Real-Time Control

Anton Cervin

Department of Automatic Control
Lund University
Sweden

Today's outline

- Part I – Introduction to Automatic Control
- Part II – Sampled-Data and Networked Control
- Part III – Integrated Control and Scheduling
- Part IV – Computer Exercise

Further Reading

- B. Wittenmark, K. J. Åström, K.-E. Årzén: "Computer Control: An Overview." IFAC Professional Brief, 2002. (93 pages, available at http://www.control.lth.se)
- K. J. Åström, Tore Hägglund: "Advanced PID Control." The Instrumentation, Systems, and Automation Society, 2005.
- A. Cervin: "Integrated Control and Real-Time Scheduling." PhD Thesis, Lund University, 2003. (Available at http://www.control.lth.se)

Real-Time Control

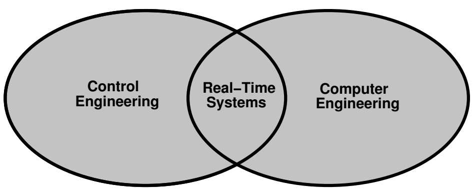
Part I – Introduction to Automatic Control

Part I – Outline

1. Introduction
2. Basic concepts
3. Modeling and design
4. Empirical PID control

1. Introduction

Real-time and control



- All control systems are real-time systems
- Many hard real-time systems are control systems

Real-time and control

- Control engineers need real-time systems to implement their systems.
- Computer engineers need control theory to build "controllable" systems
- Interesting research problems in the interface
- Much can be lost without integration

Automatic control

The silent technology:

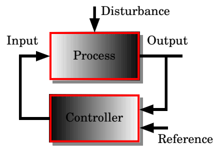
- Widely used
- Very successful
- Seldom talked about, except when disaster strikes!

Automatic control

Use of **models** and **feedback**

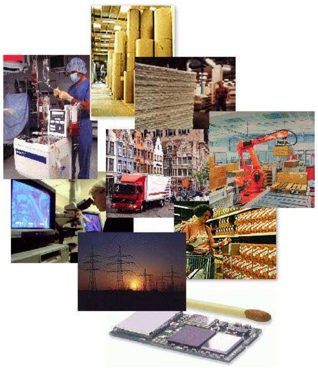
Activities:

- Modeling
- Analysis and simulation
- Control design
- Implementation

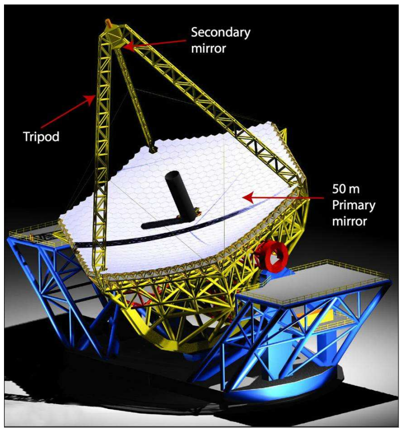


Applications

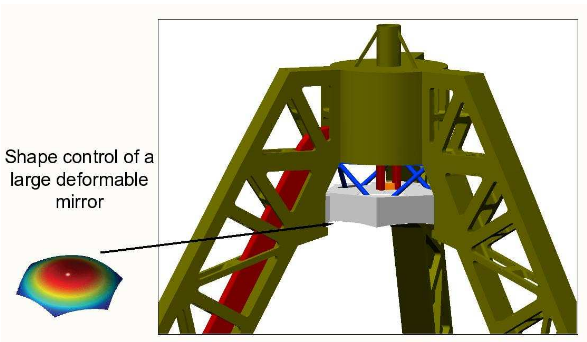
- Automotive systems
- Robotics
- Biotechnology
- Power systems
- Process control
- Communications
- Consumer electronics
- ...



Example: The EURO 50 telescope



The deformable mirror

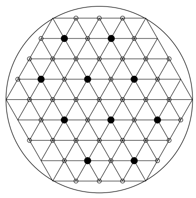


- Compensate for atmospheric disturbances 1000 times/sec

Control of deformable mirror

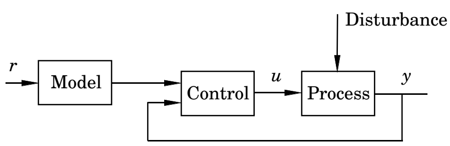
Control the system $M\ddot{x} + C\dot{x} + Kx = Fu$ to the equilibrium $Kx = Fu_r$ using measurements of $y = Ex$ and $\dot{y}$.

- Large number of sensors and actuators (2000-3000)
- Computational limitations (1kHz)
- Cannot measure and control at the same spot
- Large uncertainty in C



2. Basic Concepts

Basic setting



Must handle two tasks:

- Follow reference signals, r
- Compensate for disturbances

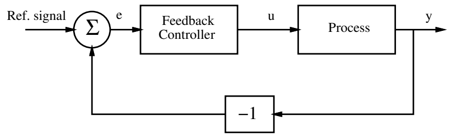
How to

- do several things with the control signal u

The feedback principle

A very powerful idea, that often leads to revolutionary changes in the way systems are designed.

The primary paradigm in automatic control.

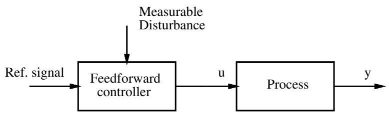


- Base corrective action on an error that has occurred
- Closed loop

Properties of feedback

+ Reduces influence of disturbances
+ Reduces effect of process variations
+ Does not require exact models
— Feeds sensor noise into the system
— May lead to instability, e.g.:
  - if the controller has too high gain
  - if the feedback loop contains too large time delays
    - from the process
    - from the controller implementation

The feedforward principle



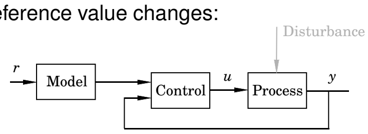
- Take corrective action before an error has occurred
- Measure the disturbance and compensate for it
- Use the fact that the reference signal is known and adjust the control signal to the reference signal
- Open loop

Properties of feedforward

+ Reduces effect of disturbances that cannot be reduced by feedback
+ Measurable signals that are related to disturbances
+ Allows faster set-point changes, without introducing control errors
— Requires good models
— Requires stable systems

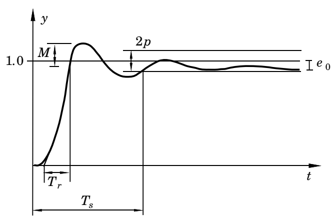
The servo problem

Focus on reference value changes:



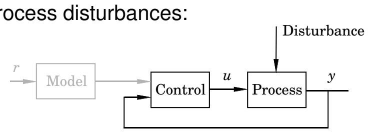
Typical design criteria:

- Rise time, T_r
- Overshoot, M
- Settling time, T_s
- Steady-state error, e_0
- ...



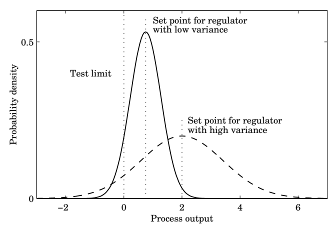
The regulator problem

Focus on process disturbances:

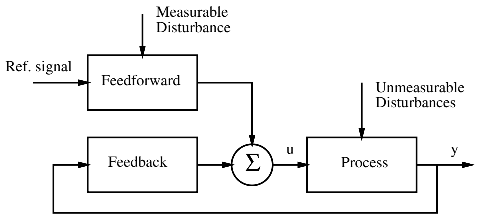


Typical design criteria:

- Output variance
- Control signal variance



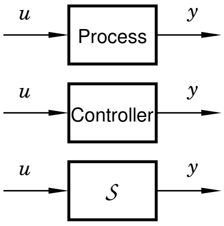
Putting it all together



Combination of **feedback** and **feedforward**

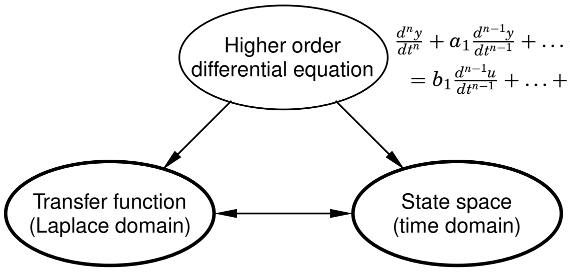
3. Modeling and Design

Modeling



- View all subsystems as “boxes” with inputs and outputs
- Linear, time-invariant (LTI) dynamical systems
- Continuous or discrete time

Continuous-time systems

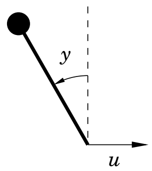


$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \ldots + a_n y = b_1 \frac{d^{n-1} u}{dt^{n-1}} + \ldots + b_n u$$

$$Y(s) = G(s)U(s)$$

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

Example: Inverted pendulum



Nonlinear differential equation from physical modeling:

$$\frac{d^2 y}{dt^2} = \omega_0^2 \sin y + ku \cos y$$

Linearized model around $y^0 = 0$ $(\sin y \approx y, \cos y \approx 1)$:

$$\frac{d^2 y}{dt^2} = \omega_0^2 y + ku$$

Inverted pendulum in state space form

Introduce state variables

- $x_1 = y$ (pendulum angle)
- $x_2 = \frac{dy}{dt}$ (pendulum angular velocity)

$$\frac{dx}{dt} = \begin{pmatrix} 0 & 1 \\ \omega_0^2 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ k \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

Inverted pendulum in transfer function form

Apply Laplace transform to differential equation:

$$s^2 Y(s) = \omega_0^2 Y(s) + kU(s)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{k}{s^2 - \omega_0^2}$$

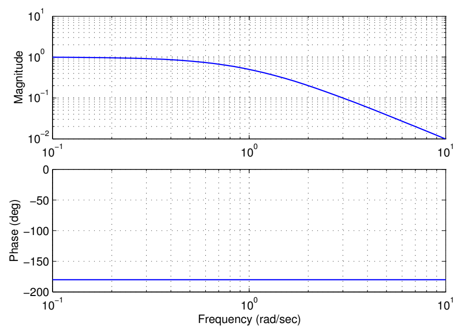
Or, from state space to transfer function:

$$G(s) = C(sI - A)^{-1}B = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} s & -1 \\ -\omega_0^2 & s \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ k \end{pmatrix} = \frac{k}{s^2 - \omega_0^2}$$

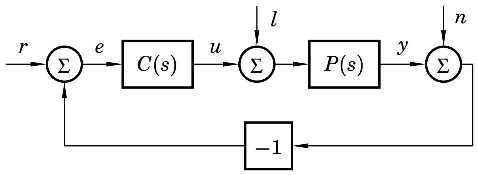
Frequency response: $G(i\omega)$

Frequency response

Plot $|G(i\omega)|$ and $\arg G(i\omega)$ for $\omega \in [0, \infty]$



Model-based design



Given $P(s)$, determine $C(s)$ such that the specifications on the closed-loop system are met. Common approaches:

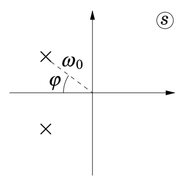
- Frequency domain design (loop shaping)
- Pole placement design
  - transfer function domain
  - state space domain
- Optimization-based methods (H_∞, LQG, ...)

Pole placement – transfer function domain

- Determine the required form of $C(s) = \frac{b_1 s^{n-1} + \ldots + b_n}{s^n + a_1 s^{n-1} + \ldots + a_n}$
- Calculate the closed loop system:

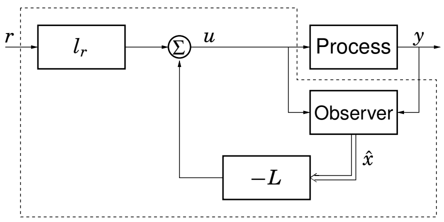
$$G_{cl}(s) = \frac{P(s)C(s)}{1 + P(s)C(s)}$$

- Choose the coefficients of $C(s)$ such that you get the desired closed-loop poles



- Large ω_0 ⇔ fast system
- Large φ ⇔ poorly damped system

Pole placement – state space domain



State feedback from an observer:

$$\frac{d\hat{x}}{dt} = A\hat{x} + Bu + K(y - C\hat{x})$$
$$u = -L\hat{x} + l_r r$$

Choose gain vectors L and K to give desired closed-loop poles

4. Empirical PID Control

PID control

- The oldest controller type
- The most widely used
  - Pulp & Paper 86%
  - Steel 93%
  - Oil refineries 93%
- Much to learn!!

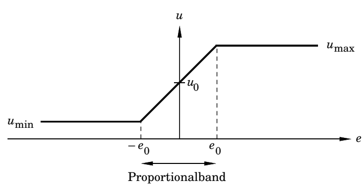
The textbook algorithm

$$u(t) = K\Big(e(t) + \frac{1}{T_i}\int^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\Big)$$

$$U(s) = K\Big(E(s) + \frac{1}{sT_i}E(s) + T_d sE(s)\Big)$$

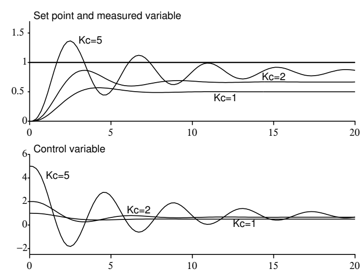
$$= P + I + D$$

Proportional term



$$u = \begin{cases} u_{\max} & e > e_0 \\ Ke + u_0 & -e_0 < e < e_0 \\ u_{\min} & e < -e_0 \end{cases}$$

Properties of P-control



- stationary error
- increased K means faster speed, increased noise sensitivity, worse stability

Errors with P-control

Control signal:

$$u = Ke + u_0$$

Error:

$$e = \frac{u - u_0}{K}$$

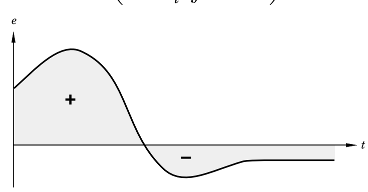
Error removed if:

- K equals infinity
- $u_0 = u$

Solution: Automatic way to obtain u_0

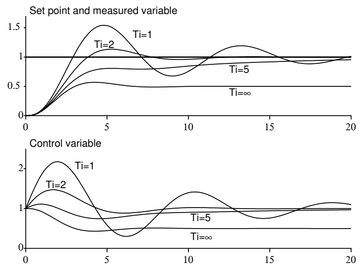
Integral term

$$u = Ke + u_0$$
$$u = K\left(e + \frac{1}{T_i}\int e(t)dt\right) \quad \text{(PI)}$$



Stationary error present $\rightarrow \int edt$ increases $\rightarrow u$ increases $\rightarrow y$ increases $\rightarrow$ the error is not stationary

Properties of PI-control

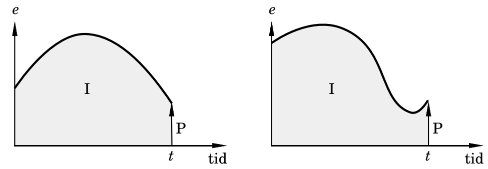


- removes stationary error
- smaller T_i implies worse stability, faster steady-state error removal

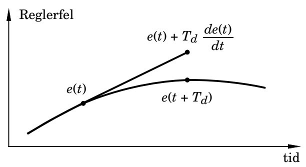
Prediction

A PI-controller contains no prediction

The same control signal is obtained for both these cases:



Derivative part



P:

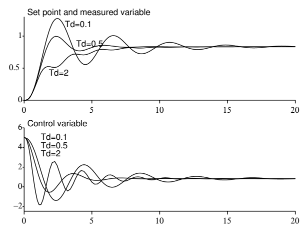
$$u(t) = K e(t)$$

PD:

$$u(t) = K \left(e(t) + T_d \frac{de(t)}{dt} \right) \approx K e(t + T_d)$$

T_d = Prediction horizon

Properties of PD-control



- T_d too small, no influence
- T_d too large, decreased performance

In industrial practice the D-term is often turned off.

Algorithm modifications

Modifications are needed to make the controller practically useful

- Limitations of derivative gain
- Derivative weighting
- Reference weighting
- Handle control signal limitations

Limitations of derivative gain

We do not want to apply derivation to high frequency measurement noise, therefore the following modification is used:

$$sT_d \approx \frac{sT_d}{1 + sT_d/N}$$

N = maximum derivative gain, often $10 - 20$

Derivative weighting

The reference is often constant for long periods of time

Reference often changed in steps → D-part becomes very large.

Derivative part applied on part of the reference or only on the measurement signal.

$$D(s) = \frac{sT_d}{1 + sT_d/N} (\gamma R(s) - Y(s))$$

Often, $\gamma = 0$

Reference weighting

An advantage to also use weighting on the reference.

$$u = K(r - y)$$

replaced by

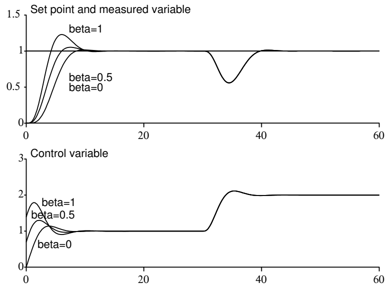
$$u = K(\beta r - y)$$

$0 \leq \beta \leq 1$

A way of introducing feedforward from the reference signal (position a closed loop zero)

Improved set-point responses.

Reference weighting



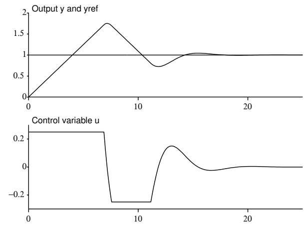
Control signal limitations

All actuators saturate.

Problems for controllers with integration.

When the control signal saturates the integral part will continue to grow – integrator windup.

When the control signal saturates the integral part will integrate up to a very large value. This may cause large overshoots.



Anti-windup

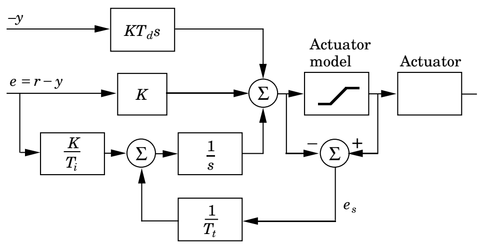
Several solutions exist:

- limit the reference variations (saturation never reached)
- conditional integration (integration is switched off when the control is far from the steady-state)
- tracking (back-calculation)

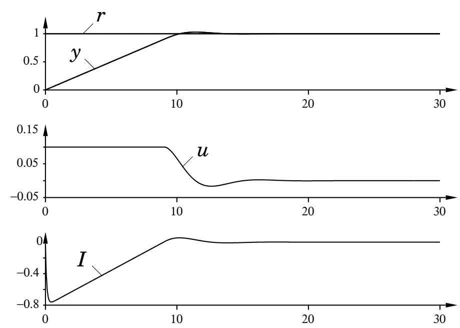
Tracking

- when the control signal saturates, the integral is recomputed so that its new value gives a control signal at the saturation limit
- to avoid resetting the integral due to, e.g., measurement noise, the recomputation is done dynamically, i.e., through a LP-filter with a time constant T_t.

Tracking



Tracking



Tuning

Parameters: $K, T_i, T_d, N, \beta, \gamma, T_t$

Methods:

- empirically, rules of thumb, tuning charts
- model-based tuning, e.g., pole-placement
- automatic tuning experiment
  - Ziegler-Nichols method
    - step response method
    - ultimate sensitivity method
  - relay method

Industrial reality

Canadian paper mill audit. Average paper mill: 2000 loops, 97% use PI, remaining 3% are PID, adaptive, ...

- default settings used
- poor performance due to bad tuning
- poor performance due to actuator problems

Real-Time Control

Part II – Sampled-Data and Networked Control

Anton Cervin

Department of Automatic Control
Lund University
Sweden

Outline – Part II

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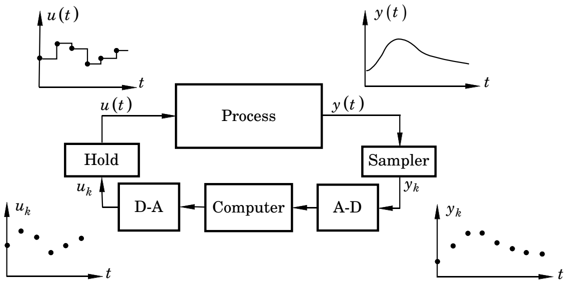
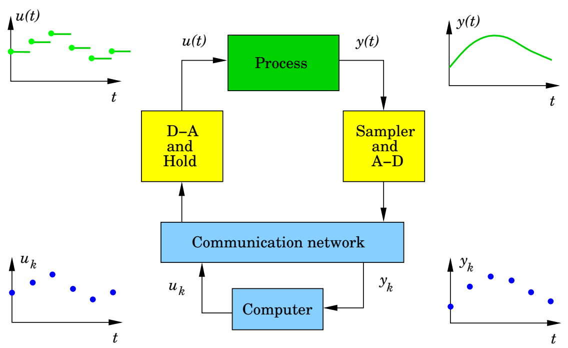
1. Introduction

Sampled-data control systems



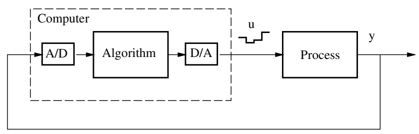
- Mix of continuous-time and discrete-time signals

Networked control systems

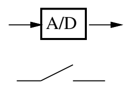


- Extra delay, possibly lost packets

Sampling



AD-converter acts as sampler



DA-converter acts as a hold device

Normally, zero-order-hold is used $\Rightarrow$ piecewise constant control signals

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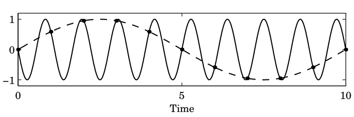
Aliasing



$\omega_s = \dfrac{2\pi}{h}$ = sampling frequency

$\omega_N = \omega_s/2$ = Nyquist frequency

Frequencies above the Nyquist frequency are folded and appear as low-frequency signals.

The fundamental alias frequency for a frequency f_1 is given by

$$f = |(f_1 + f_N) \bmod (f_s) - f_N|$$

Above: $f_1 = 0.9$, $f_s = 1$, $f_N = 0.5$, $f = 0.1$

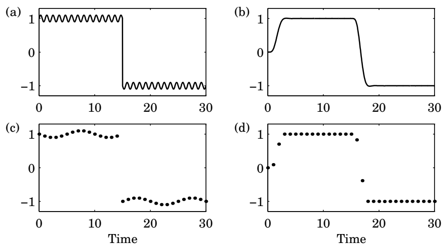
Anti-aliasing filter

Analog low-pass filter that eliminates all frequencies above the Nyquist frequency

- Analog filter
  - 2-6th order Bessel or Butterworth filter
  - Difficulties with changing h (sampling interval)
- Analog + digital filter
  - Fixed, fast sampling with fixed analog filter
  - Downsampling using digital LP-filter
  - Control algorithm at the lower rate
  - Easy to change sampling interval

The filter may have to be included in the control design

Example – Prefiltering



$\omega_d = 0.9$, $\omega_N = 0.5$, $\omega_{alias} = 0.1$

6th order Bessel with $\omega_B = 0.25$

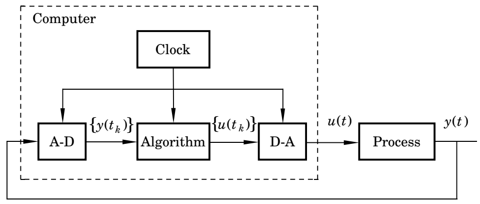
2. Design of digital controllers

Design approaches

Digital controllers can be designed in two different ways:

- Discrete-time design – sampled control theory
  - Sample the continuous system
  - Design a digital controller for the sampled system
    - Z-transform domain
    - state-space domain
- Continuous time design + discretization
  - Design a continuous controller for the continuous system
  - Approximate the continuous design
  - Use fast sampling

Sampled control theory

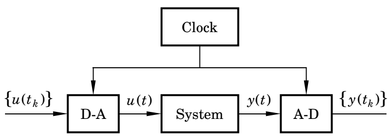


Basic idea: look at the sampling instances only

- System theory analogous to continuous-time systems
- Better performance can be achieved
- Potential problem with intersample behaviour

Sampling of systems

Look at the system from the point of view of the computer



Zero-order-hold sampling of a system

- Let the inputs be piecewise constant
- Look at the sampling points only
- Solve the system equation equation

Sampling a continuous-time system

System description

$$\begin{aligned}\frac{dx}{dt} &= Ax(t) + Bu(t)\\ y(t) &= Cx(t) + Du(t)\end{aligned}$$

Solve the system equation

$$\begin{aligned}x(t) &= e^{A(t-t_k)}x(t_k) + \int_{t_k}^{t} e^{A(t-s')}Bu(s')\,ds'\\ &= e^{A(t-t_k)}x(t_k) + \int_{t_k}^{t} e^{A(t-s')}\,ds'\,Bu(t_k) \quad (u \text{ const.})\\ &= e^{A(t-t_k)}x(t_k) + \int_{0}^{t-t_k} e^{As}\,ds\,Bu(t_k) \quad \text{(variable change)}\\ &= \Phi(t,t_k)x(t_k) + \Gamma(t,t_k)u(t_k)\end{aligned}$$

Periodic sampling

Assume periodic sampling, i.e. $t_k = k \cdot h$, then

$$\begin{aligned}x(kh+h) &= \Phi x(kh) + \Gamma u(kh)\\ y(kh) &= Cx(kh) + Du(kh)\end{aligned}$$

where

$$\begin{aligned}\Phi &= e^{Ah}\\ \Gamma &= \int_0^h e^{As}\,ds\,B\end{aligned}$$

Time-invariant linear system!

Example: Sampling of inverted pendulum

$$\begin{aligned}\frac{dx}{dt} &= \begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix} x + \begin{pmatrix} 0\\ 1\end{pmatrix} u\\ y &= \begin{pmatrix} 1 & 0\end{pmatrix} x\end{aligned}$$

We get

$$\begin{aligned}\Phi &= e^{Ah} = \begin{pmatrix} \cosh h & \sinh h\\ \sinh h & \cosh h\end{pmatrix}\\ \Gamma &= \int_0^h \begin{pmatrix} \sinh s\\ \cosh s\end{pmatrix} ds = \begin{pmatrix} \cosh h - 1\\ \sinh h\end{pmatrix}\end{aligned}$$

Several ways to calculate Φ and Γ. Matlab

Sampling a system with a time delay

Sampling the system

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t-\tau), \quad \tau \le h$$

we get the discrete-time system

$$x(kh+h) = \Phi x(kh) + \Gamma_0 u(kh) + \Gamma_1 u(kh-h)$$

where

$$\Phi = e^{Ah}$$

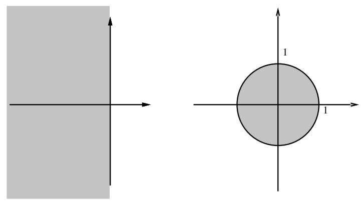
$$\Gamma_0 = \int_0^{h-\tau} e^{As}\, ds\, B$$

$$\Gamma_1 = e^{A(h-\tau)} \int_0^{\tau} e^{As}\, ds\, B$$

We get one extra state $\left(u(kh-h)\right)$ in the sampled system

Stability region

- In continuous time the stability region is the complex left half plane, i.e" the system is stable if all the poles are in the left half plane.
- In discrete time the stability region is the unit circle.



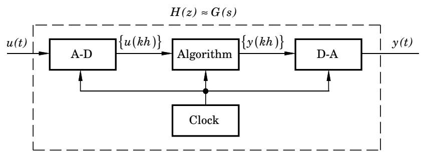
Digital control design

Similar to the continuous-time case, we can choose between

- frequency-domain design (loop shaping)
- pole-placement design
  - transfer function domain
  - state space domain
  - the poles are placed inside the unit circle
- optimal design methods (e.g. LQG)

Approximation of continuous-time design

Basic idea: Reuse the design



$G(s)$ is designed based on analog techniques

Want to get:

- A/D + Algorithm + D/A $\approx G(s)$

Methods:

- Approximate s, i.e., $H(z) = G(s')$
- Other methods (Matlab)

Approximation methods

Forward Difference (Euler's method)

$$\frac{dx(t)}{dt} \approx \frac{x(t+h)-x(t)}{h}$$

$$s' = \frac{z-1}{h}$$

Backward Difference

$$\frac{dx(t)}{dt} \approx \frac{x(t)-x(t-h)}{h}$$

$$s' = \frac{z-1}{zh}$$

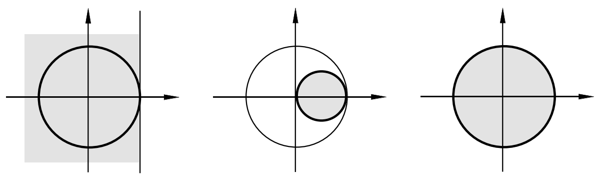
Tustin

$$\frac{\frac{dx(t)}{dt} + \frac{dx(t+h)}{dt}}{2} \approx \frac{x(t+h)-x(t)}{h}$$

$$s' = \frac{2}{h}\frac{z-1}{z+1}$$

Stability of approximations

How is the continuous-time stability region (left half plane) mapped?



Forward differences　　Backward differences　　Tustin

Discretization of the PID controller

Continuous PID controller with set-point weighting β and $\gamma = 0$:

$$U(s) = K\left(\beta R(s) - Y(s) + \frac{1}{sT_i}\left(R(s)-Y(s)\right) - \frac{sT_d}{1+sT_d/N}\,Y(s)\right)$$

Discretization

P-part:

$$P(k) = K(\beta r(k) - y(k))$$

Discretization

I-part:

$$I(t) = \frac{K}{T_I}\int_0^t (r(\tau) - y(\tau))d\tau$$

$$\frac{dI}{dt} = \frac{K}{T_I}(r(t) - y(t))$$

- Forward difference

$$\frac{I(k+1) - I(k)}{h} = \frac{K}{T_I}(r(k) - y(k))$$

```
I(k+1) := I(k) + (K*h/Ti)*(r(k)-y(k))
```

The I-part can be precalculated

- Backward difference

The I-part cannot be precalculated, $I(k) = f(r(k), y(k))$

Discretization

D-part (assume $\gamma = 0$):

$$D = K\frac{sT_D}{1 + sT_D/N}(-Y(s))$$

$$\frac{T_D}{N}\frac{dD}{dt} + D = -KT_D\frac{dy}{dt}$$

- Forward difference (unstable for small T_D)
- Backward difference

$$\frac{T_D}{N}\frac{D(k) - D(k-1)}{h} + D(k) = -KT_D\frac{y(k) - y(k-1)}{h}$$

$$D(k) = \frac{T_D}{T_D + Nh}D(k-1) - \frac{KT_DN}{T_D + Nh}(y(k) - y(k-1))$$

Discretization

Tracking:

```
v := P + I + D;
u := sat(v,umax,umin);
I := I + (K*h/Ti)*(r-y) + (h/Tt)*(u - v);
```

PID code

PID-controller with anti-windup ($\gamma = 0$).

```
r = ref.get();
y = yIn.get();
D = ad * D - bd * (y - yold);
v = K*(beta*r - y) + I + D;
u = sat(v,umax,umin);
uOut.put(u);
I = I + (K*h/Ti)*(r - y) + (h/Tt)*(u - v);
yold = y;
```

`ad` and `bd` are precalculated parameters given by the backward difference approximation of the D-term.

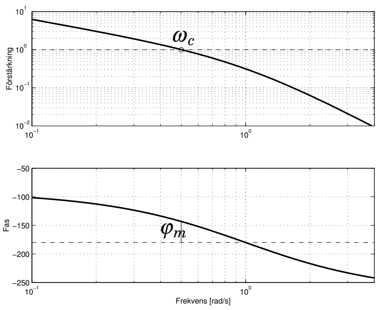
Choice of sampling interval

Nyquist's sampling theorem:

> *"We must sample at least twice as fast as the highest frequency we are interested in"*

- What frequencies are we interested in?

Typical loop transfer function $L(i\omega) = P(i\omega)C(i\omega)$:



- ω_c = cross-over frequency, φ_m = phase margin
- We should have $\omega_s \gg 2\omega_c$

Sampling interval rule of thumb

A sample-and-hold (S&H) circuit can be approximated by a delay of $h/2$.

$$G_{S\&H}(s) \approx e^{-sh/2}$$

This will decrease the phase margin by

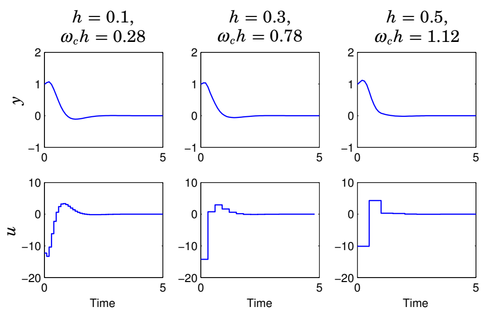
$$\arg G_{S\&H}(i\omega_c) = \arg e^{-i\omega_c h/2} = -\omega_c h/2$$

Assume we can accept a phase loss between 5° and 15°. Then

$$\boxed{0.15 < \omega_c h < 0.5}$$

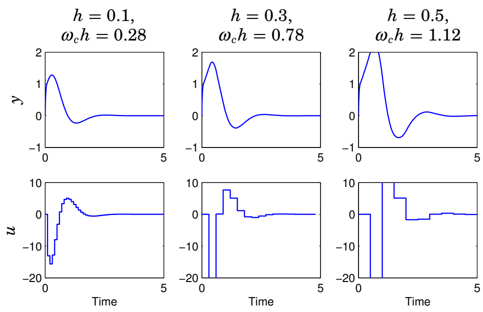
This corresponds to a Nyquist frequency about 6 to 20 times larger than the crossover frequency

Example: control of inverted pendulum



- Large $\omega_c h$ may seem OK, but beware!
  - Digital design assuming perfect model
  - Controller perfectly synchronized with initial disturbance

Pendulum with non-synchronized disturbance



Accounting for the anti-aliasing filter

Assume we also have a second-order Butterworth anti-aliasing filter with a gain of 0.1 at the Nyquist frequency. The filter gives an additional phase margin loss of $\approx 1.4\omega_c h$.

Again assume we can accept a phase loss of $5°$ to $15°$. Then

$$\boxed{0.05 < \omega_c h < 0.14}$$

This corresponds to a Nyquist frequency about 23 to 70 times larger than the crossover frequency

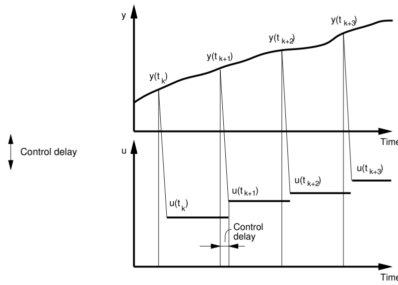
3. Delay and jitter

Computational delay

Problem: $u(k)$ cannot be generated instantaneously at time k when $y(k)$ is sampled

Computational delay (control delay) due to execution time

```
LOOP
  wait for clock interrupt;
  read analog input;
  perform calculations;
  set analog output;
END;
```



Minimizing the computational delay

General controller in state-space representation:

$$\begin{aligned} x(k+1) &= Fx(k) + Gy(k) + G_r r(k) \\ u(k) &= Cx(k) + Dy(k) + D_r r(k) \end{aligned}$$

Do as little as possible between the input and the output:

```
r = ref.get();
y = yIn.get();
/* Calculate Output */
u := u1 + D*y + Dr*r;
uOut.put(u);
/* Update State */
x := F*x + G*y + Gr*r;
u1 := C*x;
```

Sampling interval and delay rule of thumb

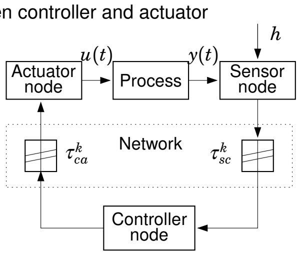
Assume that the delay is τ. This gives an additional phase margin loss of $-\omega_c \tau$. Extending our first rule of thumb we get

$$\boxed{0.15 < \omega_c (h + 2\tau) < 0.5}$$

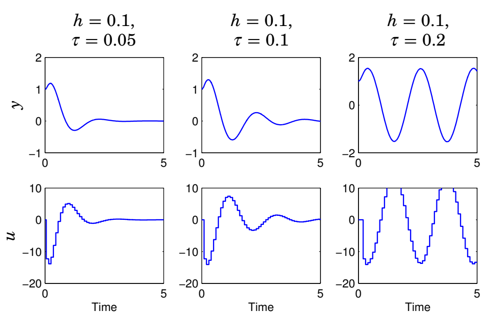
- If the delay is too large, we **must** decrease the speed of the controlled system (i.e. the cross-over frequency ω_c)
  - The delay imposes a fundamental performance limitation

Other sources of time delays

- Deadtime in the process
  - deadtime after the actuator
  - deadtime before the sensor
- Communication delays
  - between sensor and controller
  - between controller and actuator



Pendulum controller with time delay



- No delay compensation

Delay margin

Suppose the loop transfer function without delay has

- cross-over frequency ω_c
- phase margin φ_m

Phase margin loss due to delay:

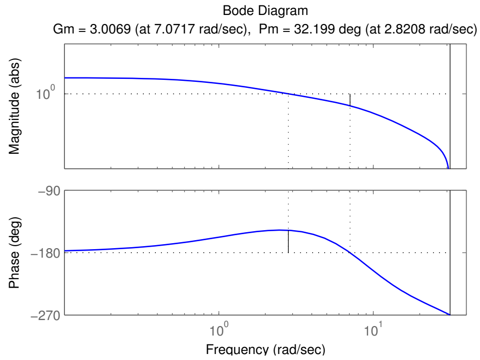
$$\arg e^{-i\omega_c\tau} = -\omega_c\tau$$

Closed-loop system stable if

$$\omega_c\tau < \varphi_m \quad \Leftrightarrow \quad \tau < \frac{\varphi_m}{\omega_c}$$

$\tau_m = \dfrac{\varphi_m}{\omega_c}$ is called the **delay margin**

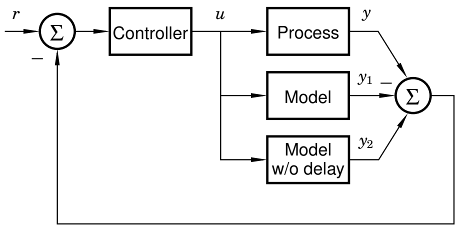
Example: delay margin for pendulum controller



$\varphi_m = 32°$, $\omega_c = 2.8$ rad/s $\Rightarrow$ $\tau_m = \frac{32\pi}{180 \cdot 2.8} = 0.2$

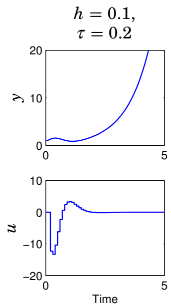
Delay compensation using Smith predictor

Idea: control against simulated model without delay:



- Requires **accurate** and **stable** model

Pendulum controller with Smith predictor



- The controller thinks that it is doing the right thing
- Based on feedforward rather than feedback

Delay compensation using pole placement

- Sample the model with the time delay
  - Gives a model with $d = \left\lceil \frac{\tau}{h} \right\rceil$ extra states
- Place all closed-loop poles within the unit circle
  - As a first attempt, place the extra poles in the origin
- Try to respect the rule of thumb

$$0.15 < \omega_c(h + 2\tau) < 0.5$$

Delay compensation in state space form

Assume a constant delay $\tau \leq h$.

Sampled model:

$$x(k+1) = \Phi x(k) + \Gamma_0 u(k) + \Gamma_1 u(k-1)$$

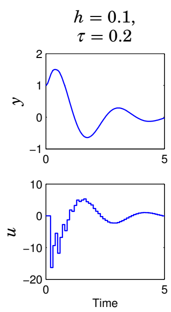
Observer and state feedback:

$$\hat{x}(k) = (I - KC)\Big(\Phi\hat{x}(k-1) + \Gamma_0 u(k-1) + \Gamma_1 u(k-2)\Big) + Ky(k)$$

$$u(k) = -L\begin{pmatrix}\hat{x}(k) \\ u(k-1)\end{pmatrix}$$

(Extension to $\tau > h$ is straightforward)

Pendulum controller with delay compensation



- Shaky response, nervous control signal
- $\omega_c(h + 2\tau) = 1.4$

Delay jitter

- In general, it is the **average** value of the delay that determines the degree of control performance degradation.
- However, jitter (i.e. time variation) in the delay makes it harder to compensate for.
- Jitter can be removed at the expense of more delay, using buffers.
  - Trade-off between delay and jitter
  - Most often, a short but jittery delay is preferred over a long but constant delay
  - But, counter-examples exist!

Jitter compensation in state space form

Assume a **time-varying** delay $\tau_k \leq h$.

Sampled model:

$$x(k+1) = \Phi x(k) + \Gamma_0(\tau_k)u(k) + \Gamma_1(\tau_k)u(k)$$

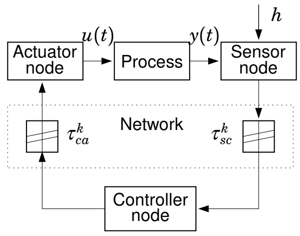
Problem: current delay τ_k is not known at time k! Solution:

- Base the L calculation on the **average** delay, $\mathrm{E}\{\tau\}$
- Measure the actual delay at the output
- Make the observer time-varying

$$\hat{x}(k) = (I - KC)\Big(\Phi\hat{x}(k-1) + \Gamma_0(\tau_k)u(k-1) + \Gamma_1(\tau_k)u(k-2)\Big) + Ky(k)$$

$$u(k) = -L \begin{pmatrix} \hat{x}(k) \\ u(k-1) \end{pmatrix}$$

Jitter compensation in networked systems



Part of the current loop delay (τ_{sc}) can now be measured!

- Time-varying state feedback L_k based on $\tau_{sc}^k + \mathrm{E}\{\tau_{ca}\}$
- Let the actuator node record the total delay
- The total delay is communicated back to the controller
- Make the observer time-varying as before

Real-Time Control

Part III – Integrated Control and Scheduling

Anton Cervin

Department of Automatic Control
Lund University
Sweden

Outline

1. Task models for control
2. Handling overruns

1. Task models for control

The simple task model

In the simple task model, a task τ_i is described by

- a fixed period T_i
- a fixed, known worst-case execution time C_i
- a hard deadline $D_i = T_i$

Is this model suitable for control tasks?

Fixed period?

Not necessarily:

- Some controllers are not sampled against time
  - Engine controllers
- Some controllers may switch between different modes with different sampling intervals
  - Hybrid controllers
- The control task could be triggered sporadically by measurements arriving over the network

Fixed and known WCET?

Not always:

- WCET analysis is a very hard problem
  - May have to use estimates or measurements
- Some controllers may switch between different modes with different execution times
  - Hybrid controllers
- Some controllers can explicitly trade off execution time for quality of control
  - "Any-time" algorithms
  - Model-predictive controllers (MPC)
  - Long execution time ⇔ High quality of control

Hard deadlines?

Most often not:

- Controller deadlines are *firm* rather than hard
  - OK to miss a few outputs, but not too many in a row
- $D_i = T_i$ is a quite arbitrary choice
- Really depends on what happens when a deadline is missed
  - Late completion – often OK
  - Aborted computation (no new output) – worse

Inputs and outputs?

Completely missing from the simple task model:

- When are the inputs (measurement signals) read?
  - Beginning of period?
  - Beginning of task execution?
- When are the outputs (control signals) written?
  - End of period?
  - End of task execution?
  - Other time?

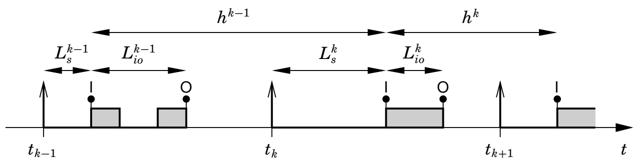
A simple control task

```
t = now();
while (1) {
  read_input();
  control_algorithm();
  write_output();
  t = t + T;
  wait_until(t);
}
```

- The input and output operations may be synchronous or asynchronous
- Trade-off between delay and jitter

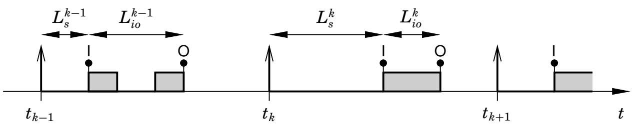
Control task timing

Assuming task-triggered inputs and outputs:



- L_s^k – sampling latency in period k
- L_{io}^k – input-output latency in period k
- h^k – actual sampling interval in period k

Jitter



- Absolute sampling jitter: $J_s \stackrel{\text{def}}{=} \max_k L_s^k - \min_k L_s^k$
- Absolute input-output jitter: $J_{io} \stackrel{\text{def}}{=} \max_k L_{io}^k - \min_k L_{io}^k$
- J_s and J_{io} can be found using scheduling theory

Computing the jitter

Under fixed-priority scheduling, exact response-time analysis can be used:

- worst-case analysis [Joseph & Pandya, 1986]
- best-case analysis [Redell & Sanfridson, 2002]:

$$R_i^b = C_i + \sum_{j \in hp(i)} \left\lceil \frac{R_i^b - T_j}{T_j} \right\rceil C_j$$

Under EDF, response-time analysis is more complicated (and the exact best-case is not known)

Computing the jitter

Sampling jitter:

- Replace task τ_i by "sampling task" $\tilde{\tau}_i$ with $\tilde{C}_i = \epsilon$
- $\max L_{si} = \tilde{R}_i$
- $\min L_{si} = 0$
- $J_{si} = \tilde{R}_i$

Input-output jitter:

- $\max L_{ioi} = R_i$
- $\min L_{ioi} = R_i^b$
- $J_{ioi} = R_i - R_i^b$

Subtask scheduling

A control algorithm normally consists of two parts:

```
while (1) {
  read_input();
  calculate_output();
  write_output();
  update_state();
  ...
}
```

Idea: schedule the two parts as separate tasks

- reduce delay
- reduce jitter

Task models

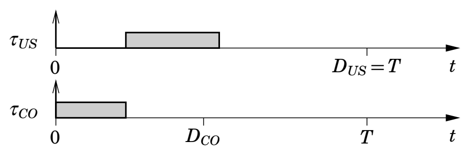
Each control task τ is divided into two subtasks:

- τ_{CO} – Calculate output
- τ_{US} – Update state
- Input and output operations are ignored in the analysis

Two possible task models:

- Priority-constrained – deadline-monotonic scheduling
- Offset model – EDF scheduling

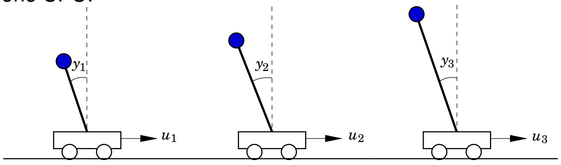
Deadline assignment

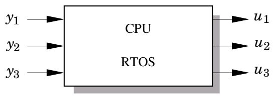


- $D_{CO} < D_{US}$
- We want to minimize D_{CO}. Iterative algorithm:
  1. Start by assigning $D_{CO} := T - C_{US}$ for all tasks
  2. Assign deadline-monotonic priorities to all subtasks
  3. Calculate the response time R of each subtask
  4. Assign $D_{CO} := R_{CO}$ for all tasks
  5. Repeat from 2 until no further improvement.

Example

Suppose you want to control three inverted pendulums using one CPU:

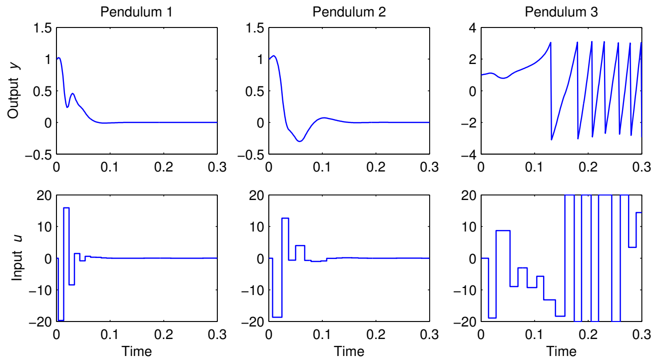




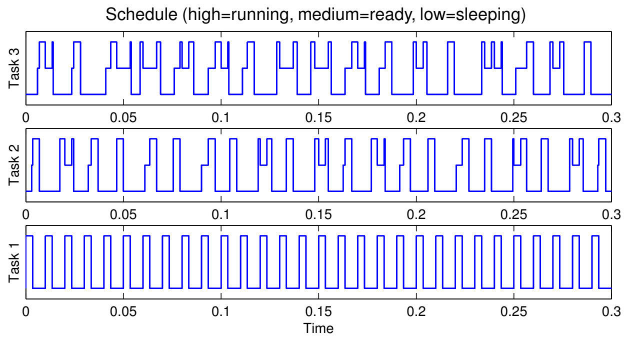
Example

- Discrete-time LQG controllers
- Execution time: $C_i = 3.5$ ms
- Sampling intervals: $(h_1, h_2, h_3) = (10, 14.5, 17.5)$ ms
- Control delay of 3.5 ms assumed in the design

Simulation under RM scheduling



Simulation under RM scheduling



- Large delay and jitter for controller 3

Subtask scheduling analysis

Each pendulum controller is divided into two subtasks:

- Calculate output: $C_{CO} = 1.5$ ms
- Update state: $C_{US} = 2.0$ ms

First iteration of algorithm:

	T	D	C	R
τ_{CO1}	10.0	8.0	1.5	1.5
τ_{US1}	10.0	10.0	2.0	3.5
τ_{CO2}	14.5	12.5	1.5	5.0
τ_{US2}	14.5	14.5	2.0	7.0
τ_{CO3}	17.5	15.5	1.5	8.5
τ_{US3}	17.5	17.5	2.0	14.0

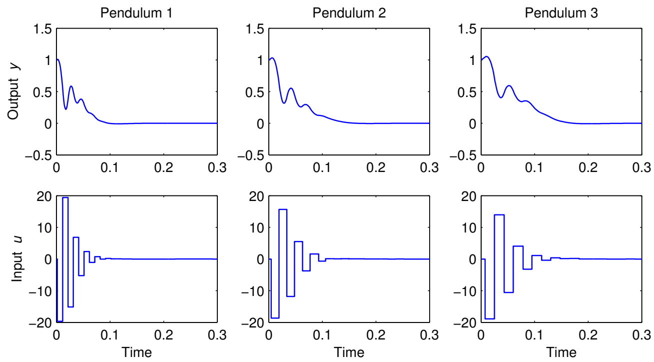
Subtask scheduling analysis

Third iteration (converged):

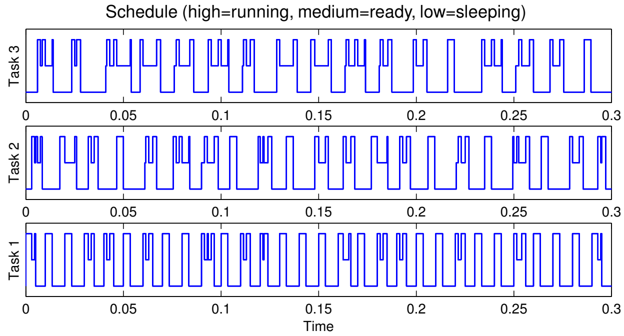
	T	D	C	R
τ_{CO1}	10.0	1.5	1.5	$\boxed{1.5}$
τ_{US1}	10.0	10.0	2.0	6.5
τ_{CO2}	14.5	3.0	1.5	$\boxed{3.0}$
τ_{US2}	14.5	14.5	2.0	8.5
τ_{CO3}	17.5	4.5	1.5	$\boxed{4.5}$
τ_{US3}	17.5	17.5	2.0	14.0

New worst-case input-output latencies: 1.5, 3.0, 4.5 ms.

Simulation subtask scheduling



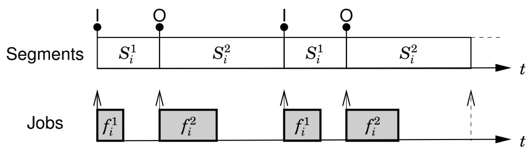
Simulation subtask scheduling



- More context switches

Extension: the Control Server model

- Schedule the inputs and outputs using kernel events (interrupts)
- Schedule the jobs of the subtasks (segments) using a modified Constant Bandwidth Server



A control server task

The control server has been implemented in Shark. Example task:

```
while (1) {
  // first segment
  r = read_input(0);
  y = read_input(1);
  u = calculate_output(r,y);
  write_output(0,u);
  task_endsegment();

  // second segment
  update_state();
  task_endsegment();
}
```

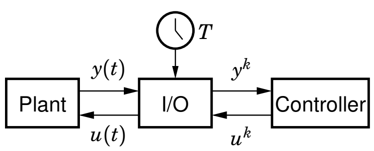
2. Handling overruns

Question

What to do in case of a controller execution overrun?

- **Abort** the computation?
- Continue the computation in the next period, but **skip** the next sample?
- Continue the computation in the next period, but **queue** the next sample?

Simple analysis of overruns



- Continuous-time plant
- Discrete-time controller with time-varying response time
- Synchronized, time-triggered inputs and outputs
  - Real-time control model assumed in Liu and Layland (1973)
  - Control server model
  - At least one sample input-output delay

Motivation

- Robustness against design faults
  - Very difficult to predict the worst-case response time
  - Treat the overrun as an exception and deal with it
  - Feedback in the computer system
- More efficient use of resources
  - The worst-case response time may be very rare
  - Trade-off between sampling period and risk of overruns

Stochastic Control Analysis

- Assume response-time probability density function $f_c(x)$
- Assume response times independent between samples
- Formulate jump linear system

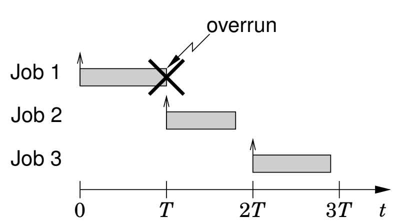
$$x(k+1) = A_i x(k) + B_i v(k)$$

  - x – plant, I/O and controller states
  - v – sampled noise process
  - i – current Markov node
- Evaluate quadratic cost function (performance index)

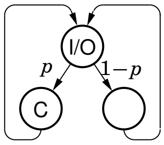
$$J = \lim_{t\to\infty} \frac{1}{t} \int_0^t x^T(\tau) Q x(\tau)\, d\tau$$

  - Well-known theory (1960s)
  - Jitterbug toolbox (Lincoln and Cervin, 2002)

The Abort Strategy



The Abort Strategy – Markov Chain



- Probability of no overrun:

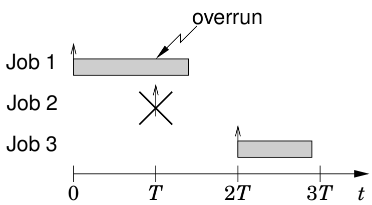
$$p = \int_0^T f_c(x)dx$$

The Abort Strategy – Implementation

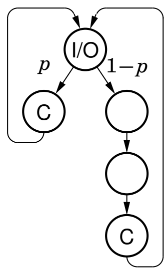
```
t := Clock();
loop
  select
    delay until t + T;
  then abort
    Read_input();
    Compute_control();
    Write_output();
  end
  t := t + T;
  delay until t;
end
```

- Possible in ADA, Real-Time Java, Shark(??)

The Skip Strategy



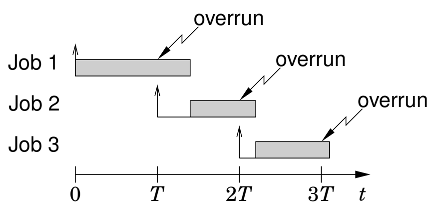
The Skip Strategy



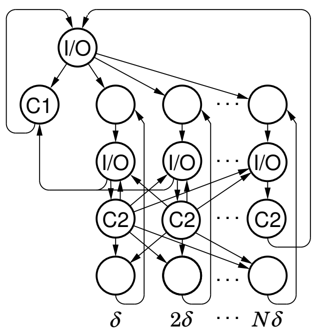
The Skip Strategy – Implementation

```
t := Clock();
late := false;
loop
  if not late then
    Read_input();
    Compute_control();
    Write_output();
  end
  t := t + T;
  if Clock() < t
    delay until t;
    late = false;
  else
    late = true;
  end
end loop
```

The Queue Strategy



The Queue Strategy – Markov Chain



- Queued execution time discretized with interval $\delta = T/N$
- C1 – ctrl using current sample, C2 – ctrl using old sample

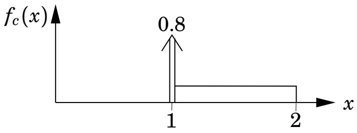
The Queue Strategy – Implementation

```
t := Clock();
loop
  Read_input();
  Compute_control();
  Write_output();
  t := t + T;
  delay until t;
end loop
```

- The textbook implementation of a control loop

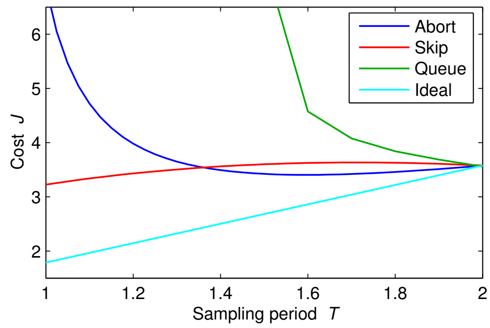
Example

- Plant: $P(s) = \frac{1}{s}$
- LQG controller with period T assuming one-sample delay and cost function $J = \lim_{t\to\infty} \frac{1}{t}\int_0^t y^2(\tau)\,d\tau$
- Response-time probability density function:



- Evaluate J for different $T \in [1, 2]$ and different strategies
- Ideal case without overruns: $J = \frac{\sqrt{3}+9}{6}T \approx 1.79T$

Example – Results



Example – Conclusions

- The Queue strategy performs the worst
  - Domino effect (c.f. EDF scheduling)
- The Skip strategy is the most robust one
  - Stable for all periods
  - Optimal period = nominal period $\Rightarrow$ easy to design the controller (holds for many examples)
  - Easy to implement
  - Easy to analyze

Real-Time Control

Part IV – Computer Control Exercise

Anton Cervin

Department of Automatic Control
Lund University
Sweden

The Problem

Control of a (simulated) ball and beam process

Instructions

1. Design a continuous-time PID controller

$$C(s) = K\left(1 + \frac{1}{sT_i} + sT_d\right)$$

   that gives a fast ($\omega_0 \approx 3$ rad/s) and well-damped step response. Use pole placement design (see below)
2. Discretize the controller, after introducing
   - maximum derivative gain N
   - reference weighting β
   - tracking with time-constant T_t
3. Select a suitable sampling interval
4. Write a program that implements the controller
5. Try the controller against the simulated ball and beam and tune its parameters

Process Description

- Input u: beam angle (e.g. actuated by a servo motor)
- Output y: ball position (measured using e.g. a camera)

The process dynamics are given by

$$P(s) = \frac{10}{s^2}$$

To make it more interesting, the simulated process also has

- control signal limitation $u \in [-5.0,\ 5.0]$
- some process and measurement disturbances
- some unmodeled dynamics

Pole placement design

1. Compute the closed-loop transfer function

$$G_{cl}(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} = \frac{p(s)}{q(s)}$$

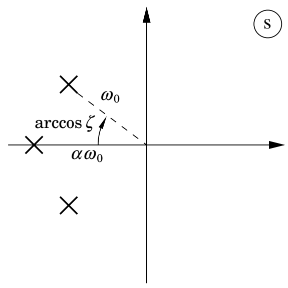
   (i.e. simplify the expression for $G_{cl}(s)$ to get a quotient between two polynomials in s)
2. The desired characteristic polynomial can be expressed as

$$(s + \alpha\omega_0)(s^2 + 2\zeta\omega_0 s + \omega_0^2)$$

   Set this equal to $q(s)$ and solve for K, T_i and T_d as expressions in ω_0, ζ and α!

Pole placement

Geometrical interpretation of the characteristic polynomial $(s + \alpha\omega_0)(s^2 + 2\zeta\omega_0 s + \omega_0^2)$:



Controller Implementation

- Implement the controller as a periodic thread in Shark
- After `#include <ballbeam.h>`, the following commands are available:

```
double get_position();  // read position of the ball
void set_angle(double); // set the angle of the beam
double get_reference(); // read the reference value
```