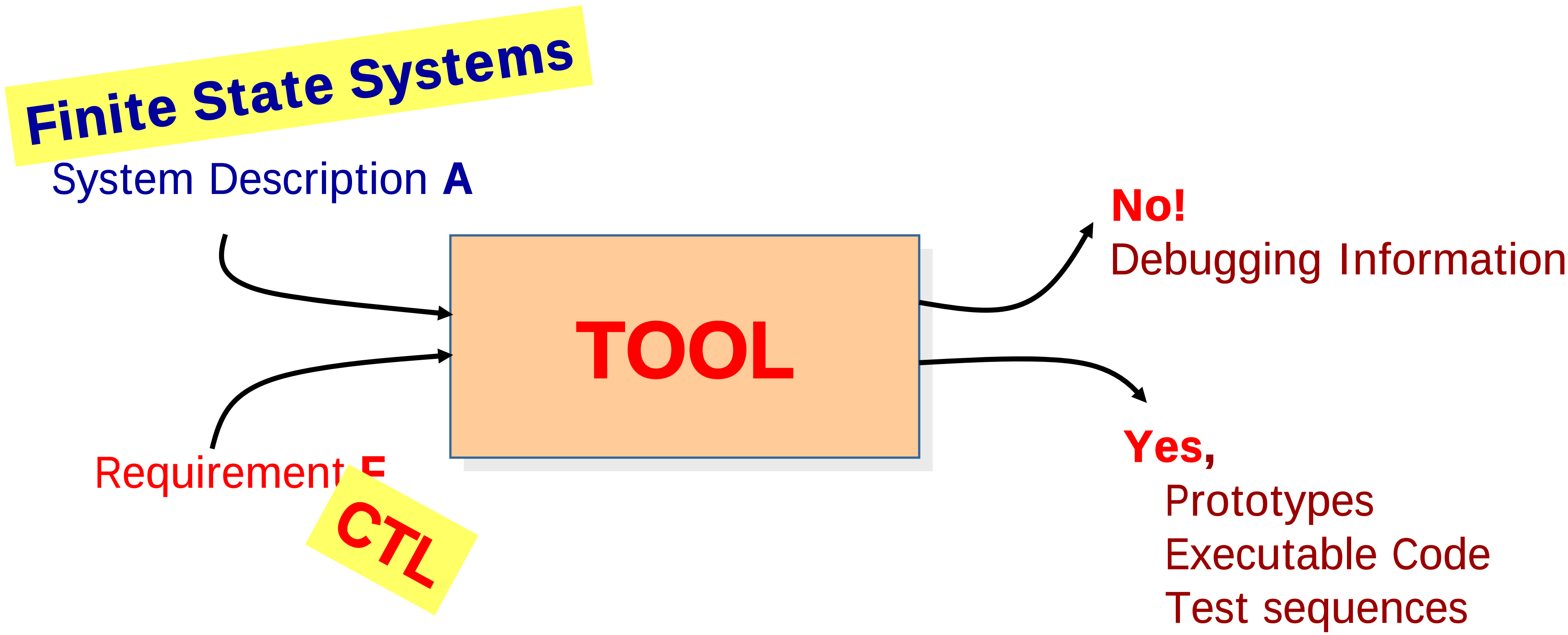
Finite State Model Checking

AALBORG UNIVERSITY DENMARK

BRICS
Basic Research in Computer Science

CISS
CENTER FOR INDLEJREDE SOFTWARE SYSTEMER

Finite State Model Checking

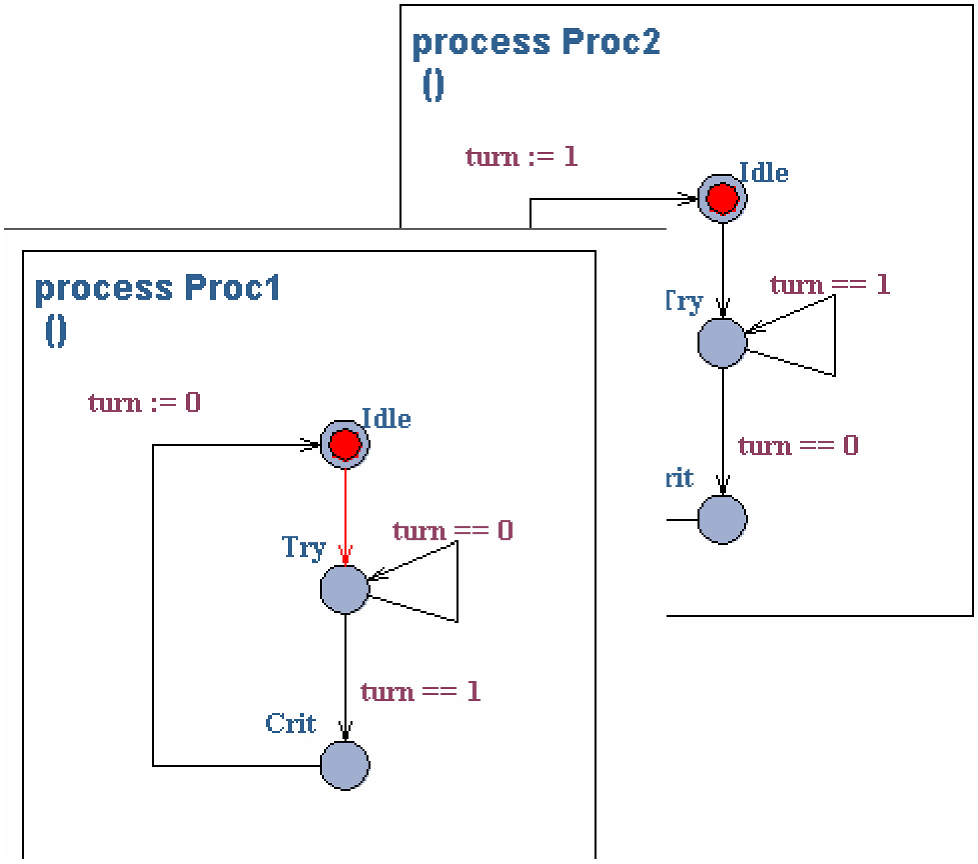


Tools: visualSTATE, SPIN, Statemate, Verilog, Formalcheck,...

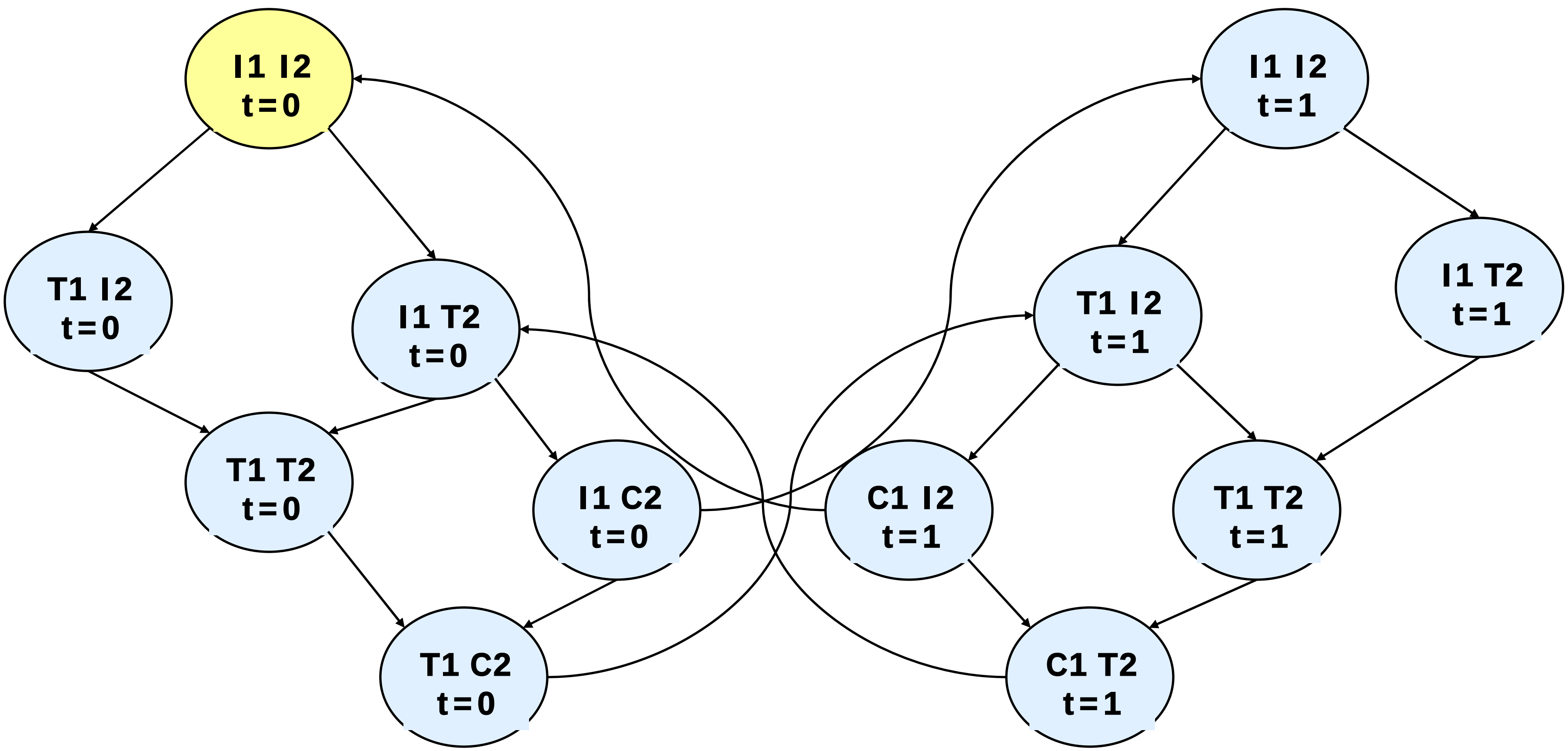
From Programs to **Networks**

```
P1 :: while True do
      T1 : wait(turn=1)
      C1 : turn:=0
      endwhile
||
P2 :: while True do
      T2 : wait(turn=0)
      C2 : turn:=1
      endwhile
```

Mutual Exclusion Program



From Network Models to
Kripke Structures



CTL Models =
Kripke Structures

A CTL-model is a triple $\mathcal{M} = (S, R, Label)$ where

- S is a non-empty set of states,
- $R \subseteq S \times S$ is a total relation on S, which relates to $s \in S$ its possible successor states,
- $Label : S \longrightarrow 2^{AP}$, assigns to each state $s \in S$ the atomic propositions $Label(s)$ that are valid in s.

Computation Tree Logic, CTL

Clarke & Emerson 1980

Syntax

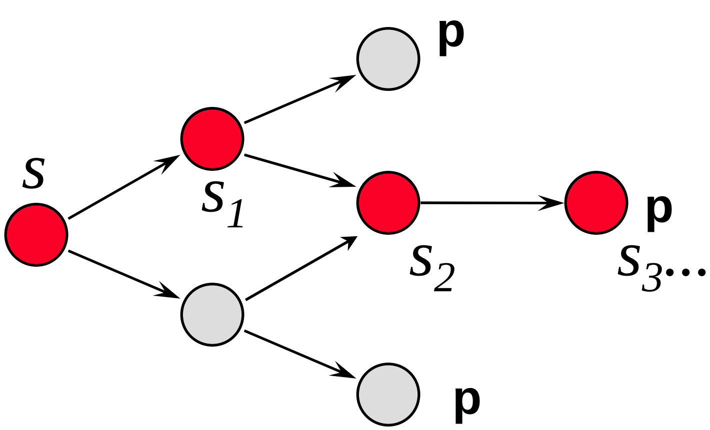
$$\phi \quad ::= \quad p \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{EX}\,\phi \mid \mathsf{E}\,[\phi\,\mathsf{U}\,\phi] \mid \mathsf{A}\,[\phi\,\mathsf{U}\,\phi].$$

- EX (pronounced "for some path next")
- E (pronounced "for some path")
- A (pronounced "for all paths") and
- U (pronounced "until").

Path

Definition 20. (Path)

A *path* is an infinite sequence of states $s_0\, s_1\, s_2 \ldots$ such that $(s_i, s_{i+1}) \in R$ for all $i \geqslant 0$.



The set of path starting in s

$P_{\mathcal{M}}(s)$

Formal Semantics

(satisfaction relation $\models$)

$$
\begin{array}{ll}
s \models p & \text{iff } p \in Label(s) \\
s \models \neg \phi & \text{iff } \neg (s \models \phi) \\
s \models \phi \vee \psi & \text{iff } (s \models \phi) \vee (s \models \psi) \\
s \models \mathsf{EX}\,\phi & \text{iff } \exists \sigma \in P_{\mathcal{M}}(s).\, \sigma[1] \models \phi \\
s \models \mathsf{E}\,[\phi\, \mathsf{U}\, \psi] & \text{iff } \exists \sigma \in P_{\mathcal{M}}(s).(\exists j \geqslant 0.\, \sigma[j] \models \psi \wedge (\forall 0 \leqslant k < j.\, \sigma[k] \models \phi)) \\
s \models \mathsf{A}\,[\phi\, \mathsf{U}\, \psi] & \text{iff } \forall \sigma \in P_{\mathcal{M}}(s).(\exists j \geqslant 0.\, \sigma[j] \models \psi \wedge (\forall 0 \leqslant k < j.\, \sigma[k] \models \phi)).
\end{array}
$$

CTL, Derived Operators

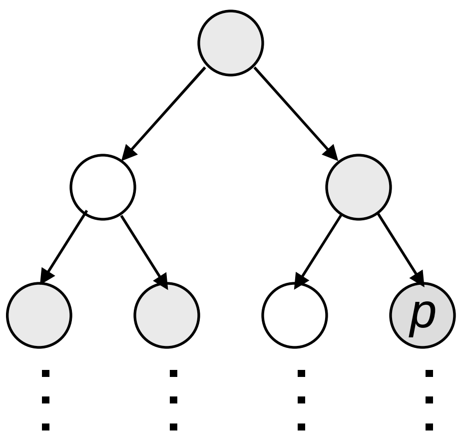
$$\mathsf{EF}\,\phi \equiv \mathsf{E}\,[\mathrm{true}\,\mathsf{U}\,\phi]$$

possible

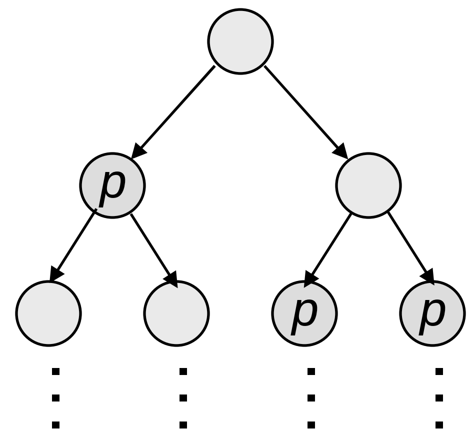
$$\mathsf{AF}\,\phi \equiv \mathsf{A}\,[\mathrm{true}\,\mathsf{U}\,\phi].$$

inevitable

EF p



AF p



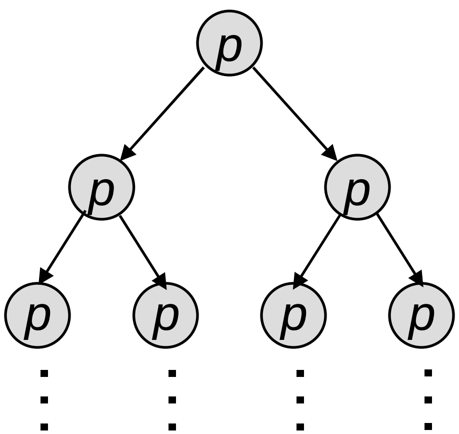
CTL, Derived Operators

$$\mathsf{EG}\,\phi \equiv \neg\,\mathsf{AF}\,\neg\,\phi$$ *potentially always*

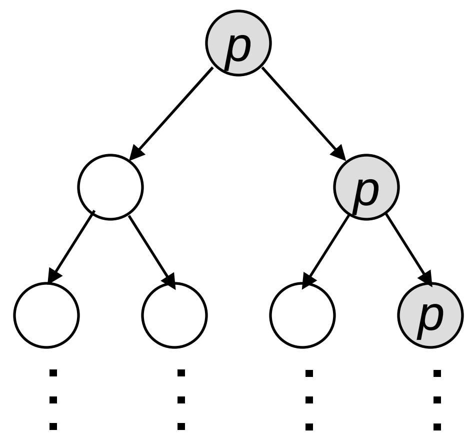
$$\mathsf{AG}\,\phi \equiv \neg\,\mathsf{EF}\,\neg\,\phi$$ *always*

$$\mathsf{AX}\,\phi \equiv \neg\,\mathsf{EX}\,\neg\,\phi.$$

AG p



EG p



Theorem

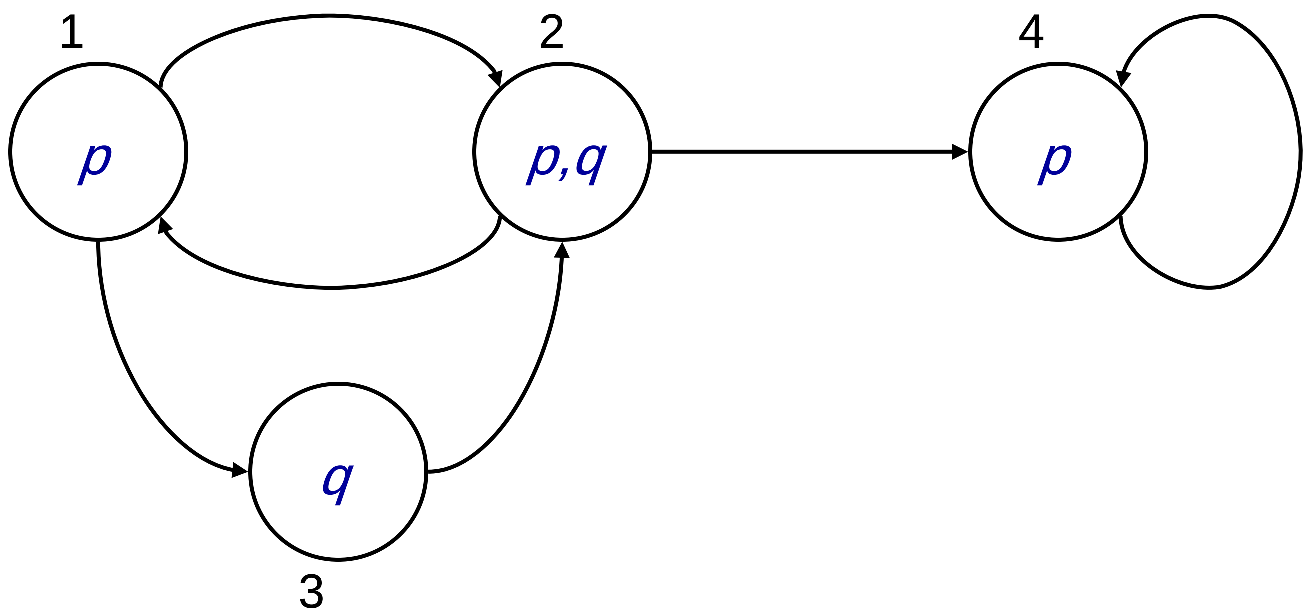
All operators are derivable from

- EX f
- EG f
- E[f U g]

and boolean connectives

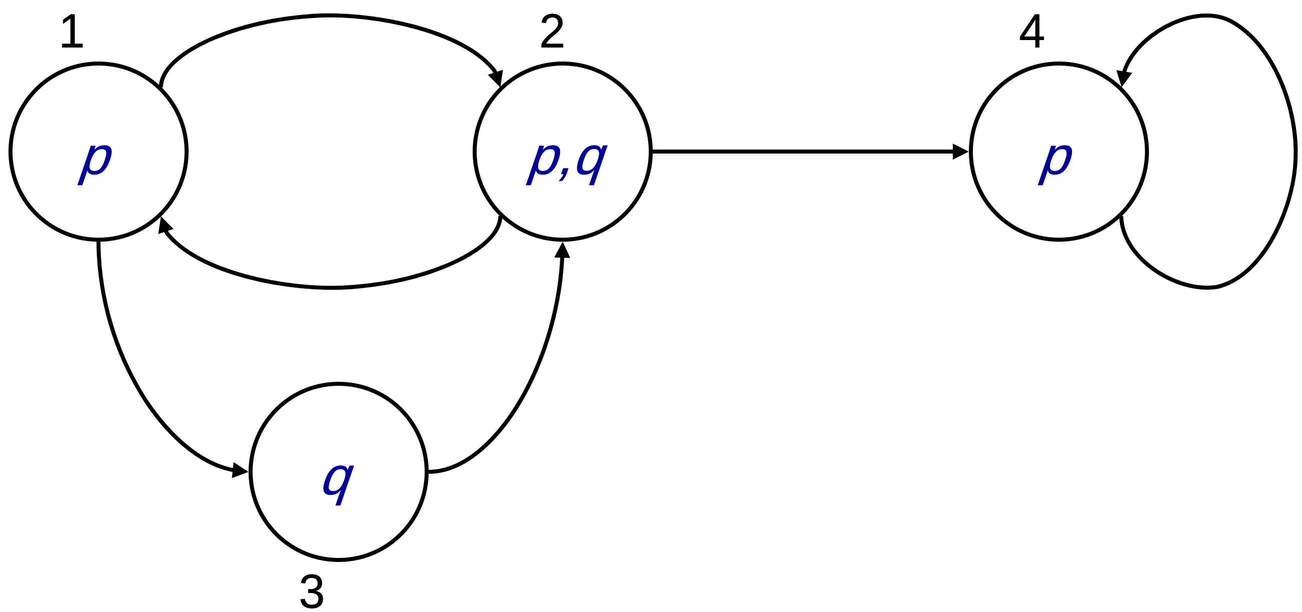
$$A[f \; U \; g] \equiv \neg E[\neg g U(\neg f \wedge \neg g)] \wedge \neg EG \neg g$$

Example



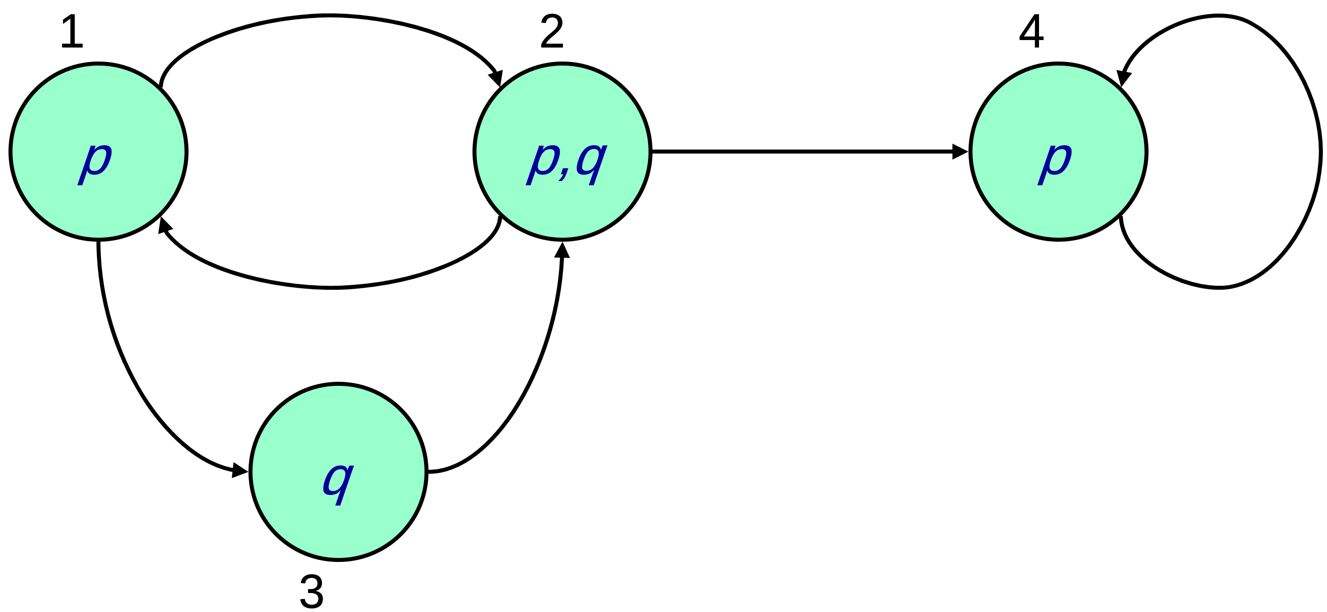
Example

EX *p*



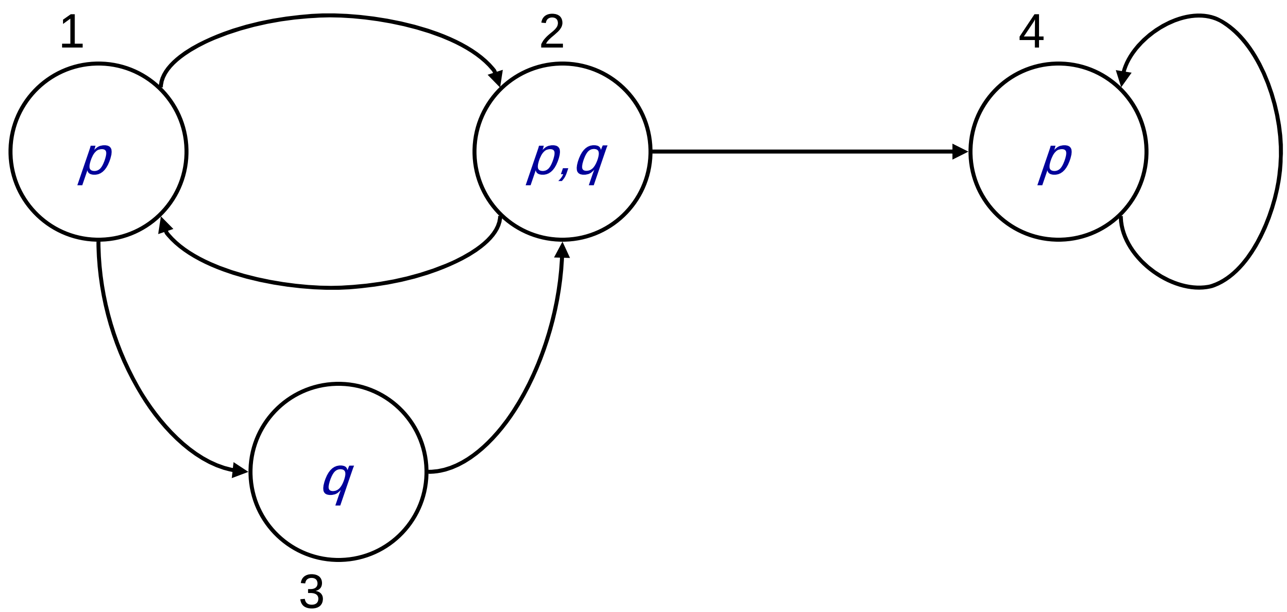
Example

EX *p*



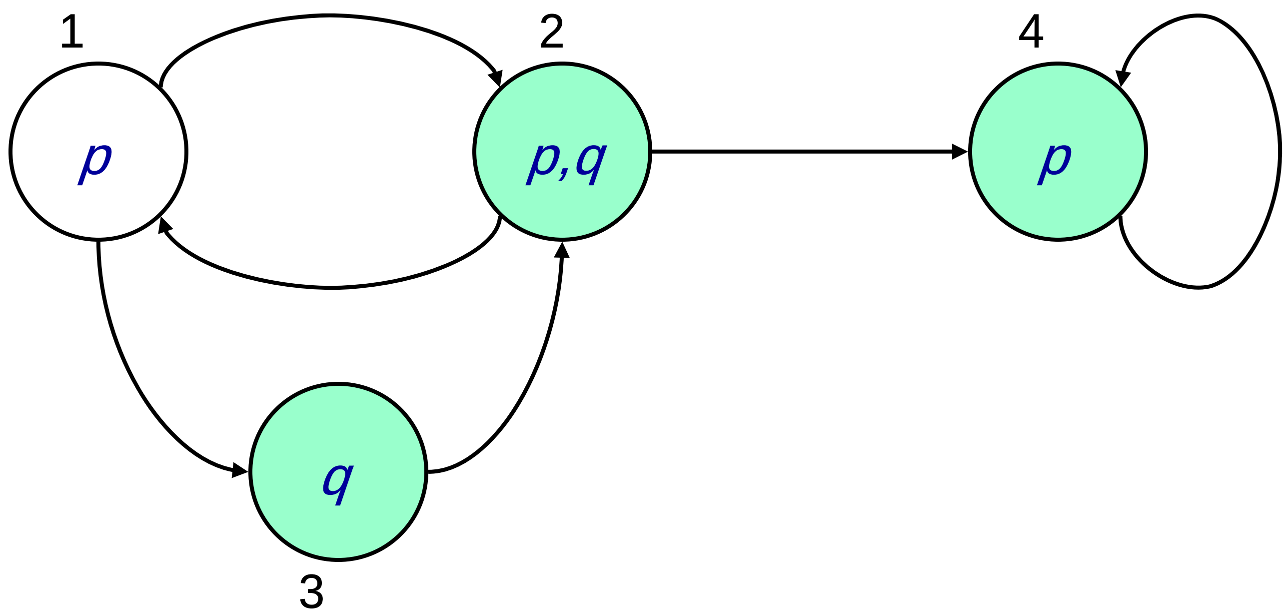
Example

AX *p*



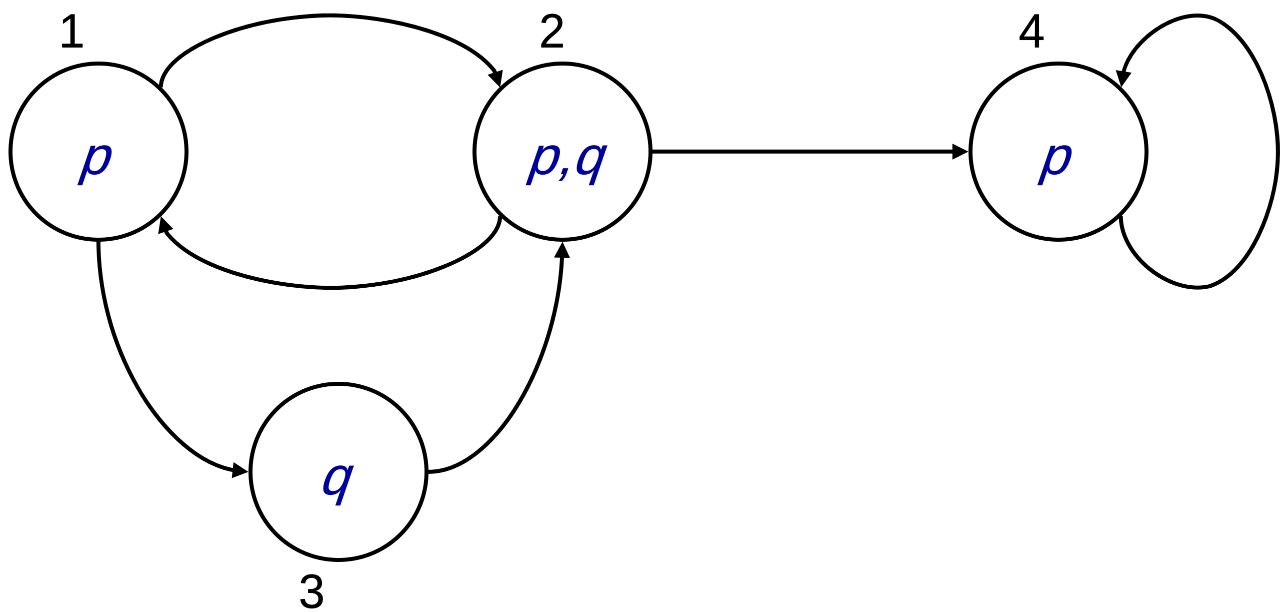
Example

AX *p*



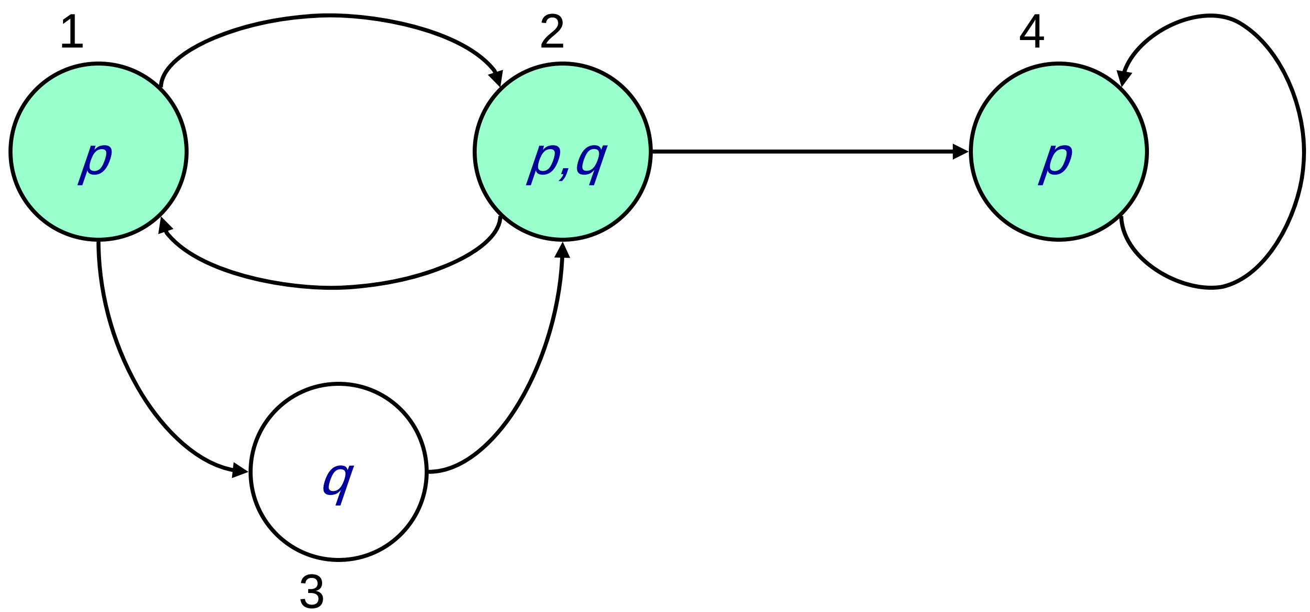
Example

EG *p*



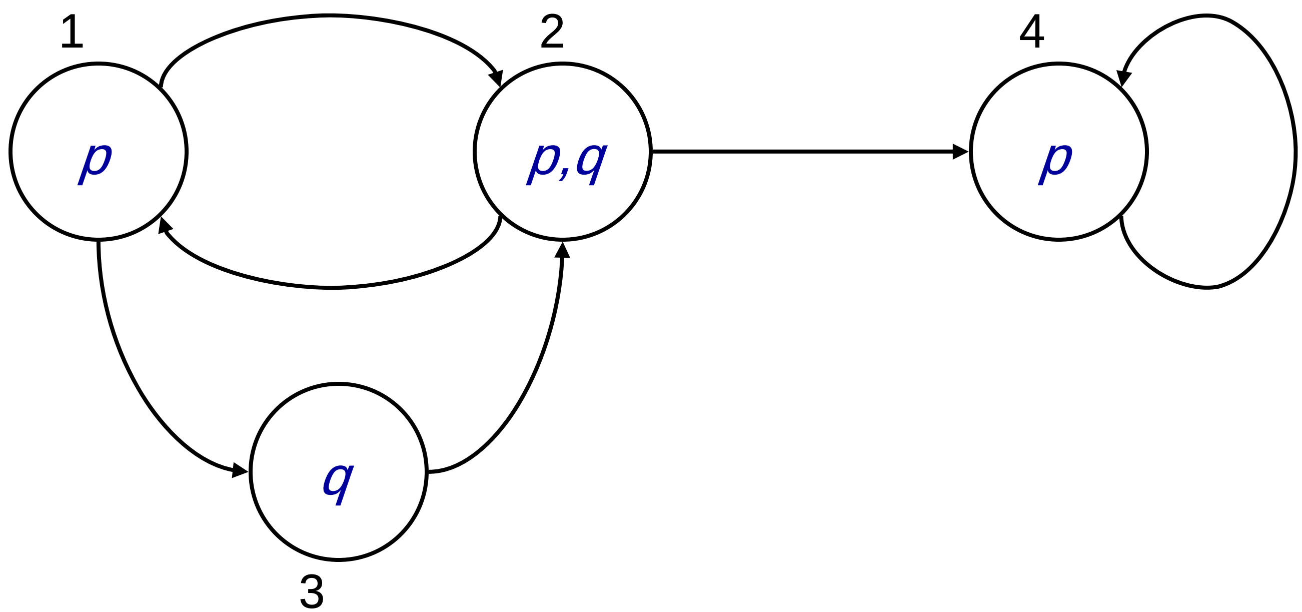
Example

EG p



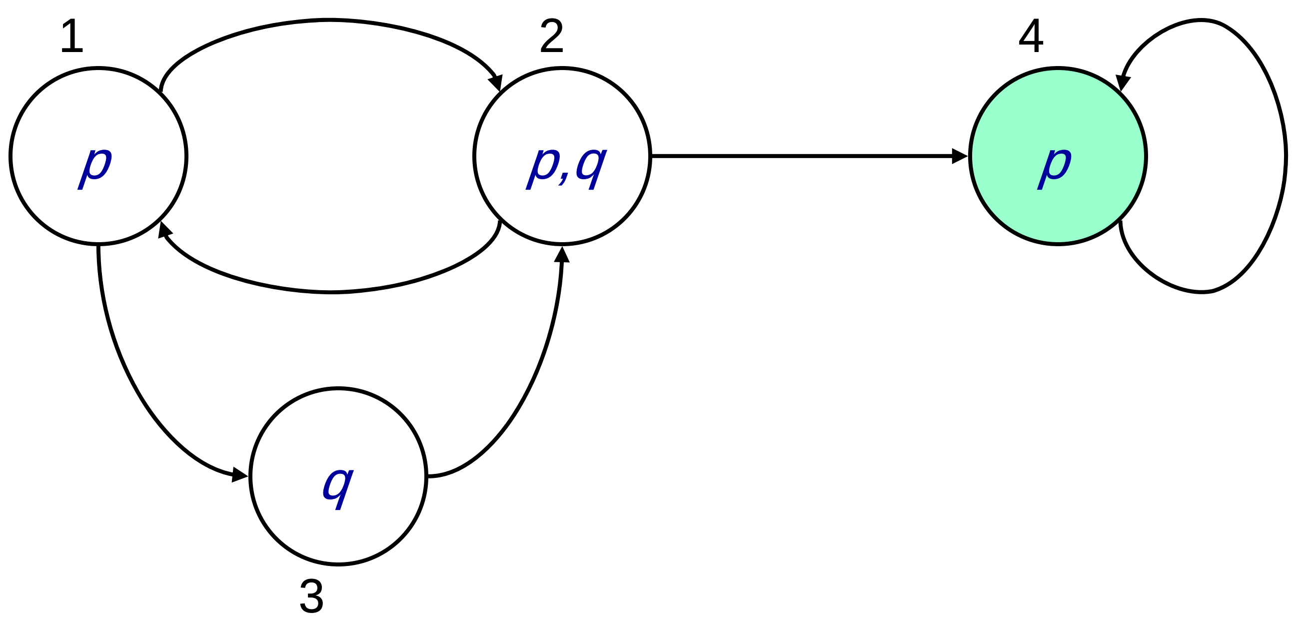
Example

AG p



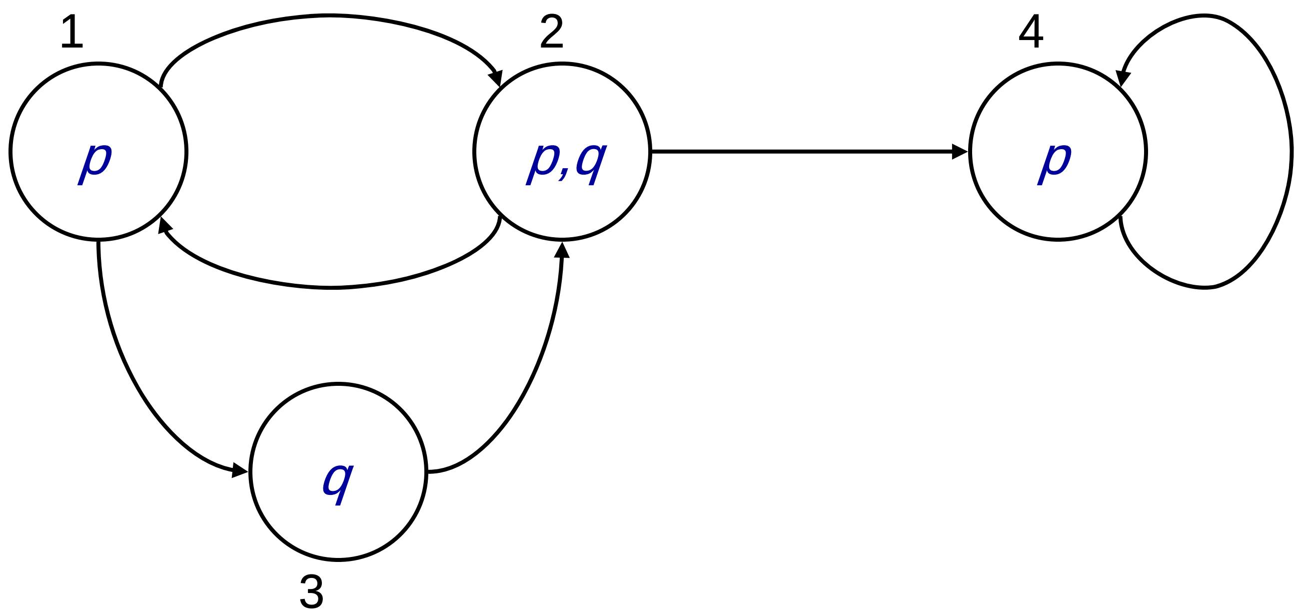
Example

AG *p*



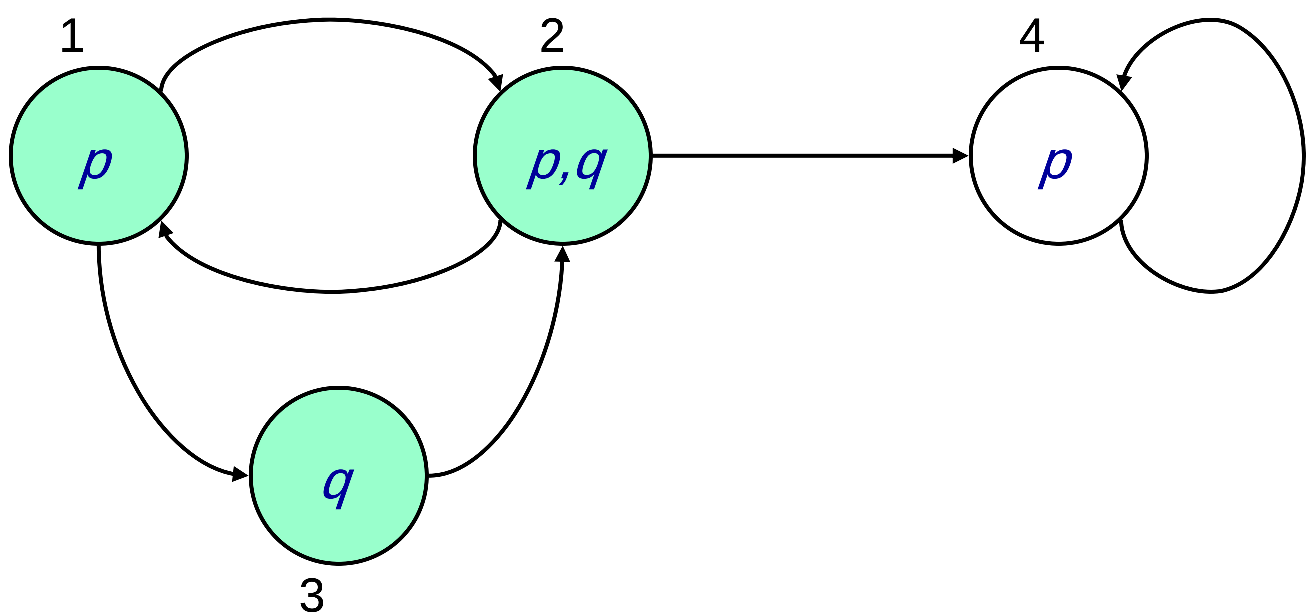
Example

A[*p* U *q*]



Example

A[p U q]



Properties of MUTEX example ?

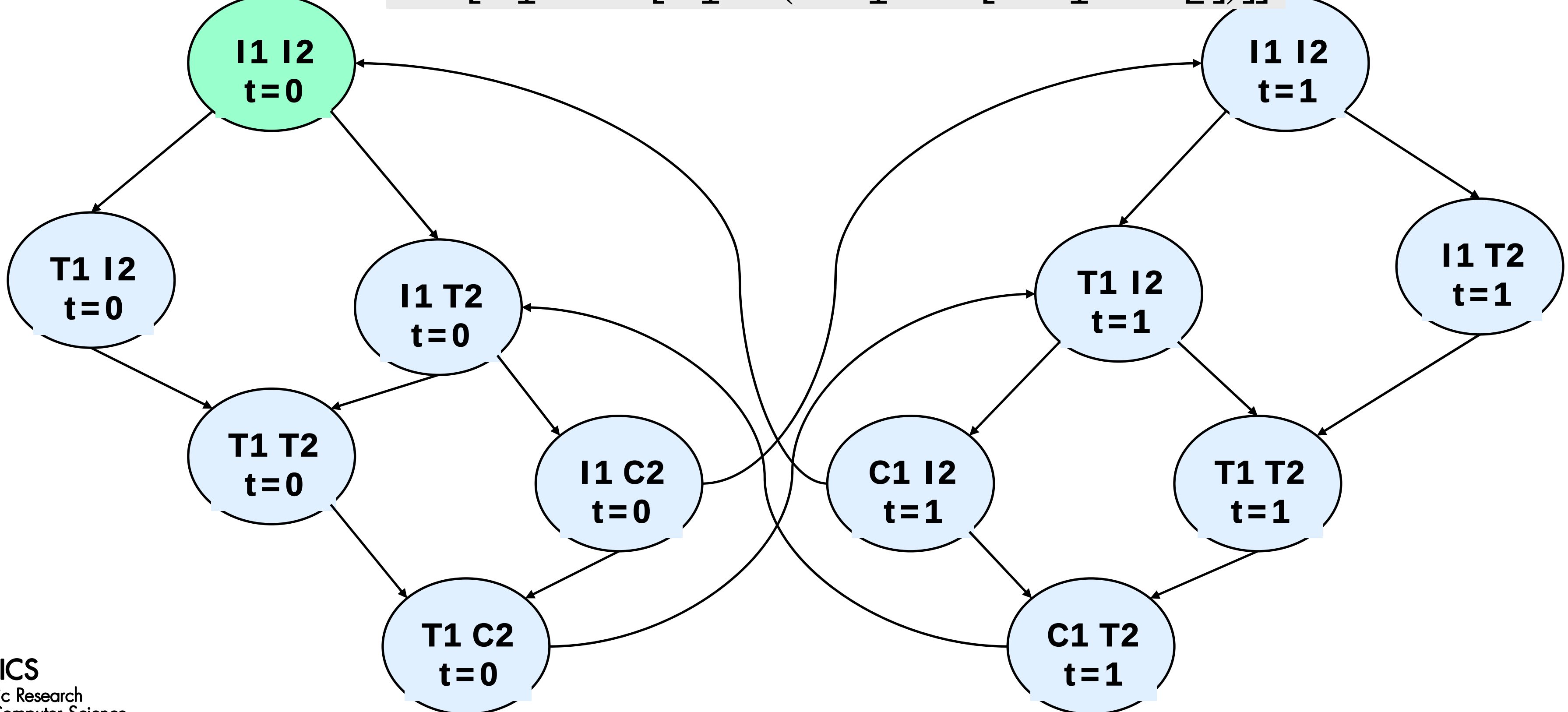
$$AG\,\neg(C_1 \wedge C_2)$$

$$AG[\,T_1 \Rightarrow AF(C_1)]$$

$$EG[\neg C_1]$$

$$AG[C_1 \Rightarrow A[C_1\,U\,(\neg C_1 \wedge A[\neg C_1\,U\,C_2\,])]]$$

HOW to DECIDE
IN GENERAL



CTL Model Checking Algorithms

Fixpoint Characterizations

$$\mathrm{EF}\,p \;\equiv\; p \vee \mathrm{EX}\,\mathrm{EF}\,p$$

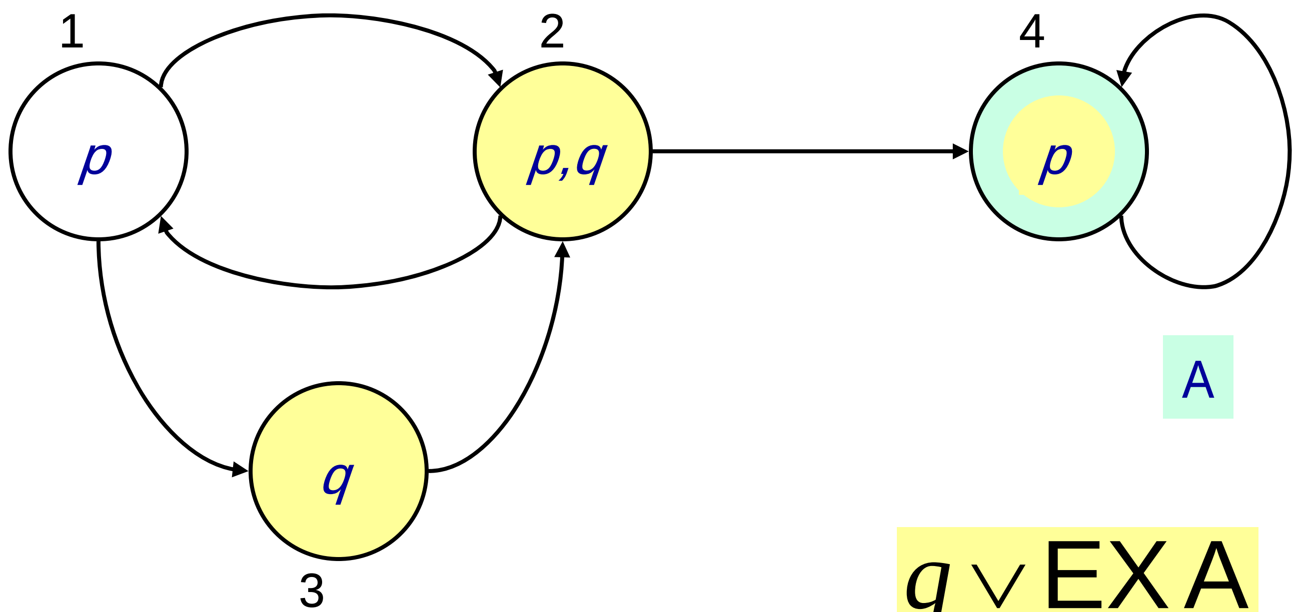
or let A **be the set of states satisfying** EF p **then**

$$\mathrm{A} \;\equiv\; p \vee \mathrm{EX}\,\mathrm{A}$$

in fact A **is the smallest such set (the least fixpoint)**

Example

EF q



$q \vee \mathrm{EX}\,\mathrm{A}$

Fixed points of monotonic functions

- Let τ be a function $2^S \to 2^S$
- Say τ is *monotonic* when

$$x \subseteq y \text{ implies } \tau(x) \subseteq \tau(y)$$

- Fixed point of τ is y such that

$$\tau(y) = y$$

- If τ monotonic, then it has
  - least fixed point $\mu y.\ \tau(y)$
  - greatest fixed point $\nu y.\ \tau(y)$

Iteratively computing fixed points

- Suppose S is finite
  - The least fixed point $\mu y.\ \tau(y)$ is the limit of

$$\text{false} \quad \subseteq \quad \tau(\text{false}) \quad \subseteq \quad \tau(\tau(\text{false})) \quad \subseteq \cdots$$

  - The greatest fixed point $\nu y.\ \tau(y)$ is the limit of

$$\text{true} \quad \supseteq \quad \tau(\text{true}) \quad \supseteq \quad \tau(\tau(\text{true})) \quad \supseteq \cdots$$

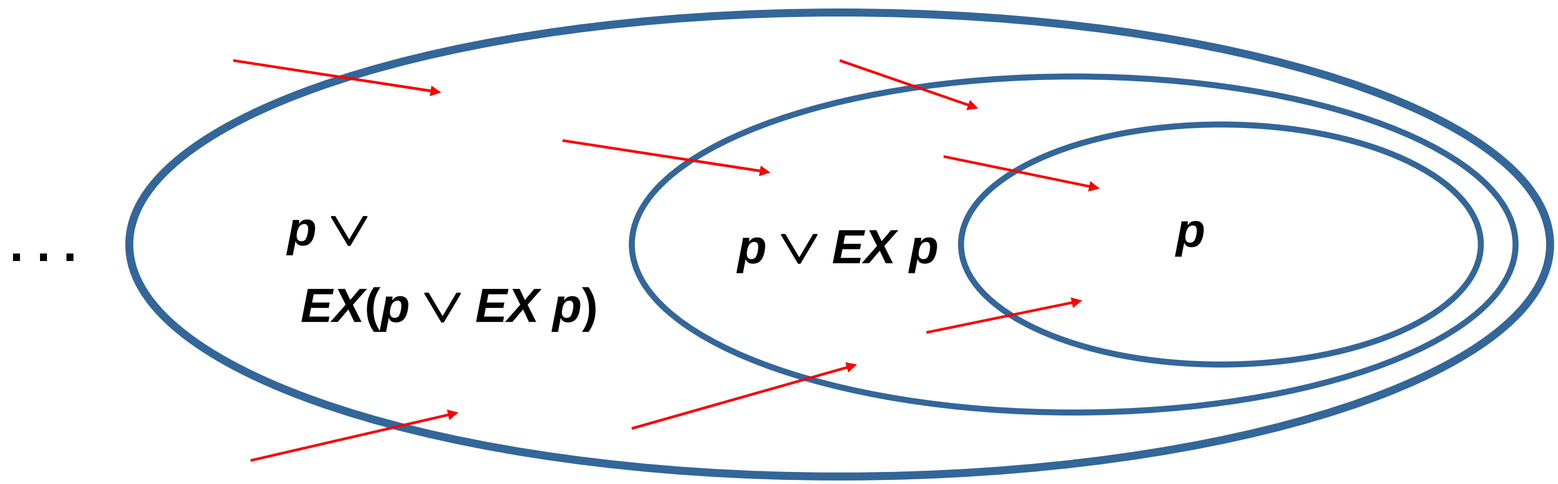
Note, since S is finite, convergence is finite

Example: *EF p*

- *EF p* is characterized by

$$EF\ \ p = \mu y.\,(\,p \vee EX\ \ y\,)$$

- Thus, it is the limit of the increasing series...

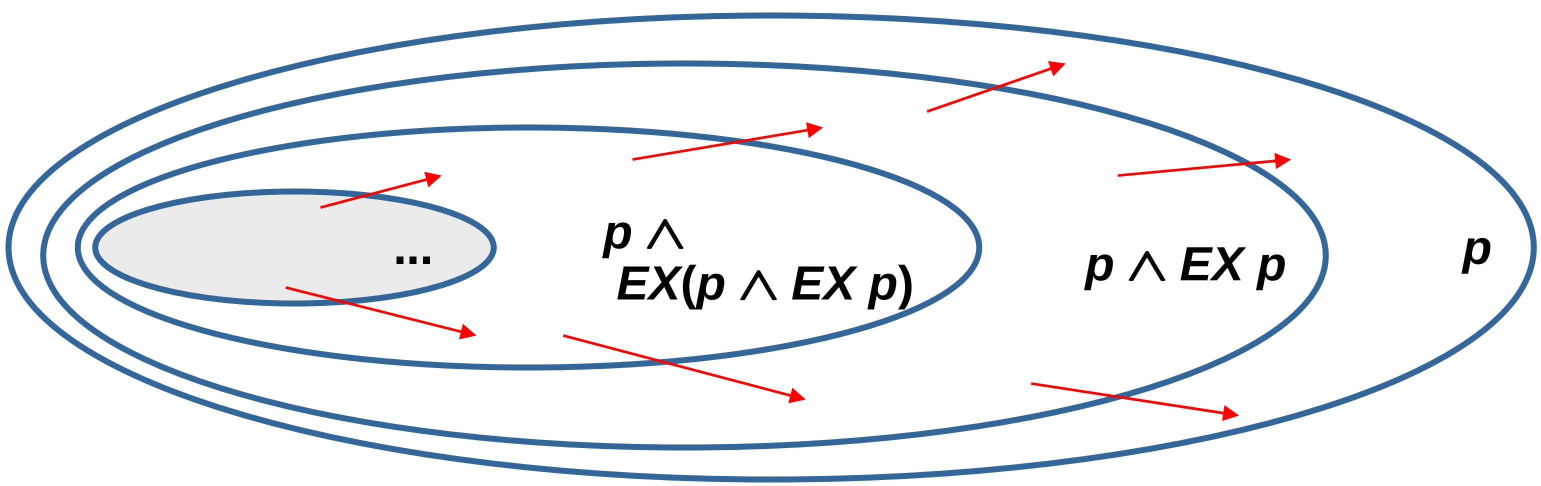


Example: *EG p*

- *EG p* is characterized by

$$EG\ p = \nu y.\ (p \wedge EX\ y)$$

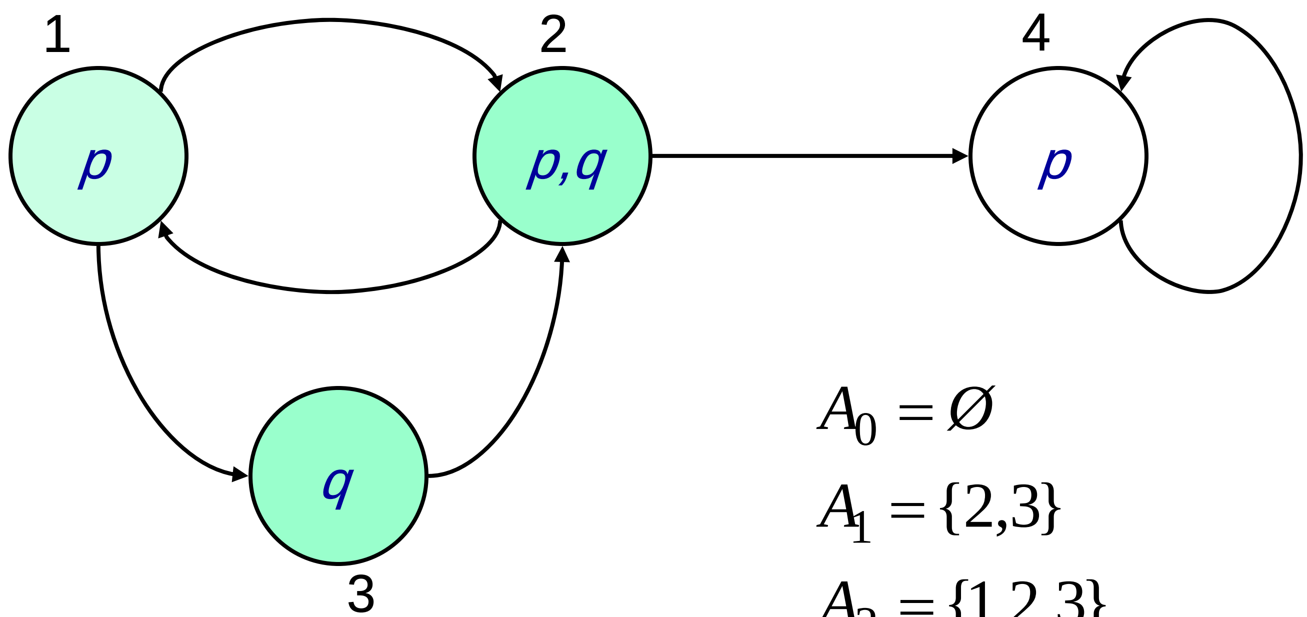
- Thus, it is the limit of the decreasing series...



Example, continued

EF *q*

$$EF\ q = \mu y.(q \vee EX\ y)$$



$$A_0 = \emptyset$$

$$A_1 = \{2,3\}$$

$$A_2 = \{1,2,3\}$$

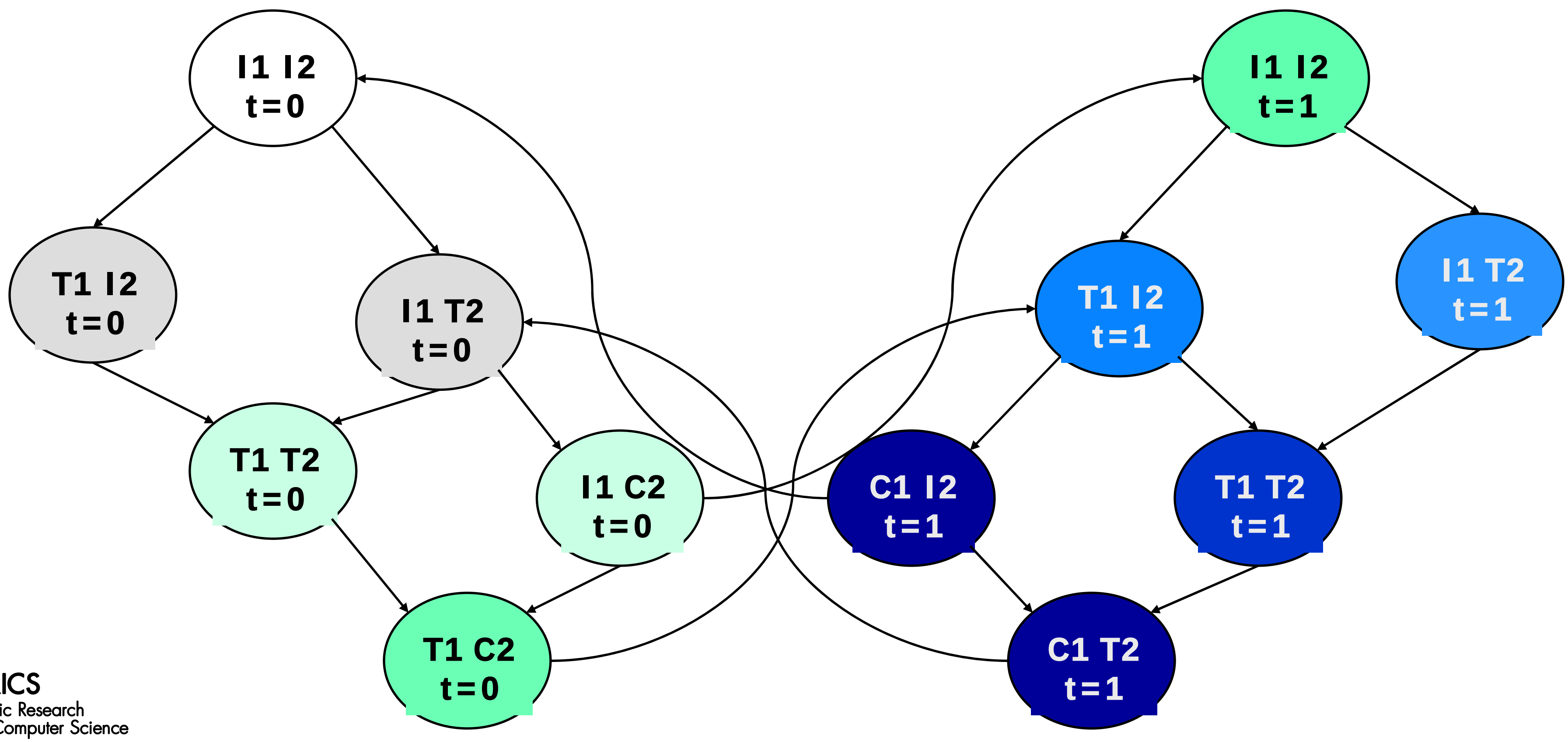
$$A_3 = \{1,2,3\}$$

Remaining operators

$$
\begin{aligned}
AF\, p &= \mu y.(p \vee AX\ y) \\
AG\, p &= \nu y.(p \wedge AX\ y) \\
E(p\, U\, q) &= \mu y.(q \vee (p \wedge EX\ y)) \\
A(p\, U\, q) &= \mu y.(q \vee (p \wedge AX\ y))
\end{aligned}
$$

Properties of MUTEX example ?

$$AG[\, T_1 \Rightarrow AF(C_1)]$$
$$AF(C_1)]$$



function $Sat\,(\phi : Formula)$: **set of** $State$;

(* precondition: true *)

begin

if $\phi = \text{true} \longrightarrow$ **return** S

$[]\ \phi = \text{false} \longrightarrow$ **return** $\varnothing$

$[]\ \phi \in AP \longrightarrow$ **return** $\{\, s \mid \phi \in Label(s) \,\}$

$[]\ \phi = \neg\, \phi_1 \longrightarrow$ **return** $S - Sat(\phi_1)$

$[]\ \phi = \phi_1 \ \vee\ \phi_2 \longrightarrow$ **return** $(Sat(\phi_1) \cup Sat(\phi_2))$

$[]\ \phi = \mathsf{EX}\, \phi_1 \longrightarrow$ **return** $\{\, s \in S \mid (s, s') \in R \ \wedge\ s' \in Sat(\phi_1) \,\}$

$[]\ \phi = \mathsf{E}\,[\phi_1\, \mathsf{U}\, \phi_2] \longrightarrow$ **return** $Sat_{EU}(\phi_1, \phi_2)$

$[]\ \phi = \mathsf{A}\,[\phi_1\, \mathsf{U}\, \phi_2] \longrightarrow$ **return** $Sat_{AU}(\phi_1, \phi_2)$

fi

(* postcondition: $Sat(\phi) = \{\, s \mid \mathcal{M}, s \models \phi \,\}$ *)

end

function $Sat_{EU}\,(\phi, \psi : Formula)$: **set of** $State$;

(* precondition: true *)

begin var Q, Q' : **set of** $State$;

$Q, Q' := Sat(\psi), \varnothing$;

do $Q \neq Q' \longrightarrow$

$Q' := Q$;

$Q := Q \cup (\{\, s \mid \exists\, s' \in Q.\, (s, s') \in R \,\} \cap Sat(\phi))$

od;

return Q

(* postcondition: $Sat_{EU}(\phi, \psi) = \{\, s \in S \mid \mathcal{M}, s \models \mathsf{E}\,[\phi\, \mathsf{U}\, \psi] \,\}$ *)

end

Table 3.4: Labelling procedure for $\mathsf{E}\,[\phi\, \mathsf{U}\, \psi]$

function $Sat_{AU}\,(\phi, \psi : Formula)$ **: set of** $State$;

(* precondition: true *)

begin var Q, Q' **: set of** $State$;

$Q, Q' := Sat(\psi), \varnothing$;

do $Q \neq Q' \longrightarrow$

$Q' := Q$;

$Q := Q \cup (\{s \mid \forall s'.(s, s') \in R \Rightarrow s' \in Q\} \cap Sat(\phi))$

od;

return Q

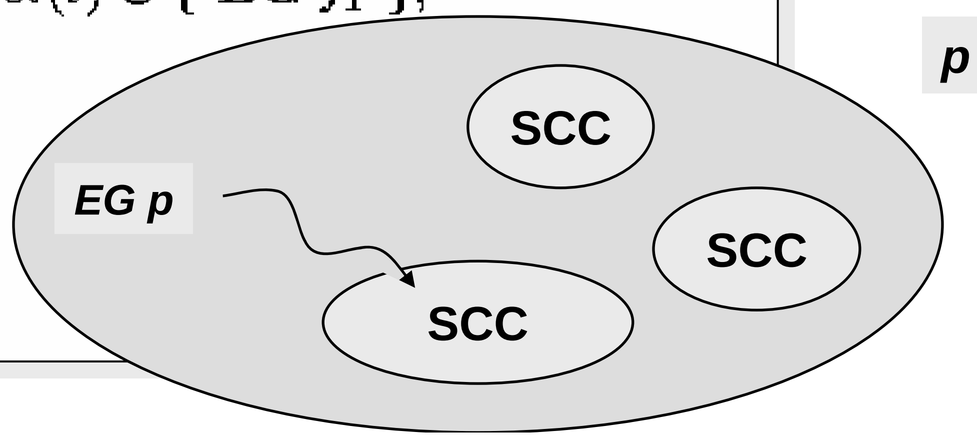
(* postcondition: $Sat_{AU}(\phi, \psi) = \{\, s \in S \mid \mathcal{M}, s \models \mathsf{A}\,[\phi\,\mathsf{U}\,\psi]\,\}$ *)

end

Table 3.5: Labelling procedure for $\mathsf{A}\,[\phi\,\mathsf{U}\,\psi]$

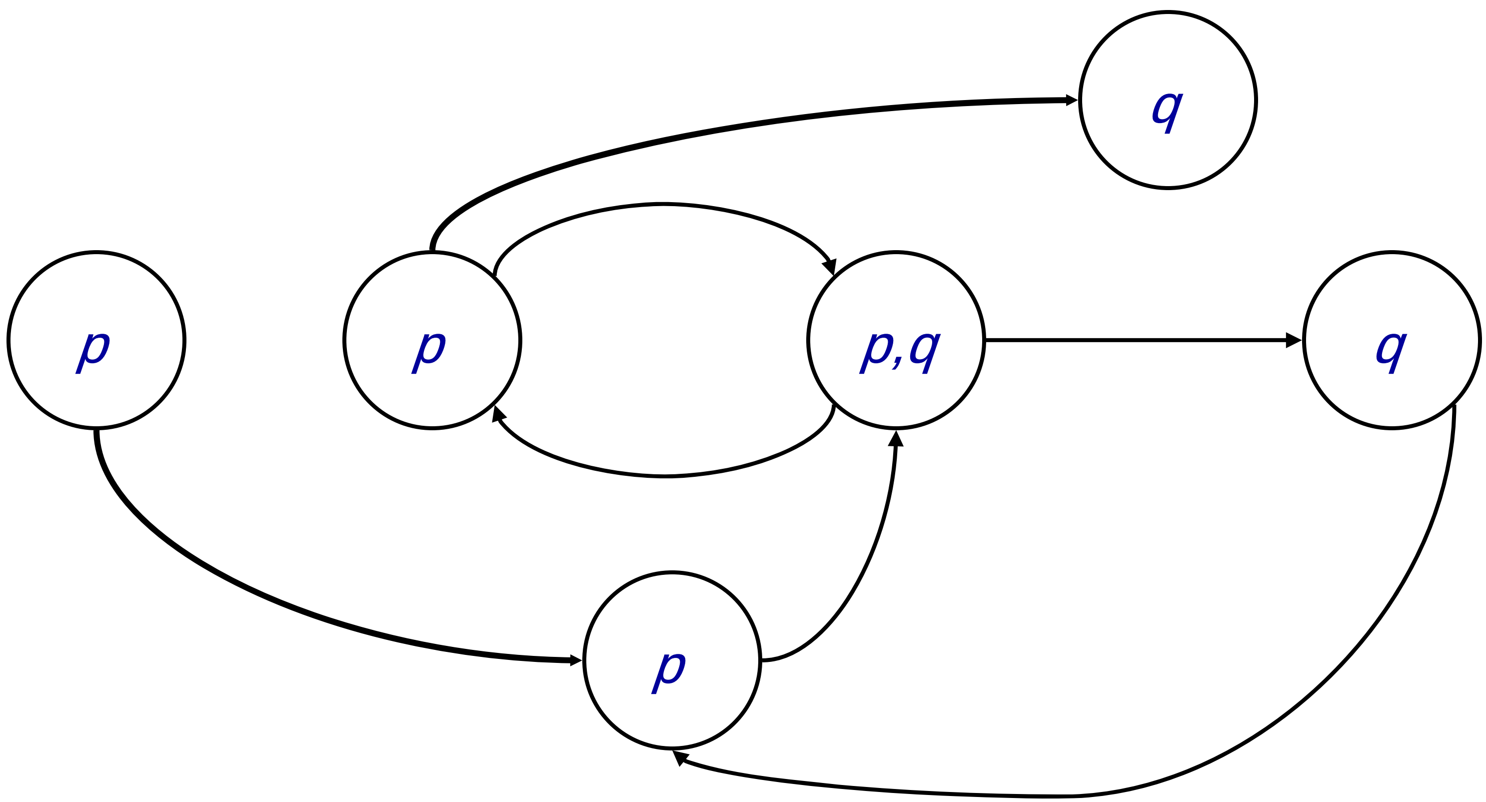
More Efficient Check

procedure $CheckEG(f_1)$
begin
  $S' := \{\, s \mid f_1 \in label(s) \,\}$;
  $SCC := \{\, C \mid C$ is a nontrivial SCC of $S' \,\}$;
  $T := \bigcup_{C \in SCC} \{\, s \mid s \in C \,\}$;
  for every $s \in T$ **do** $label(s) := label(s) \cup \{\, \mathbf{EG}\ f_1 \,\}$;
  while $T \neq \emptyset$ **do**
  begin
    choose $s \in T$;
    $T := T \setminus \{s\}$;
    for every t **such that** $t \in S'$ **and** $R(t, s)$ **do**
    begin
      if $\mathbf{EG}\ f_1 \notin label(t)$ **do**
      begin
        $label(t) := label(t) \cup \{\, \mathbf{EG}\ f_1 \,\}$;
        $T := T \cup \{t\}$
      end
    end
  end
end



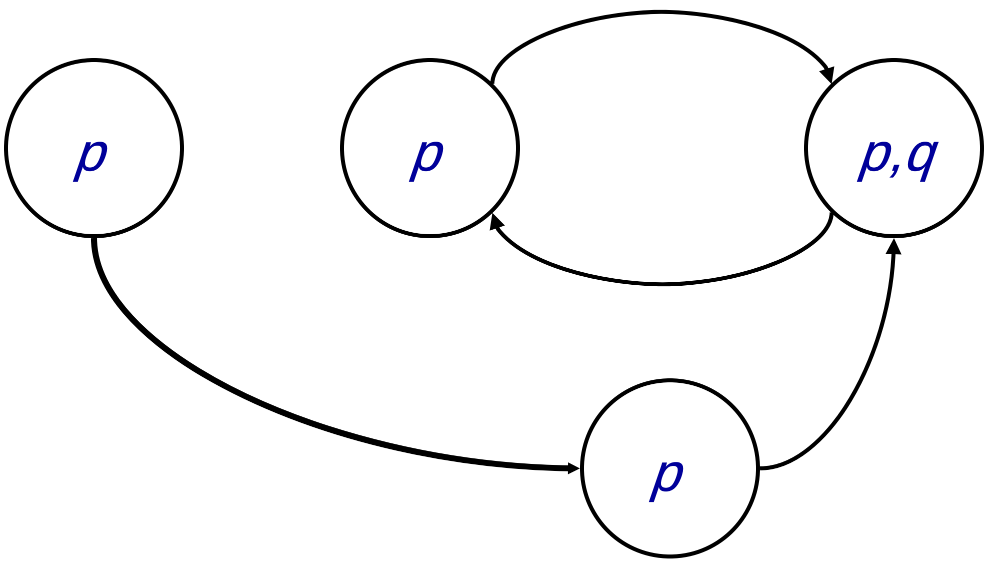
Example

EG p



Example

EG *p*



Reduced Model

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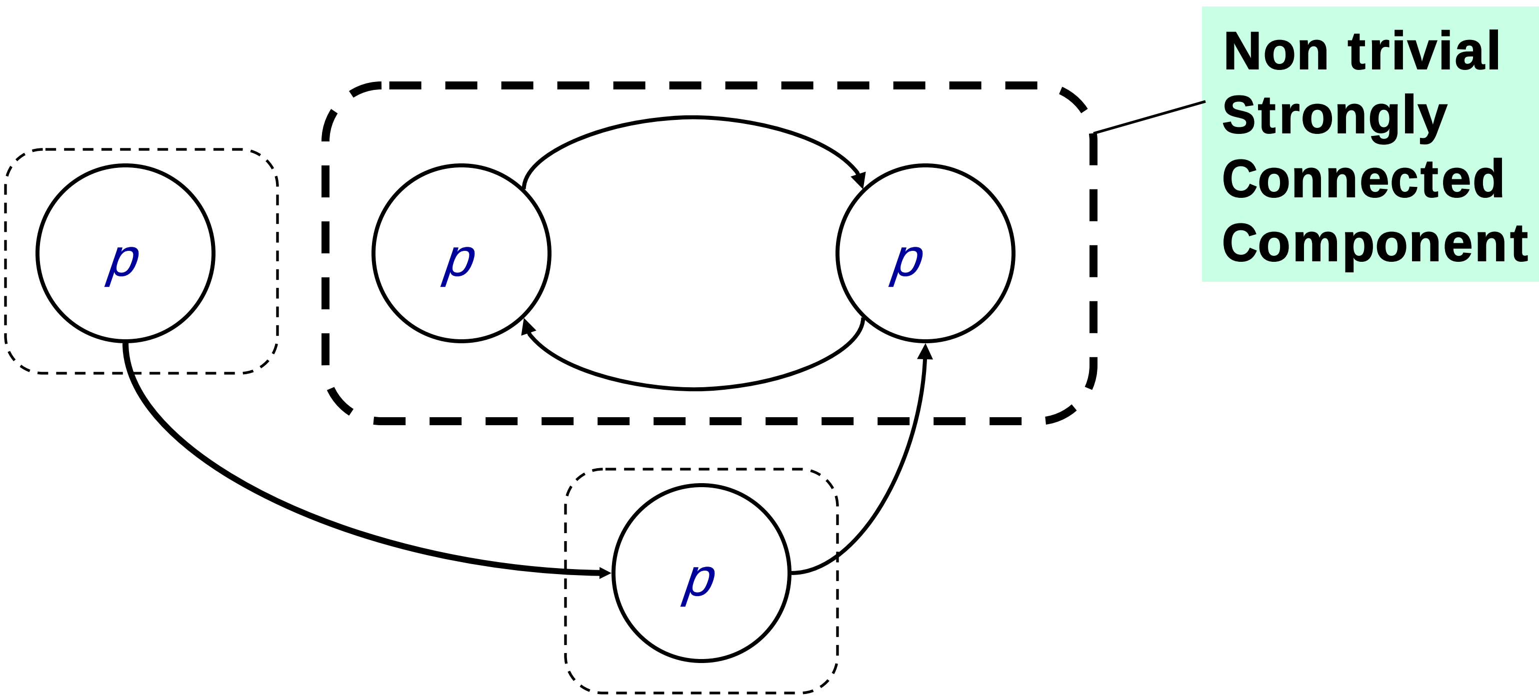
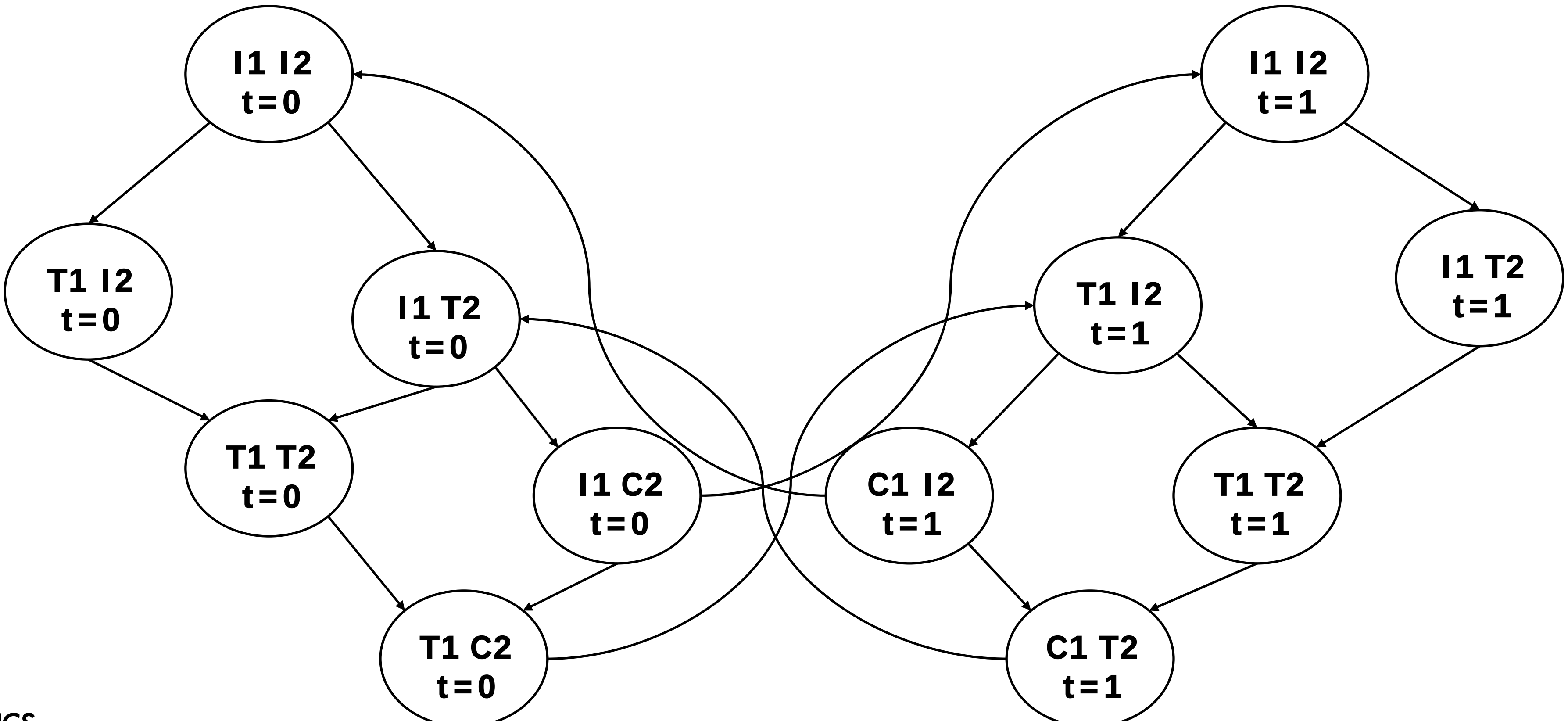
Example

EG *p*



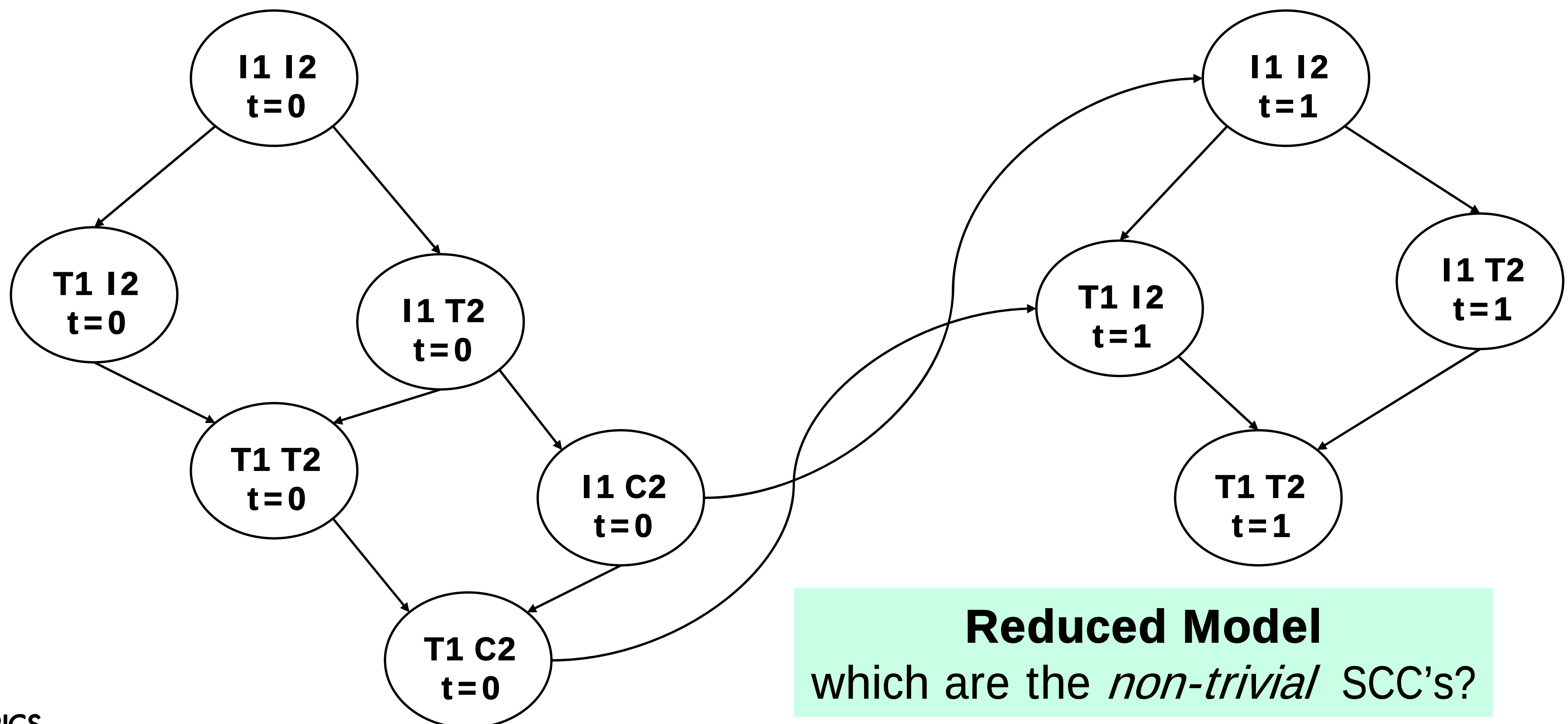
Properties of MUTEX example ?

$EG[\neg C_1]$



Properties of MUTEX example ?

$EG[\neg C_1]$



Reduced Model

which are the *non-trivial* SCC's?

Complexity

The worst-case time complexity of checking whether system-model sys satisfies the CTL*-formula* ϕ *is* $\mathcal{O}(| S_{sys} |^2 \times | \phi |)$

However S_{sys} may be EXPONENTIAL in number of parallel components!
--
FIXPOINT COMPUTATIONS may be carried out using
ROBDD's
(Reduced Ordered Binary Decision Diagrams)
Bryant, 86