

When reachability-based secrecy implies equivalence-based secrecy in security protocols

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Context

Verifying security protocols

- programs that ensure secure communications
- notoriously difficult to design

The intruder controls the network

- can see all messages
- can modify and send new messages
- can intercept messages

Two standard notions of secrecy

Reachability-based (syntactic) secrecy	Equivalence-based (strong) secrecy
$P \rightarrow^* s$	$P(M) \approx P(M') \quad \forall M, M'$
decidable classes	stronger security notion
many available tools	closer to computational secrecy

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Goal: Relating the two notions of secrecy

Motivations:

- Few results for strong secrecy
- [ESOP'05] in a cryptographic setting, accessibility-based secrecy implies indistinguishability, for asymmetric encryption.

Goal

syntactic secrecy implies strong secrecy
 $P \xrightarrow{*} s \quad \Rightarrow \quad P(M) \approx P(M')$

Passive case:

- probabilistic encryption
- the secret does not occur in keys

Active case:

- probabilistic encryption
- ground keys
- no tests on the secret

Messages

Messages are modeled by **terms**.

- concatenation: $\langle m_1, m_2 \rangle$
- probabilistic symmetric encryption: $\text{enc}(m, k, r)$
- probabilistic asymmetric encryption: $\text{enca}(m, \text{pub}(a), r)$
- digital signatures: $\text{sign}(m, \text{priv}(a))$
- + **constants**, **variables** and **names** (from a set \mathcal{N}).

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We equip the algebra with an **equational theory**:

$$\left\{ \begin{array}{l} \pi_1(\langle z_1, z_2 \rangle) = z_1 \\ \pi_2(\langle z_1, z_2 \rangle) = z_2 \\ \text{dec}(\text{enc}(z_1, z_2, z_3), z_2) = z_1 \\ \text{deca}(\text{enca}(z_1, \text{pub}(z_2), z_3), \text{priv}(z_2)) = z_1 \\ \text{check}(z_1, \text{sign}(z_1, \text{priv}(z_2)), \text{pub}(z_2)) = \text{ok} \\ \text{retrieve}(\text{sign}(z_1, z_2)) = z_1 \end{array} \right.$$

Deducibility

Frame:

$$\underbrace{\nu\tilde{n}}_{\text{fresh values}} \left\{ \underbrace{M_1/x_1, \dots, M_l/x_l}_{\text{sequence of messages (terms)}} \right\}$$

Deduction

System:

$$\frac{}{\nu\tilde{n}.\sigma \vdash x\sigma} \quad x \in \text{dom}(\sigma) \qquad \frac{}{\nu\tilde{n}.\sigma \vdash s} \quad s \in \mathcal{N} \setminus \tilde{n}$$
$$\frac{\nu\tilde{n}.\sigma \vdash t_1 \quad \dots \quad \nu\tilde{n}.\sigma \vdash t_r}{\nu\tilde{n}.\sigma \vdash f(t_1, \dots, t_r)} \qquad \frac{\nu\tilde{n}.\sigma \vdash t \quad t =_E t'}{\nu\tilde{n}.\sigma \vdash t'}$$

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Example: k and $\langle k, k' \rangle$ are deducible from the frame

$$\nu k, k', r. \left\{ \text{enc}(k, k', r)/x, k'/y \right\}.$$

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Definition: A term M is *syntactically secret* in φ if $\varphi \not\vdash M$.

Static equivalence \approx_s

Definition: $\phi_1 \approx_s \phi_2$ iff $\text{dom}(\phi_1) = \text{dom}(\phi_2)$ and for every couple of terms (M, N) such that $\text{var}(M, N) \subseteq \text{dom}(\phi_1)$,

$$M\phi_1 = N\phi_1 \quad \Leftrightarrow \quad M\phi_2 = N\phi_2.$$

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$\phi_1 = \nu k \{ \text{enc}(\text{yes}, k)/x, k/y \}$ and $\phi_2 = \nu k \{ \text{enc}(\text{no}, k)/x, k/y \}$

are not statically equivalent

since $\text{dec}(x, y) = \text{yes}$ for ϕ_1 , while $\text{dec}(x, y) = \text{no}$ for ϕ_2 .

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Definition: s is *strongly secret* in ϕ if

$$\phi(M/s) \approx_s \phi(M'/s) \quad \forall M, M'$$

Syntactic secrecy is weaker than strong secrecy!

Examples of ψ_i s.t. $\psi_i \not\equiv \mathbf{s}$ and $\psi_i(M) \not\approx_s \psi_i(M')$ for some M, M' .

Probabilistic
encryption

$$\psi_1 = \nu k, r. \{ \text{enc}(\mathbf{s}, k, r) / x, \text{enc}(n, k, r) / y \} \quad x = y$$

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Retrieve rule

$$\psi_4 = \{ \text{sign}(\mathbf{s}, \text{priv}(a)) / x, \text{pub}(a) / y \} \quad \text{check}(n, x, y) = \text{ok}$$

Syntactic secrecy vs strong secrecy (passive case)

A frame $\varphi = \nu\tilde{n}.\sigma$ is *well-formed* if

- encryption is **probabilistic**,
- **s** is not part of a **key** and,
- φ does not contain **destructor** symbols.

Theorem 1 For any well-formed frame, weak secrecy is equivalent to strong secrecy, that is

$$\varphi \not\vdash s \quad \text{iff} \quad \varphi(M/s) \approx_s \varphi(M'/s)$$

for all M, M' closed terms public wrt φ .

Proof sketch (passive case)

Base case Lemma: $u\sigma(M/s) = v\sigma(M/s)$ implies $u\sigma = v\sigma$.

Transfer Lemma: $\varphi = \nu\tilde{n}.\sigma$ well-formed frame, $\varphi \not\vdash s$.

If $u\sigma(M/s) \rightarrow w$, then

- there exists $\varphi' = \nu\tilde{n}.\sigma'$ extending φ , preserving deducible terms and,
- such that $w = w'\sigma'(M/s)$ and $u\sigma \rightarrow w'\sigma'$.

Hence, $u\sigma(M/s)\downarrow = u'\sigma'(M/s)$ and $v\sigma(M/s)\downarrow = v'\sigma'(M/s)$.

Applied-pi calculus (1) [Abadi Fournet]

(Plain) processes are defined by the grammar:

$P, Q, R :=$	processes
$\mathbf{0}$	null process
$P Q$	parallel composition
$!P$	replication
$\nu n.P$	name restriction
$[M = N].P$	conditional
$p(z).P$	message input
$\bar{p}\langle M \rangle.P$	message output

Structural equivalence rules:

$A \equiv A \mathbf{0}$
$A (B C) \equiv (A B) C$
$A B \equiv B A$
$!P \equiv P !P$
$\nu n.\mathbf{0} \equiv \mathbf{0}$
$\nu u.\nu v.A \equiv \nu v.\nu u.A$
$A \nu u.B \equiv \nu u.(A B)$ if $u \notin \text{fv}(A) \cup \text{fn}(A)$

Internal reduction is given by the rules:

COMM	$\bar{p}\langle x \rangle.P p(x).Q \rightarrow P Q$
COND	$[M = M].P \rightarrow P$

Applied-pi calculus (2)

Extended processes are defined by the grammar:

$A, B :=$	extended processes
P	plain process
$A B$	parallel composition
$\nu n.A$	name restriction
$\nu x.A$	variable restriction
$\{M/x\}$	active substitution

Additional structural equivalence rules:

ALIAS	$\nu x.\{M/x\} \equiv \mathbf{0}$
SUBST	$\{M/x\} A \equiv \{M/x\} A\{M/x\}$
REWRITE	$\{M/x\} \equiv \{N/x\}$ if $M =_E N$

Labeled reduction is defined by the following rules:

IN	$p(x).P \xrightarrow{p(M)} P\{M/x\}$	SCOPE	$\frac{A \xrightarrow{\alpha} A'}{\nu u.A \xrightarrow{\alpha} \nu u.A'} \quad u \text{ not in } \alpha$
OUT-ATOM	$\bar{p}\langle u \rangle.P \xrightarrow{\bar{p}\langle u \rangle} P$	PAR	$\frac{A \xrightarrow{\alpha} A'}{A B \xrightarrow{\alpha} A' B} \quad \text{condition (*)}$
OPEN-ATOM	$\frac{A \xrightarrow{\bar{p}\langle u \rangle} A'}{\nu u.A \xrightarrow{\nu u.\bar{p}\langle u \rangle} A'} \quad u \neq p$	STRUCT	$\frac{A \equiv B \quad B \xrightarrow{\alpha} B' \quad B' \equiv A'}{A \xrightarrow{\alpha} A'}$

where u is a metavariable that ranges over names and variables.

Modeling protocols

The Yahalom protocol:

$$A \Rightarrow B : A, N_a$$

$$B \Rightarrow S : B, \{A, N_a, N_b\}_{K_{bs}}$$

$$S \Rightarrow A : \{B, K_{ab}, N_a, N_b\}_{K_{as}}, \{A, K_{ab}\}_{K_{bs}}$$

$$A \Rightarrow B : \{A, K_{ab}\}_{K_{bs}}$$

$$P_A = \nu n_a. \bar{p}\langle a, n_a \rangle. p(z_a). [b = \pi_1(\text{dec}(\pi_1(z_a), k_{as}))].$$

$$[n_a = \pi_1(\pi_2(\pi_2(\text{dec}(\pi_1(z_a), k_{as})))]. \bar{p}\langle \pi_2(z_a) \rangle$$

$$\xrightarrow{\nu z. \bar{p}\langle z \rangle} \nu n_a. (\{ \langle a, n_a \rangle / z \} \mid p(z_a). [b = u_b]. [n_a = u_{n_a}]. \bar{p}\langle \pi_2(z_a) \rangle)$$

$$\xrightarrow{p(\langle b, z \rangle)} \nu n_a. (\{ \langle a, n_a \rangle / z \} \mid [b = \pi_1(\text{dec}(b, k_{as}))]. [n_a = u'_{n_a}]. \bar{p}\langle a, n_a \rangle)$$

Definitions of secrecy

We say that s is *syntactically secret* in P if, for every P' such that $P \Rightarrow^* P'$, s is not deducible from P' , that is $\varphi(P') \not\vdash s$.

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Labeled bisimilarity (\approx_l) is the largest symmetric relation \mathcal{R} on closed extended processes such that $A \mathcal{R} B$ implies:

1. $\varphi(A) \approx \varphi(B)$;
2. if $A \rightarrow A'$ then $B \rightarrow^* B'$ and $A' \mathcal{R} B'$, for some B' ;
3. if $A \xrightarrow{\alpha} A'$ then $B \rightarrow^* \xrightarrow{\alpha} \rightarrow^* B'$ and $A' \mathcal{R} B'$, for some B' .

We say that \mathbf{s} is *strongly secret* in P if $P^{(M/\mathbf{s})} \approx_l P^{(M'/\mathbf{s})}$ for any closed terms M, M' public wrt P .

Hypotheses

A process P is *well-formed* if:

- encryption is **probabilistic**
- there are **no destructors** above constructors, nor above **s**
- the **keys are ground**
- for any **test** $[M = N]$, the terms M, N are
 - name,
 - constant,
 - or of the form $\pi^1(\text{dec}(\dots \pi^n(\text{dec}(\pi^{n+1}(z), k_n)) \dots, k_1))$,
where the π^i are words on $\{\pi_1, \pi_2\}$.

Ground keys

Counter-example for non ground keys:

$$P = \nu k, r, r'. (\bar{c}\langle \text{enc}(\mathbf{s}, k, r) \rangle \mid c(z). \bar{c}\langle \text{enc}(a, \text{dec}(z, k), r') \rangle)$$

$$\rightarrow \nu k, r, r'. (\{\text{enc}(\mathbf{s}, k, r) / z\} \mid \bar{c}\langle \text{enc}(a, \mathbf{s}, r') \rangle)$$

$$\xrightarrow{\nu z'. \bar{c}\langle z' \rangle} \nu k, r, r'. \{\text{enc}(\mathbf{s}, k, r) / z, \text{enc}(a, \mathbf{s}, r') / z'\}$$

Tests over s

Conditionals should not test on the secret:

$$\begin{aligned} P &= \nu k, r. (\bar{p} \langle \text{enc}(s, k, r) \rangle \mid p(z). [\text{dec}(z, k) = a]. \bar{p} \langle \text{ok} \rangle) \\ &\rightarrow \nu k, r. (\{ \text{enc}(s, k, r) / z \} \mid [s = a]. \bar{p} \langle \text{ok} \rangle) \\ P^{(a/s)} &\not\approx_l P^{(b/s)}. \end{aligned}$$

There may be hidden tests on the secret. Yahalom protocol, again:

$$\begin{aligned} A \Rightarrow B &: A, N_a \\ B \Rightarrow S &: B, \{A, N_a, N_b\}_{K_{bs}} \\ S \Rightarrow A &: \{B, K_{ab}, N_a, N_b\}_{K_{as}}, \{A, K_{ab}\}_{K_{bs}} \\ A \Rightarrow B &: \{A, K_{ab}\}_{K_{bs}} \end{aligned}$$

Hence we mark the test $[n_a = \pi_1(\pi_2(\pi_2(\text{dec}(\pi_1(z_a), k_{as}))))]$.

→ We construct a set of "potentially dangerous" tests \mathcal{M}_t .

Syntactic secrecy vs strong secrecy (active case)

Definition: A protocol *does not test over* \mathfrak{s} if for any test $[M = N]$ or $[N = M]$ such that $M \in \mathcal{M}_t$, N is a restricted name.

Theorem 2 For well-formed processes

- which do not test over \mathfrak{s}
- + some **syntactic condition** to ensure that messages sent through the network do not contain destructor directly above the secret.

then **syntactic secrecy is equivalent with strong secrecy.**

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Proof elements:

- Any frame produced by the protocol is an extended well-formed frame.
- If $[T_1 = T_2]$ is a test in P , then $T_1\sigma(M/\mathbf{s}) =_E T_2\sigma(M/\mathbf{s})$ implies $T_1\sigma(M'/\mathbf{s}) =_E T_2\sigma(M'/\mathbf{s})$.

Conclusion

We have proved that **syntactic secrecy implies strong secrecy**

- in the passive case, for symmetric and asymmetric encryption and digital signatures;
- in the active case, for symmetric encryption, under some (rather tight) conditions;

Application: Yahalom, Wide Mouthed Frog, symmetric key
Needham-Schroeder protocols are strongly secret.

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Further work:

- analyse more primitives (symmetric encryption, ...)
- relax some conditions (allow more tests, ...)

Related work:

- H. Hüttel. *Deciding framed bisimilarity*.
- B. Blanchet. *Automatic Proof of Strong Secrecy for Security Protocols*.