUTLINE

- A Brief Introduction
  - Motivation ... what are the problems to solve
  - CTL, LTL and basic model-checking algorithms
- Timed Systems
  - Timed automata and verification problems
  - UPPAAL tutorial (1): data stuctures & algorithms
  - UPPAAL tutorial (2): input languages
  - TIMES: From models to code "guaranteeing" timing constraints
- Further topics/Recent Work
  - Systems with buffers/queues [CAV 2006]

Lecture 3

Timed Automata and TCTL

syntax, semantics, and verification problems

Timed Automata

=

Finite Automata + Clock Constraints + Clock resets

Clock Constraints

For set C of clocks with $x, y \in C$, the set of *clock constraints* over C, $\Psi(C)$, is defined by

$$\alpha ::= x \prec c \;\big|\; x - y \prec c \;\big|\; \neg\alpha \;\big|\; (\alpha \wedge \alpha)$$

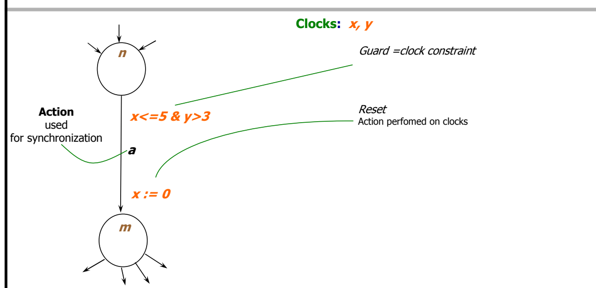
where $c \in \mathbb{N}$ and $\prec \in \{<, \leqslant\}$.

Timed Automata

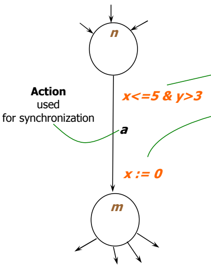
A *timed automaton* $\mathcal{A}$ is a tuple $(L, l_0, E, \mathit{Label}, C, \mathit{clocks}, \mathit{guard}, \mathit{inv})$ with

- L, a non-empty, finite set of locations with initial location $l_0 \in L$
- $E \subseteq L \times L$, a set of edges
- $\mathit{Label} : L \longrightarrow 2^{AP}$, a function that assigns to each location $l \in L$ a set $\mathit{Label}(l)$ of atomic propositions
- C, a finite set of clocks
- $\mathit{clocks} : E \longrightarrow 2^C$, a function that assigns to each edge $e \in E$ a set of clocks $\mathit{clocks}(e)$
- $\mathit{guard} : E \longrightarrow \Psi(C)$, a function that labels each edge $e \in E$ with a clock constraint $\mathit{guard}(e)$ over C, and
- $\mathit{inv} : L \longrightarrow \Psi(C)$, a function that assigns to each location an *invariant*.

Timed Automata: Syntax



Timed Automata: Semantics



Clocks: *x, y*

Guard =clock constraint

Reset
Action perfomed on clocks

State

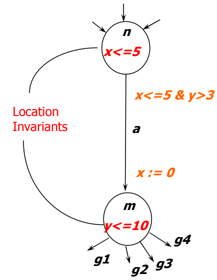
(*location* , *x*=v , *y*=u) where v,u are in $\mathbb{R}$

Transitions

(*n* , *x*=2.4 , *y*=3.1415) $\xrightarrow{a}$ (*m* , *x*=0 , *y*=3.1415)

(*n* , *x*=2.4 , *y*=3.1415) $\xrightarrow{e(1.1)}$ (*n* , *x*=3.5 , *y*=4.2415)

Timed Automata with *Invariants*



Clocks: *x, y*

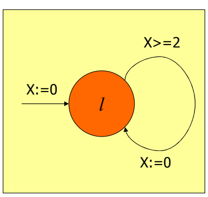
Transitions

(*n* , *x*=2.4 , *y*=3.1415) ~~$\xrightarrow{3.2}$~~

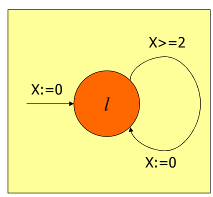
(*n* , *x*=2.4 , *y*=3.1415) $\xrightarrow{1.1}$ (*n* , *x*=3.5 , *y*=4.2415)

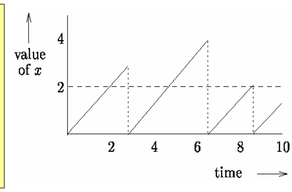
Invariants insure progress!!

Timed Automata: Example

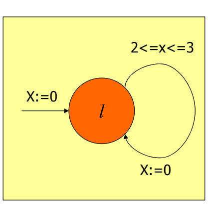


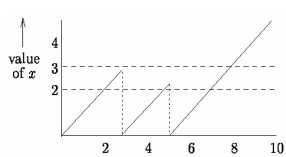
Timed Automata: Example



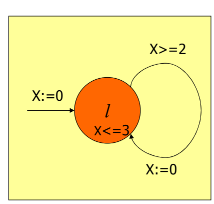


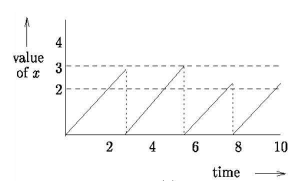
Timed Automata: Example



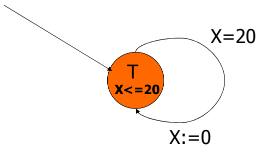


Timed Automata: Example

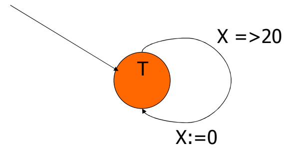




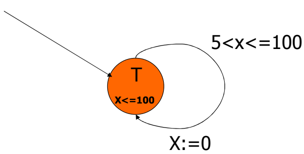
Timed Automata: Example
(periodic task)



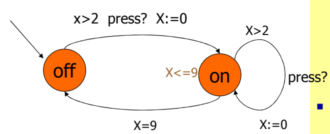
Timed Automata: Example
(sporadic task)



Timed Automata: Example
(aperiodic task)



Timed Automata: Light Switch

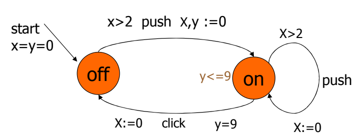


- Switch may be turned on whenever at least 2 time units has elapsed since last "turn off"
- Light automatically switches off after 9 time units if it is not pressed.

Semantics (definition)

- *clock valuations*: $V(C) \quad v : C \rightarrow R_{\geq 0}$
- *state*: $(l, v) \quad where \quad l \in L \quad and \quad v \in V(C)$
- *action transition* $(l,v) \xrightarrow{a} (l',v') \text{ iff } \; l \xrightarrow{g\;a\;r} l'$
  $g(v) \;\; and \;\; v' = v[r] \;\; and \;\; Inv(l')(v')$
- *delay Transition* $(l,v) \xrightarrow{d} (l, v+d) \text{ iff}$
  $Inv(l)(v+d') \text{ whenever } d' \leq d \in R_{\geq 0}$

Timed Automata: Example



$(off, x=y=0) \xrightarrow{3.5} (off, x=y=3.5) \xrightarrow{push}$

$(on, x=y=0) \xrightarrow{\pi} (on, x=y=\pi) \xrightarrow{push}$

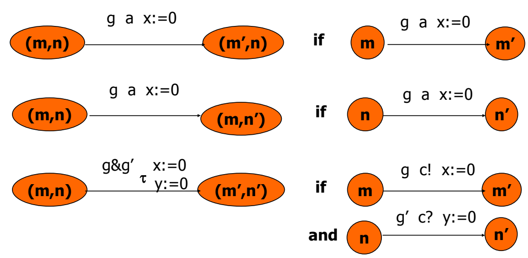
$(on, x=0, y=\pi) \xrightarrow{3} (on, x=3, y=\pi+3) \xrightarrow{9-(\pi+3)}$

$(on, x=9-(\pi+3), y=9) \xrightarrow{click} (off, x=0, y=9) \ldots$

Modeling Concurrency

- Products of automata
- Parallel composition

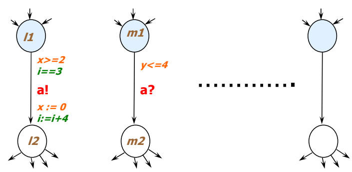
CCS Parallel Composition (implemented in UPPAAL)



Where a is an action c! or c? or τ
c is a channel name

The UPPAAL Model

= Networks of Timed Automata + Integer Variables +….



Two-way synchronization on *complementary* actions.

Closed Systems!

Example transitions

$(l1, m1, \ldots, x=2, y=3.5, i=3, \ldots) \xrightarrow{\tau} (l2, m2, \ldots, x=0, y=3.5, i=7, \ldots)$

Verification Problems

Location Reachability (def.)

n is reachable from m if there is a sequence of transitions:

$(m, u) \longrightarrow^{*} (n, v)$

(Timed) Language Inclusion, $L(A) \subseteq L(B)$

$(a_0, t_0)\ (a_1, t_1) \ldots\ldots (a_n, t_n) \in L(A)$

If

"A can perform a_0 at t_0, a_1 at t_1 … … a_n at t_n"

$(l_0, u_0) \xrightarrow{t_0} (l_0, u_0+t_0) \xrightarrow{a_0} (l_1, u_1) \ldots\ldots$

Verification Problems

- Timed Language Inclusion ☹
  - 1-clock, finite traces, decidable [Ouaknine & Worrell 04]
  - 1-clock, infinite traces & Buchi-conditions, undecidable [Abdulla et al 05]
- Untimed Language Inclusion ☺
- (Un)Timed Bisimulation ☺
- Reachability Analysis ☺
- Optimal Reachability (synthesis problem) ☺
  - If a location is reachable, what is the minimal delay before reaching the location?

Timed CTL = CTL + clock constraints

Note that The semantics of TA defines a transition system where each state has a Computation Tree

Computation Tree Logic, CTL

Clarke & Emerson 1980

Syntax

ϕ ::= P | ¬ ϕ | ϕ ∨ ϕ | EX ϕ | E[ϕ U ϕ] | A[ϕ U ϕ]

where **P** ∈ AP (atomic propositions)

TCTL

Henzinger, Sifakis et al 1992

Syntax

ϕ ::= P | g | ¬ ϕ | ϕ ∨ ϕ | z.ϕ | E[ϕ U ϕ] | A[ϕ U ϕ]

where **P** ∈ AP (atomic propositions) and g is a Clock constraint

AG (P imply z.(z<10 or q))

Timed CTL (a simplified version of TCTL)

Syntax

ϕ ::= p | ¬ ϕ | ϕ ∨ ϕ | EX ϕ | E[ϕ U ϕ] | A[ϕ U ϕ]

where **p** ∈ AP (atomic propositions) **or** a Clock constraint

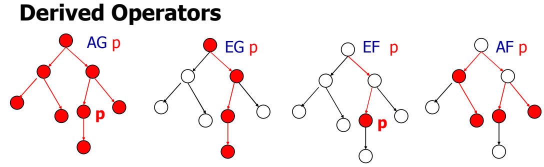
Timed CTL

Syntax

ϕ ::= p | ¬ ϕ | ϕ ∨ ϕ | EX ϕ | E[ϕ U ϕ] | A[ϕ U ϕ]

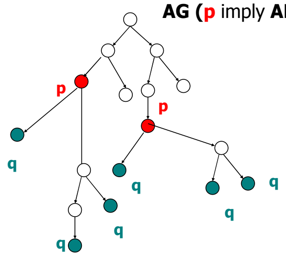
where **p** ∈ AP (atomic propositions) **or** Clock constraint

Derived Operators



Liveness: p - -> q *"p leads to q"*

AG (p imply AF q)



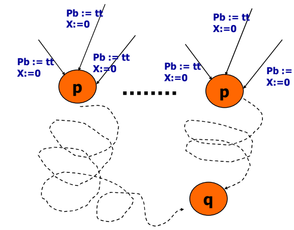
Bounded Liveness/Response

[TACAS 98]

Verify: "whenver p is true, q should be true within 10 sec

AG ((P_b and x>10) imply q)

Use extra clock x and boolean P_b
Add P_b := tt and x:=0 on all edges leading to location P



Bounded Liveness/Response

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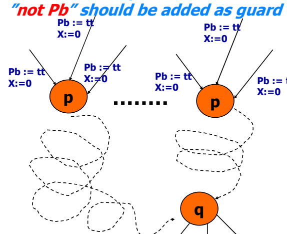
Verify: "whenver p is true, q should be true within 10 sec

AG ((P_b and x>10) imply q)

Use extra clock x and boolean P_b
Add P_b := tt and x:=0 on all edges leading to location P

This is not really correct; "not Pb" should be added as guard



Pb:=ff should be On all eadges leaving q

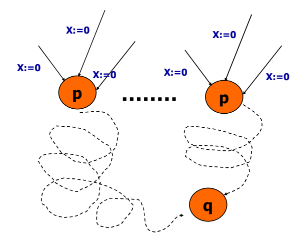
Bounded Liveness

[TACAS 98]

Verify: "whenver p is true, q should be true within 10 sec

P - - > (q and x<10)

Use extra clock x
Add x:=0 on all edges leading to P



Timed CTL in UPPAAL

EF p | AG p | EG p | AF p | p - -> q

P ::= A.l | g_c | g_d | not p | p or p | p and p | p imply p

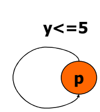
Process Location (a location in automaton A)

Clock constraint

predicate over data variables

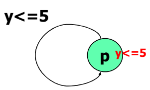
p leads to q *denotes* **AG (p imply AF q)**

Problem with Zenoness



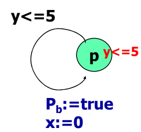
A Zeno-automaton may satisfy the formula Without containing a state where q is true

EXAMPLE



We want to specify "whenever P is true, Q should be true within 10 time units

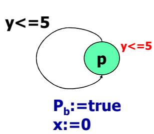
EXAMPLE



We want to specify "whenever P is true, Q should be true within 10 time units

AG (P_b and x>10 imply Q)

EXAMPLE



We want to specify "whenever P is true, Q should be true within 10 time units

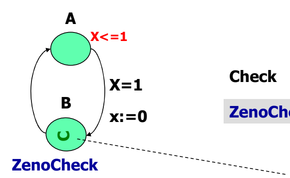
AG ((P_b and x>10) imply q)

is satisfied !!!

Solution with UPPAAL

Check Zeno-freeness by an extra observer

System | | ZenoCheck



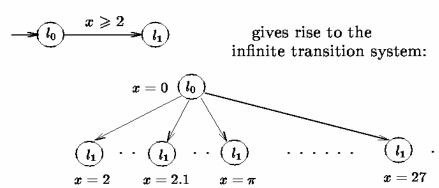
Check

ZenoCheck.A - - > ZenoCheck.B

Committed location!

REACHABILITY ANALYSIS
using Regions

Infinite State Space!

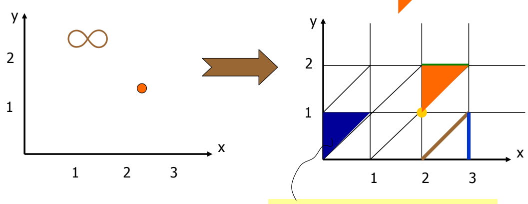


However, the reachability problem is decidable ☺ Alur&Dill 1991

Region: From infinite to finite

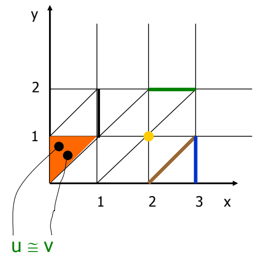
Concrete State
(n, x=2.2, y=1.5)

Symbolic state (region)
(n, )



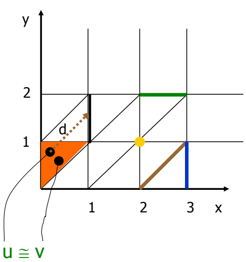
An equivalence class (i.e. a *region*)
There are only *finite* many such!!

Region equivalence (Intuition)



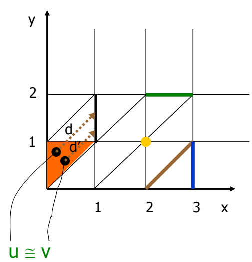
u ≅ v iff (l,u) and (l,v) may reach the same set of eqivalence classes

Region equivalence (Intuition)



u ≅ v iff (l,u) and (l,v) may reach the same set of eqivalence classes

Region equivalence (Intuition)

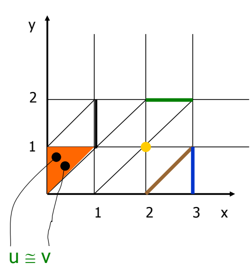


u ≅ v iff (l,u) and (l,v) may reach the same set of eqivalence classes

Region equivalence *[Alur and Dill 1990]*

- u,v are clock assignments
- u≈v iff
  - For all clocks x,
    either (1) u(x)>Cx and v(x)>Cx
    or (2) ⌊u(x)⌋=⌊v(x)⌋
  - For all clocks x, if u(x)<=Cx,
    {u(x)}=0 iff {v(x)}=0
  - For all clocks x, y, if u(x)<=Cx and u(y)<=Cy
    {u(x)}<= {u(y)} iff {v(x)}<= {v(y)}

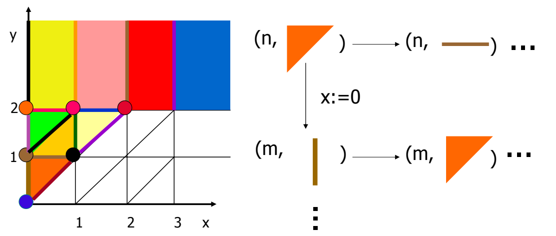
Region equivalence (alternatively)



u ≅ v iff u and v satisfy exactly the same set of constraints in the form of
xi ~ m and xi-xj ~ n
where ~ is in {<,>,≤,≥}
and m,n < MAX

This is not quite correct;
we need to consider the MAX more carefully

Region Graph
Finite-State Transition System!!



OBS: there are only Finite many regions

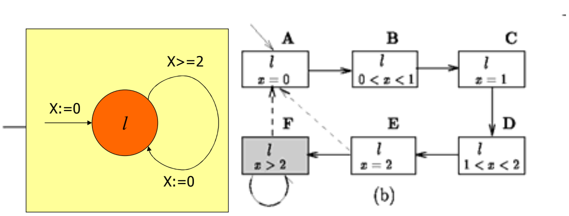
(m, [u]) ⟶ (n, [v]) if (m, u) ⟶ (n,v)

Theorem

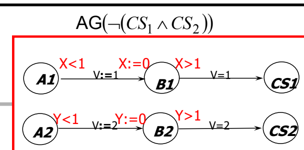
u≈v implies
- u(x:=0) ≈ v(x:=0)
- u+n ≈ v+n for all natural number n
- for all d<1: u+d ≈ v+d' for some d'<1

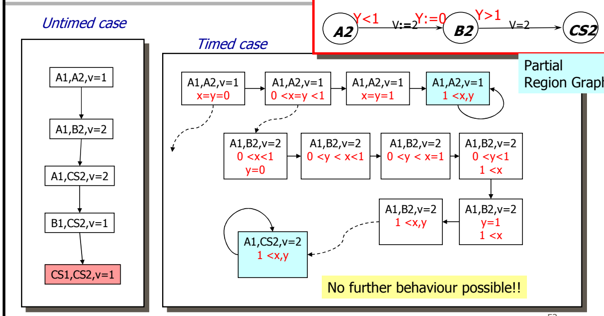
"Region equivalence' is preserved by "addition" and reset.
(also preserved by "subtraction" if clock values are "bounded")

Region graph of a simple timed automata



Fischers again



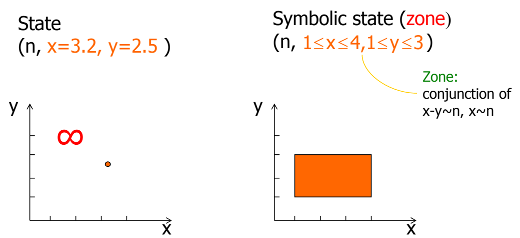


Problems with Region Construction

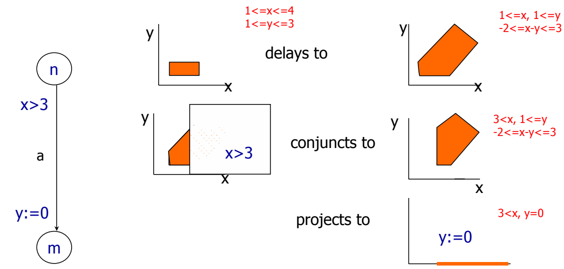
- Too many 'regions'
  - Sensitive to the maximal constants
  - e.g. x>1,000,000, y>1,000,000 as guards in TA
- The number of regions is highly exponential in the number of clocks and the maximal constants.

REACHABILITY ANALYSIS
using ZONES

Zones: From infinite to finite

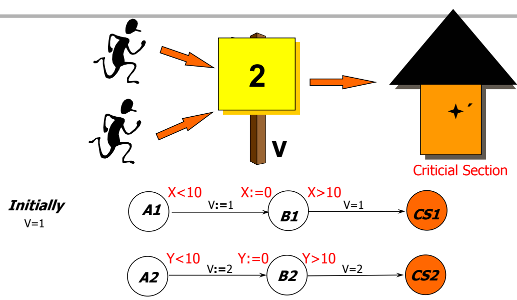


Symbolic Transitions

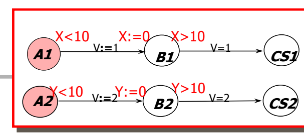


Thus (n, 1<=x<=4,1<=y<=3) =a=> (m, 3<x, y=0)

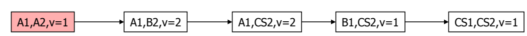
Fischer's Protocol
analysis using zones



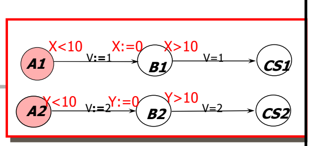
Fischers cont.



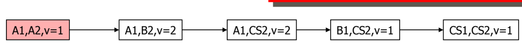
Untimed case



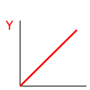
Fischers cont.



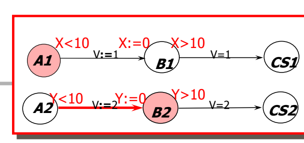
Untimed case



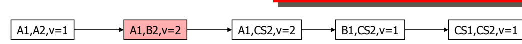
Taking time into account



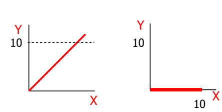
Fischers cont.



Untimed case



Taking time into account



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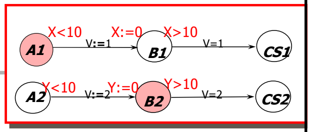

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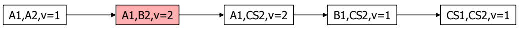
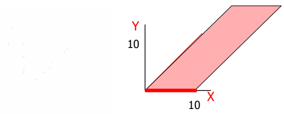


Taking time into account



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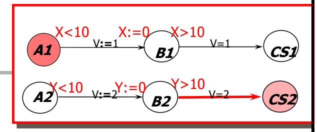

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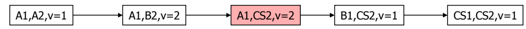

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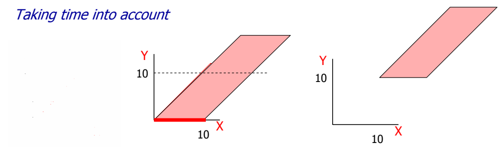

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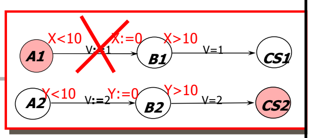
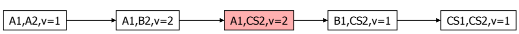


Untimed case



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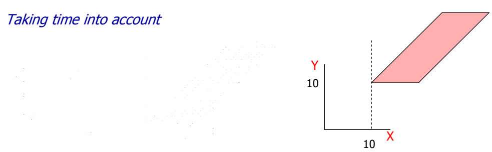


Zones = Conjuctive constraints

- A zone Z is a conjunctive formula:
  g_1 & g_2 & ... & g_n
  where g_i may be $x_i \sim b_i$ or $x_i - x_j \sim b_{ij}$
- Use a zero-clock x_0 (constant 0), we have
  $\{x_i - x_j \sim b_{ij} \mid \sim \text{ is } < \text{ or } \leq, i,j \leq n\}$
- This can be represented as a MATRIX, DBM
  (Difference Bound Matrices)

Solution set as semantics

- Let Z be a zone (a set of constraints)
- Let [Z]={u | u is a solution of Z}

(We shall simply write Z instead [Z])

Operations on Zones

- Strongest post-condition (Delay): SP(Z) or Z↑
  - [Z↑] = {u+d| d ∈ R, u∈[Z]}
- Weakest pre-condition: WP(Z) or Z↓ (the dual of Z↑)
  - [Z↓] = {u| u+d∈[Z] for some d∈R}
- Reset: {x}Z or Z(x:=0)
  - [{x}Z] = {u[0/x] | u ∈[Z]}
- Conjunction
  - [Z&g]= [Z]∩[g]

Two more operations on Zones

- Inclusion checking: $Z_1 \subseteq Z_2$
  - solution sets
- Emptiness checking: $Z = \emptyset$
  - no solution

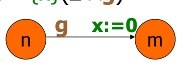
Theorem on Zones

The set of zones is closed under all zone operations

- That is, the result of the operations on a zone is a zone
- Thus, there will be a zone to represent the sets: $[Z\uparrow]$, $[Z\downarrow]$, $[\{x\}Z]$

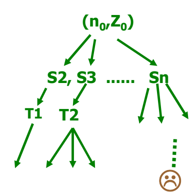
One-step reachability: Si ⁓ Sj

- Delay: (n,Z) → (n,Z') where $Z' = Z\uparrow \wedge inv(n)$
- Action: (n,Z) → (m,Z') where $Z' = \{x\}(Z \wedge \mathbf{g})$

if 

- **Reach**: (n,Z) ⁓ (m,Z') if (n,Z) →→(m,Z')
- **Successors**(n,Z)={(m,Z') | (n,Z) ⁓ (m,Z'), $Z' \neq \emptyset$}

Now, we have a search problem



EF ☹