Static Analysis

of

Dynamic Communication Systems

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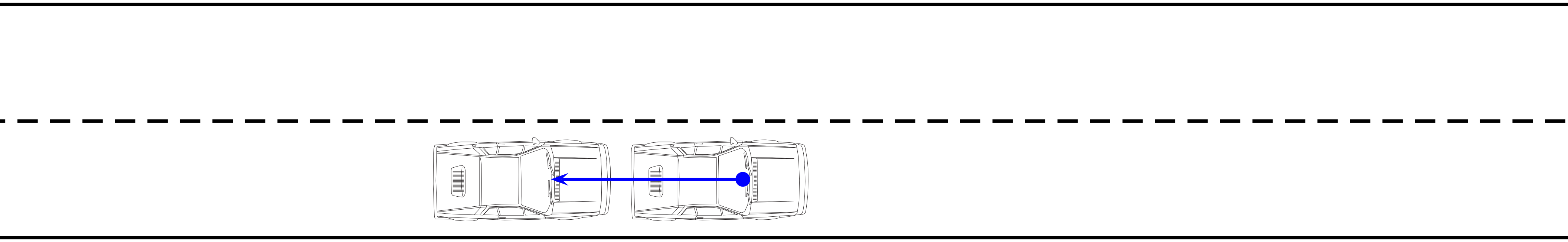
Motivation – Car Platooning

Coordinated groups of fully automated vehicles, called platoons, can double or triple highway capacity when operating in their own dedicated lanes.

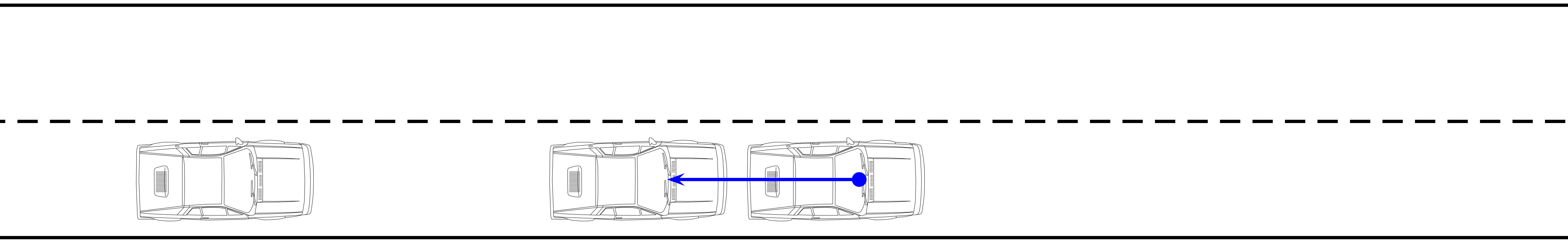
DEMO

Motivation – Car Platooning



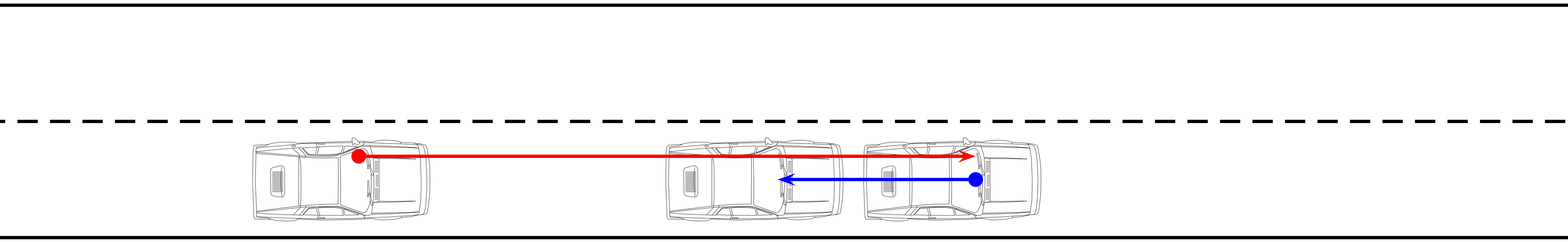
A car platoon of size two.

Motivation – Car Platooning



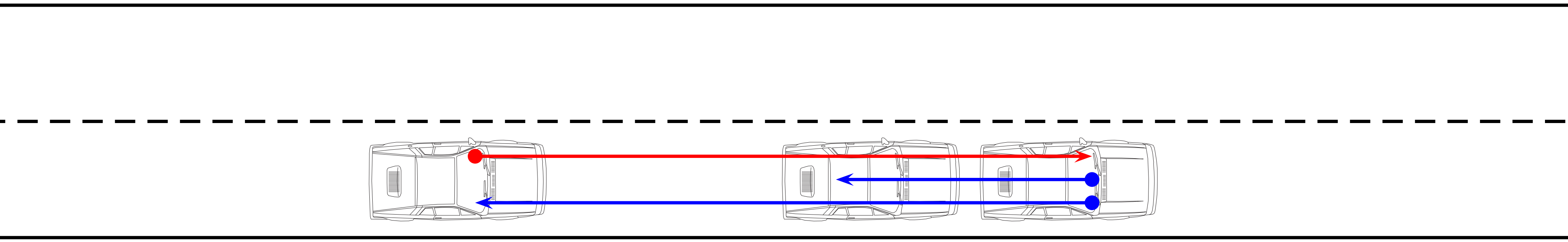
A free agent appears and approaches the platoon.

Motivation – Car Platooning



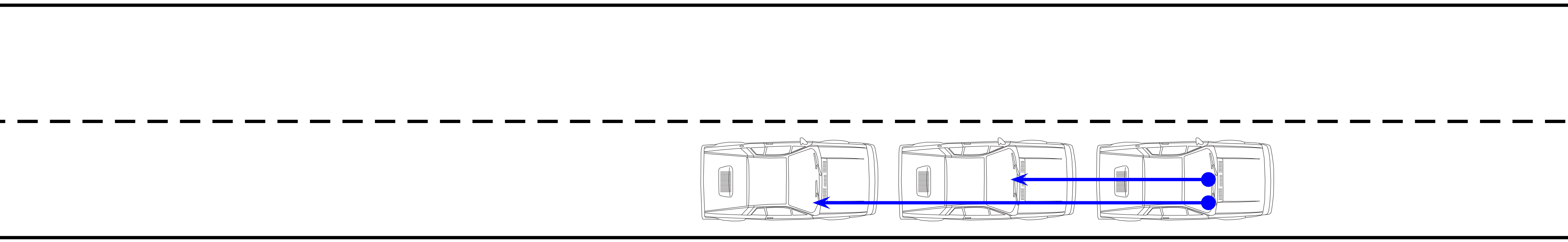
The free agent request a merge from the platoon leader.

Motivation – Car Platooning



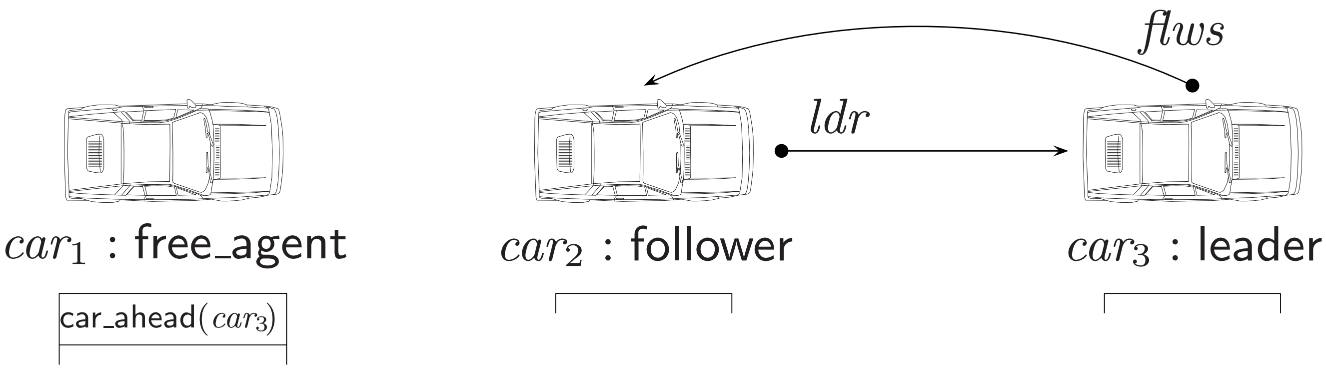
The leader accepts the free agent as a new follower.

Motivation – Car Platooning



A car platoon of size three is established.

Communication Topology



- Local states: free_agent, follower, leader.
- Channels: ldr, flws.
- Message queues.

Challenge

Specification and verification of systems with

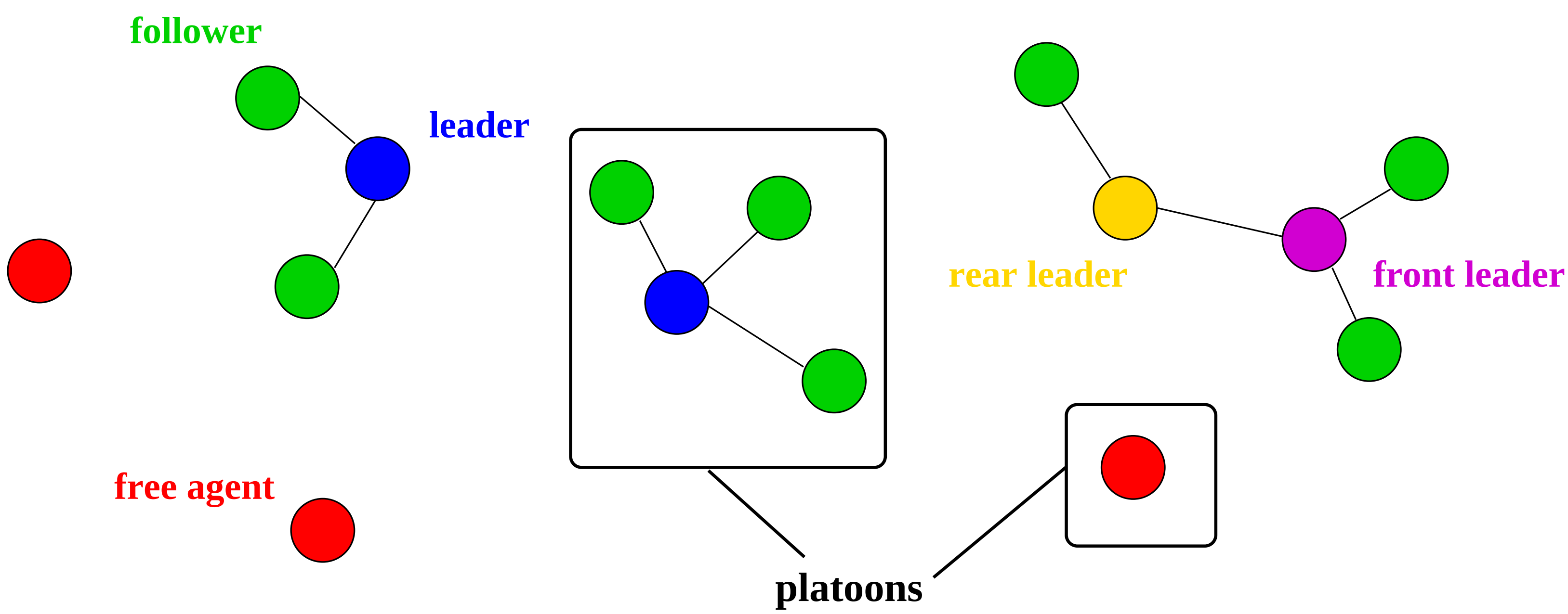
- Dynamically evolving communication topologies.
- Changing and unbounded number of objects.
- Properties of interest (platoons):
  - Topology Properties: unique leaders, no lonely followers, no leader cycles, . . .
  - Temporal properties: Merge will finish (successfully), cars will always be able to leave the platoon.

Static Analysis of DCS

- Outline:
  - Partner graph grammars (PGG)
  - Abstract interpretation of PGG's
  - Platoon Case Study: Results
  - Extensions, conclusions, further reading

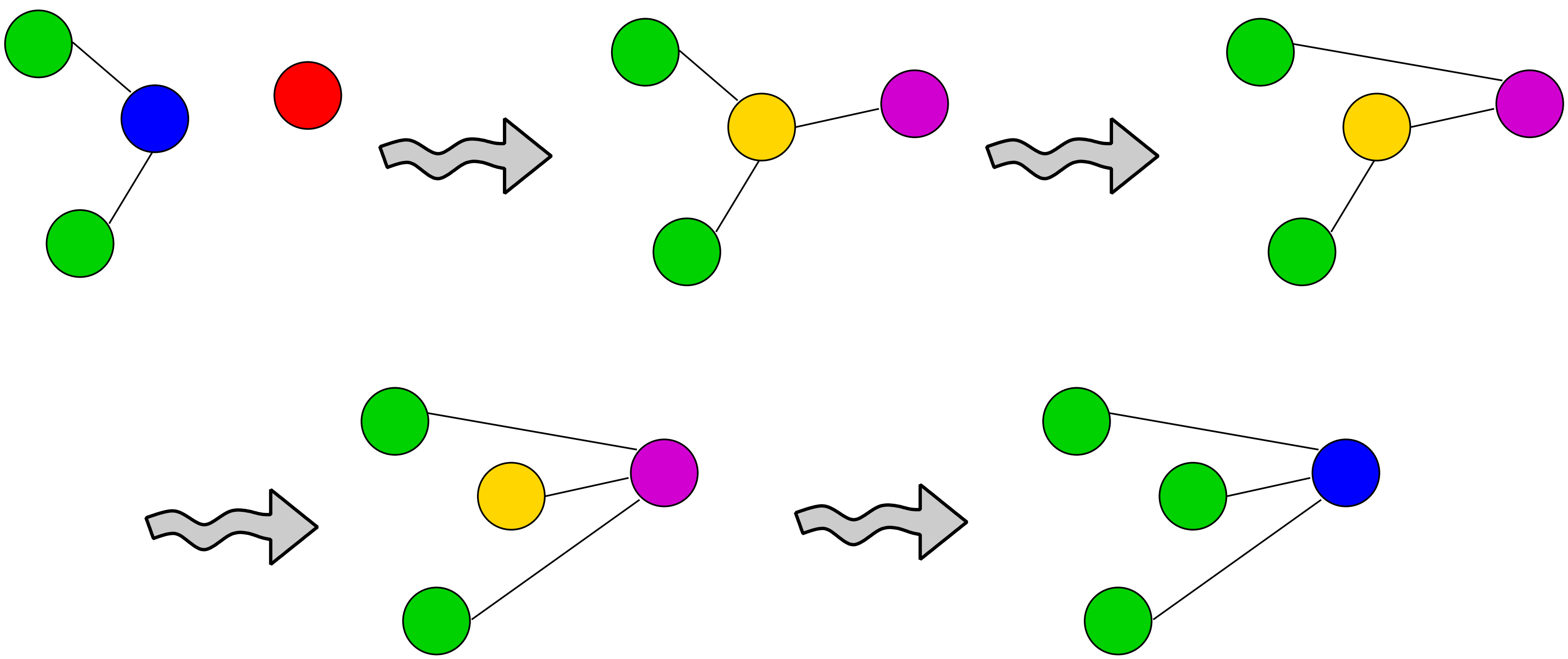
Platoons as Graphs – Idealized

- Abstract from queues and explicit messages.
- Perfect communication.
- Platoon terminology



Platoon Merge

- Non-deterministically start a merge.
- Rear leader hands over followers to front leaders.
- Finish merge after the hand-over.



Partner Graph Grammars

- Assume undirected, node-labeled graph $G = (V_G, E_G, \ell_G)$ over finite set $\mathcal{N}$ of node labels
- A PGG is a pair $(\mathcal{R}, \mathcal{I})$ of rules and initial graph.
- A rule is a four-tuple (L, h, p, R), where
  - L is the left graph
  - $h : V_L \to V_R$ is a (partial) mapping
  - $p : V_L \to \mathcal{P}(\mathcal{N})$ are partner constraints
  - R is the right graph
- SPO approach with injective matching and a mild form of negative application conditions.

Semantics

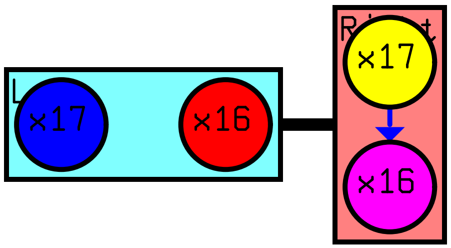
- Application of rule (L, h, p, R) to graph G:
  1. Match: an injective morphism $m : V_L \to V_G$
  2. Partner constraint satisfaction: For all $u \in dom(p)$ holds:

$$p(u) = \{\ell_G(v) \mid \{u, v\} \in E_G\}$$

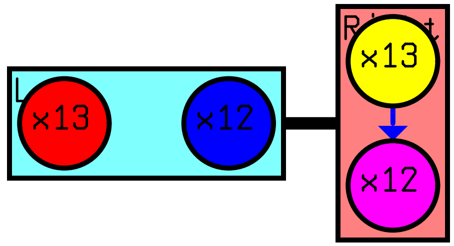
  3. Replace image of m by R.
- Graph semantics $[\![(\mathcal{R}, \mathcal{I})]\!]$: set of all graphs obtainable by iterated rule application from $\mathcal{I}$. (undecidable)

Platoon Partner Graph Grammar

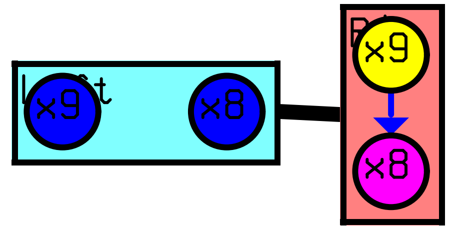
- Arbitrary creation and destruction of free agents



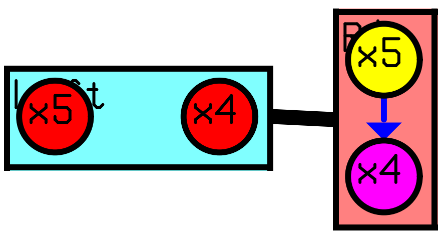
free agent and leader start to merge...



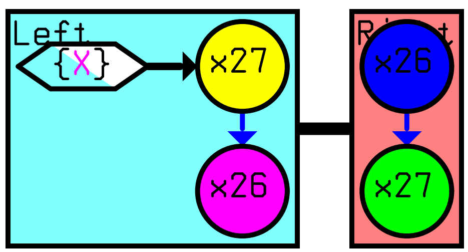
...to be rear leader and front leader



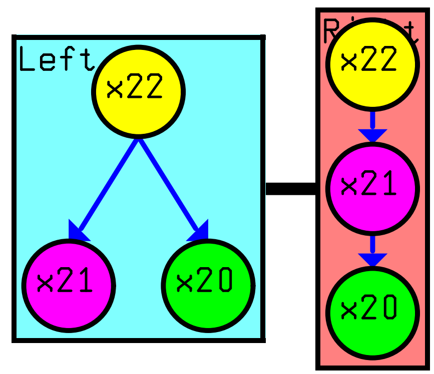
two leaders start to merge



two free agents start to merge



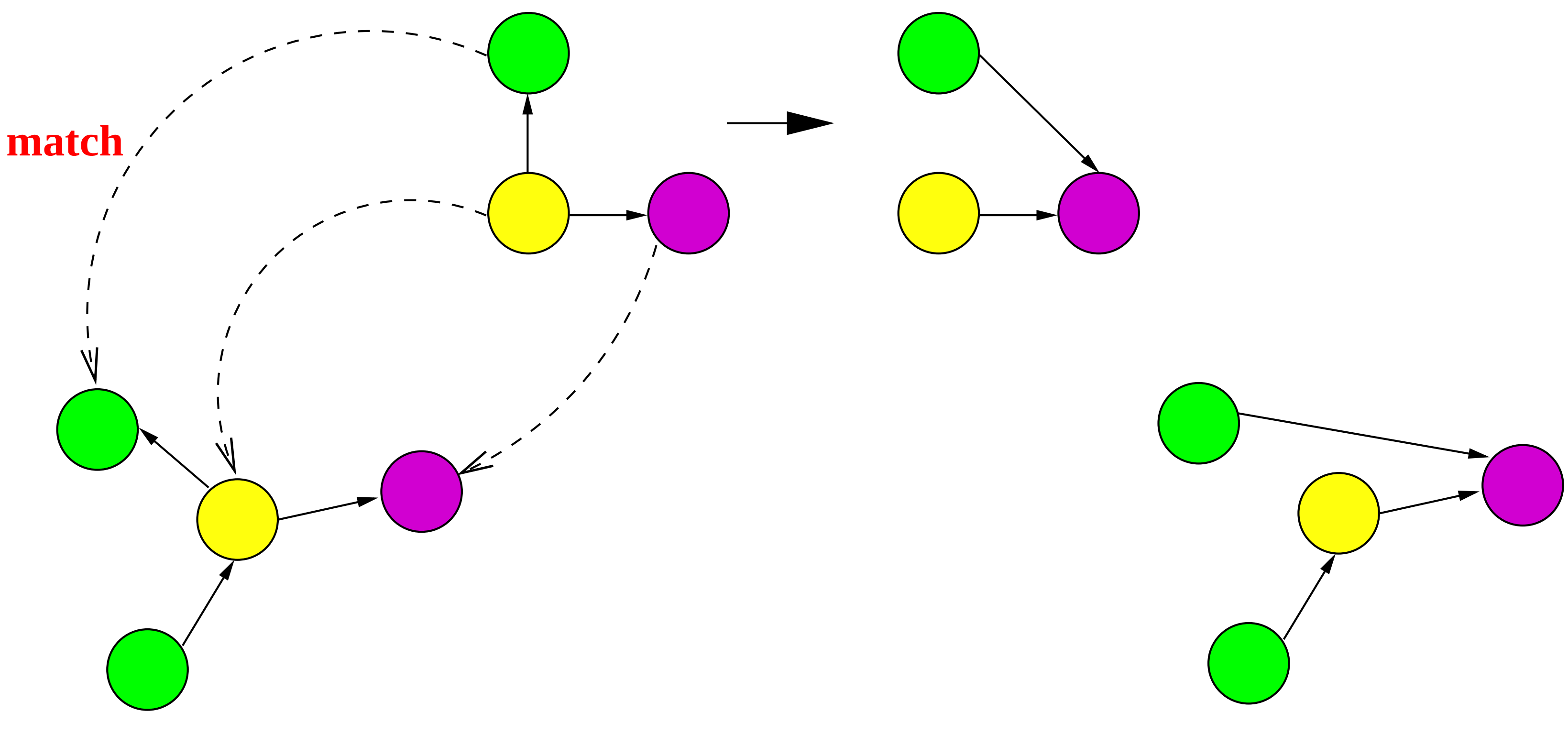
end of merge



hand over follower from rear to front

Sample Rule Application

Follower Hand-Over



Overview

- Partner graph grammars (PGG)
- Abstract interpretation of PGG's
  - Abstraction function
  - Abstract transformers
- Results
- Extensions, conclusions, further reading

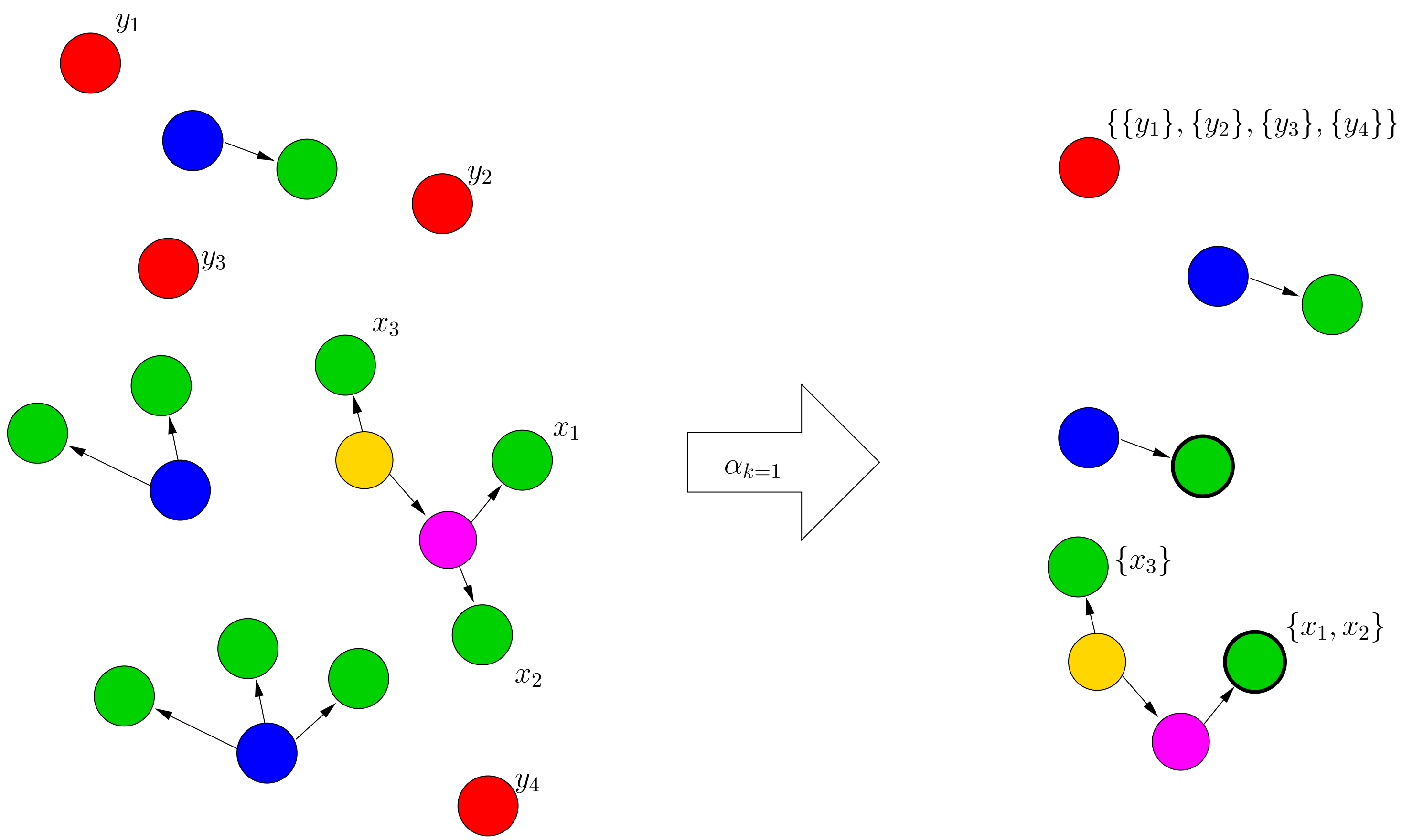
Abstraction of Graphs

- Two-layered abstraction.
- For each connected component (cluster) of graph (V, E, ℓ), two nodes are partner equivalent iff

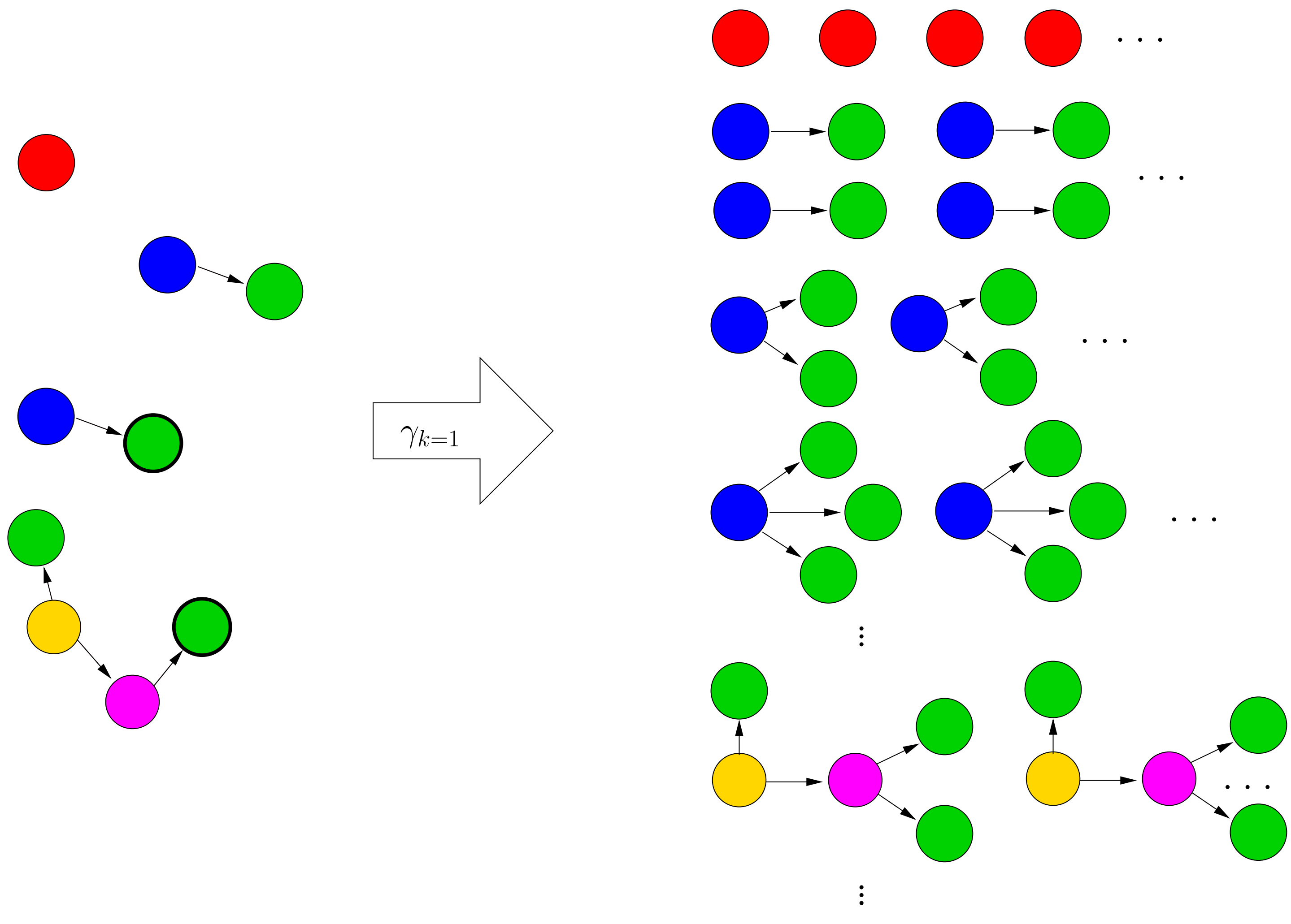
$$\ell(u) = \ell(v) \ \wedge \ \{\ell(u') \mid \{u', u\} \in E\} = \{\ell(v') \mid \{v', v\} \in E\}$$

1. Build quotient graph cluster-wise wrt. partner equivalence while tracking multiplicity up to some k.
2. Summarize isomorphic connected components: abstract clusters

Example Abstraction



Concretization

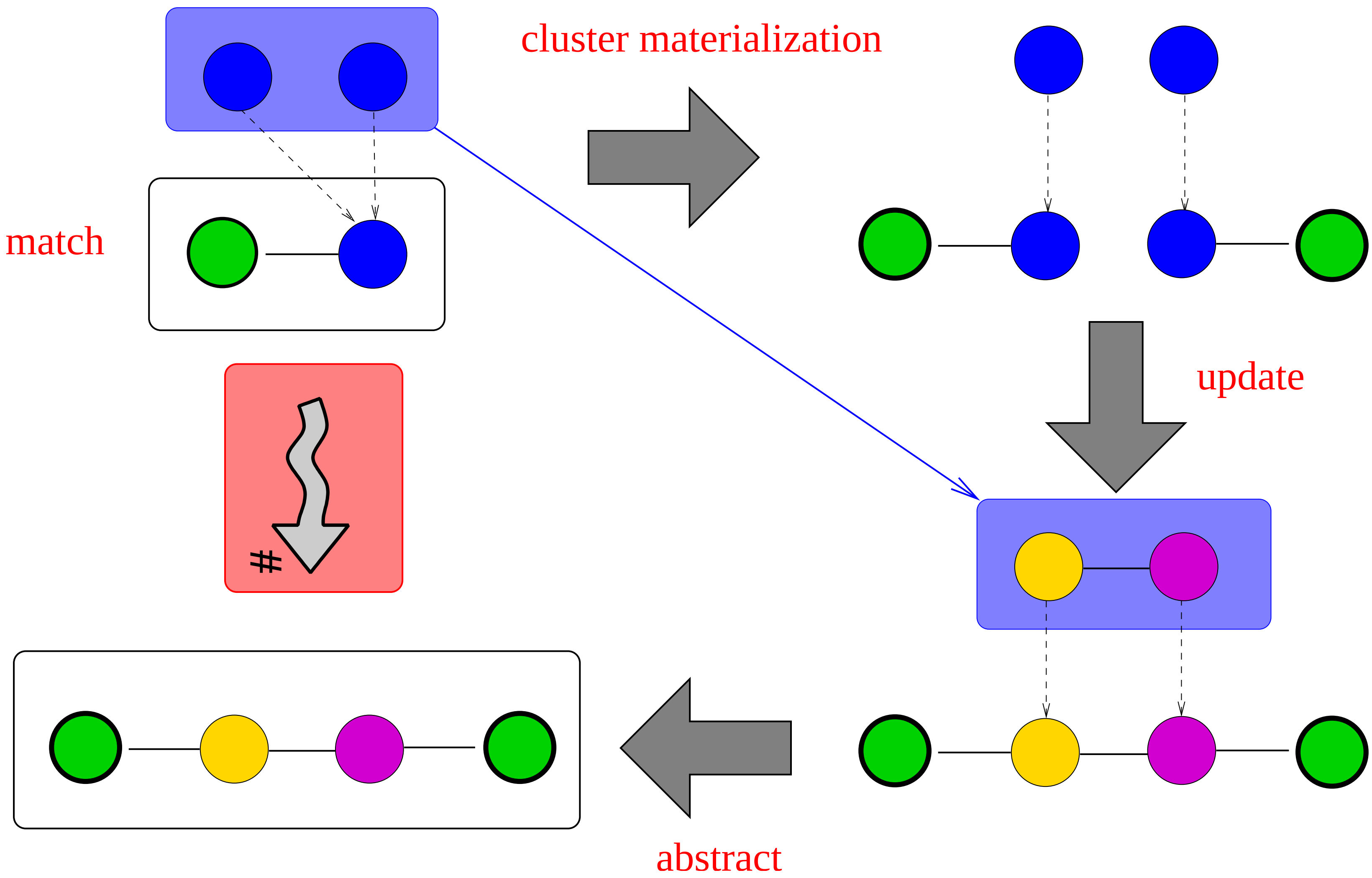


Abstract Updates—Best Abstract Transformers

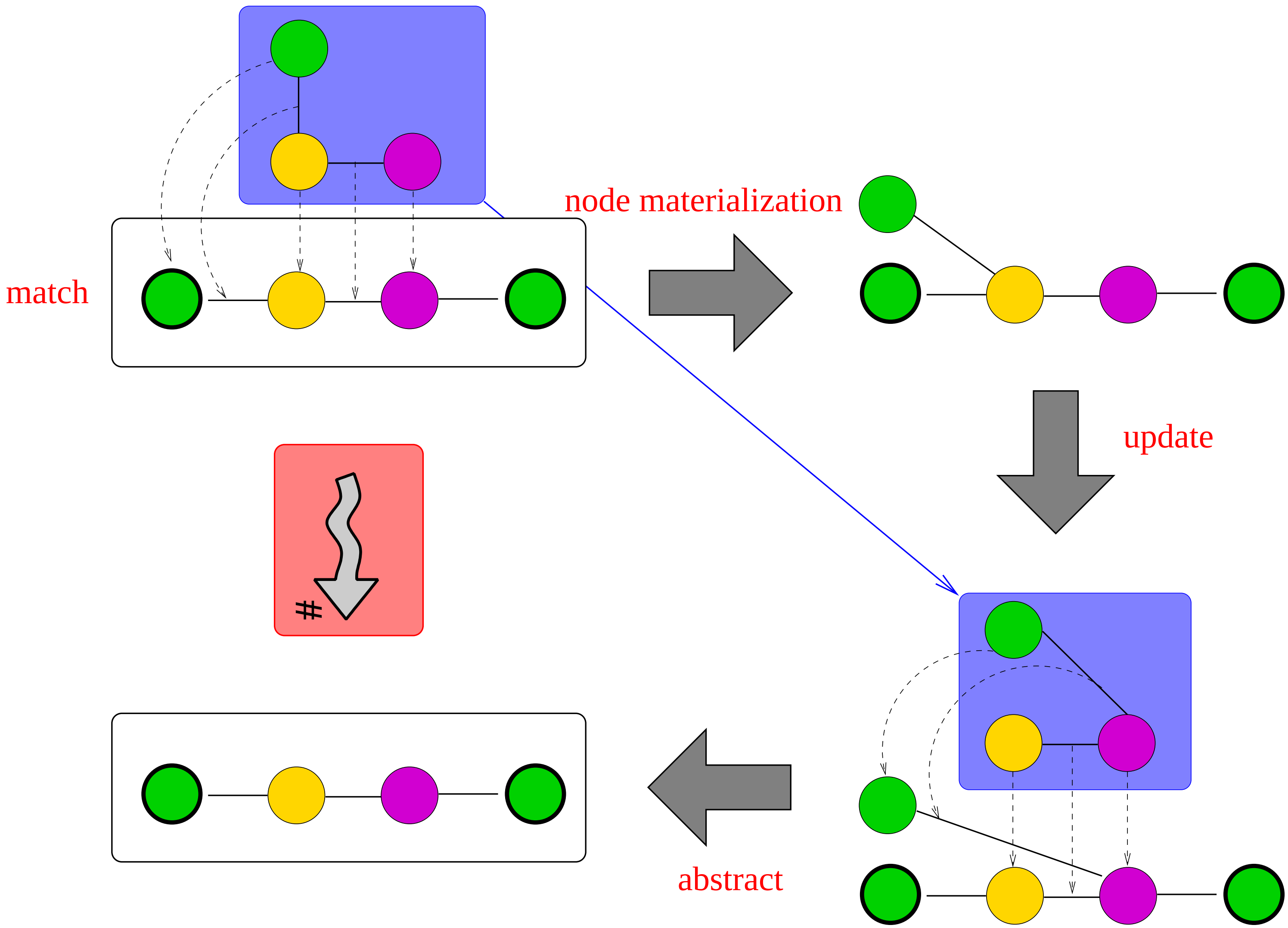
1. Non-injective matches due to abstraction.
2. Materialization locally undoes abstraction: injective match
   - Node materialization
   - Cluster materialization
3. Update like in the concrete case.
4. Abstract to guarantee boundedness.

- Abstract graph semantics: finite, bounded set $[\![\mathcal{R}]\!]^k$ of abstract clusters

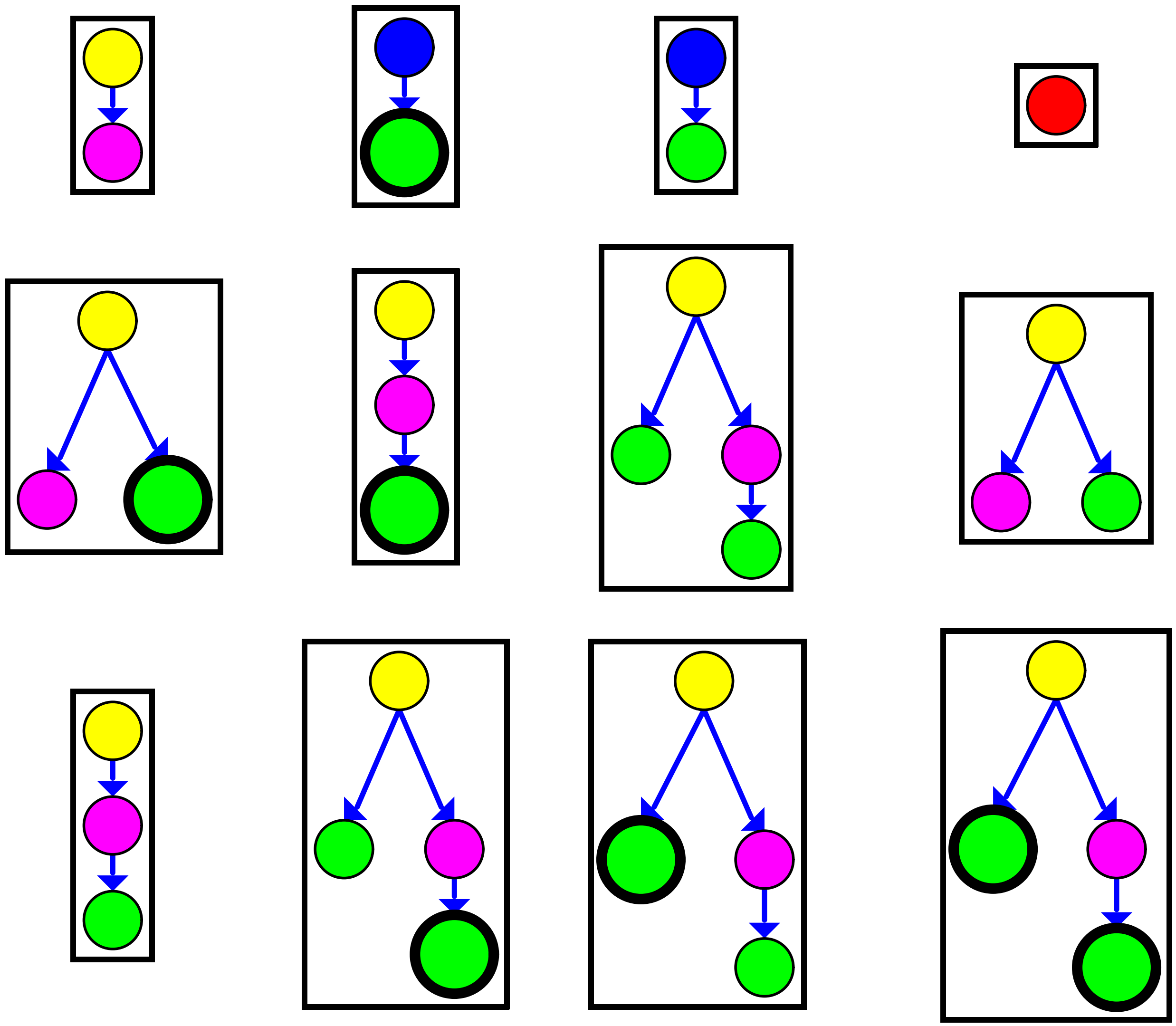
Sample Abstract Update



Sample Abstract Update



Platoon Abstract Graph Semantics



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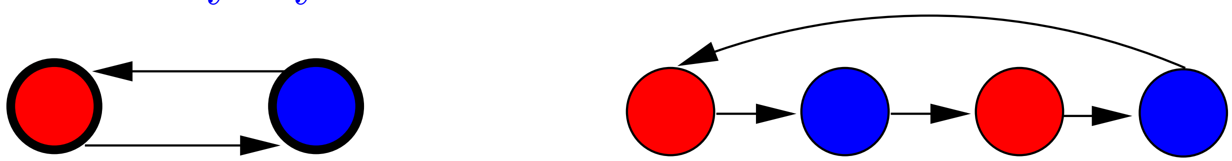
- Partner graph grammars (PGG)
- Abstract interpretation of PGG's
- Results
  - Soundness
  - Completeness
  - Experiments
- Extensions, conclusions, further reading

Soundness

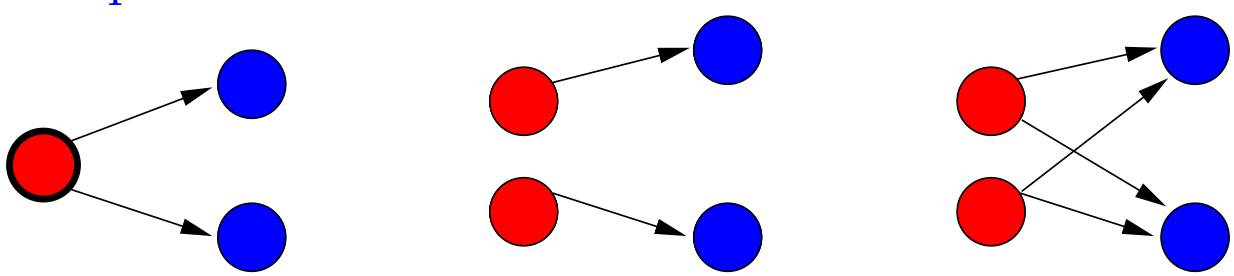
- The abstract graph semantics is an over-approximation of the concrete graph semantics: $\bigcup_{G \in [\![\mathcal{R}]\!]} \alpha_k(G) \subseteq [\![\mathcal{R}]\!]^k$
- Forbidden Subgraphs
  - Given a forbidden subgraph F
  - $\mathcal{N} := \mathcal{N} \dot{\cup} \{\mathit{evil}\}$
  - $\mathcal{R} := \mathcal{R} \dot{\cup} \{F \to \mathit{evil}\}$
  - If $\mathit{evil} \notin [\![\mathcal{R}]\!]^k$, then F does not occur in $[\![\mathcal{R}]\!]$

Towards Completeness

- Friendly Grammars: No initial graph implies cluster multiplicity.
- Summary Cycles.



- Unique Partners.



- Lone Summaries: No adjacent summary nodes.
- Inductive Summaries: all concretizations possible.

Completeness

- No summary cycles + unique partner imply Matching Theorem: Abstract matches are equivalent to concrete matches.
- Lone summaries + unique partner + inductive summaries imply cluster completeness: $\bigcup_{G \in [\![\mathcal{R}]\!]} \alpha_k(G) = [\![\mathcal{R}]\!]^k$.
- Property preservation: Lone summaries + unique partners imply (for connected graphs C) $C \models \psi \iff \alpha_k(C) \models \psi$, for first-order formulas ψ without equality.

Experimental Evaluation of Platoons

Example	N. Labels	E. Labels	Rules	Constraints	Clusters	Rule App.	Iter
Merge	5	1	8	1	12	19	5
Split	6	1	4	4	13	15	4
Combined	6	1	12	5	169	302	10
Combined+	6	1	12	9	22	34	7
Queues	18	4	30	34	159	163	40
Faulty1	5	1	32	1	20	53	6
Faulty3	5	2	32	1	450	1206	10

- Relevant topology properties proven, sometimes completeness.
- Useful for protocol design, too.
- Partner graph grammars are highly sensitive to changes, in particular to partner constraints.

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Existing Extensions

- Syntactic sugar for PGG's; directed, edge-labeled graphs.
- Counting clusters.
- General clusters.
- A logic for reasoning about PGG's.

Potential Extensions

- Attributed graphs for queue implementations.
- Unified framework with general, property-driven . notions of (partner) equivalence.
- More hierarchy than just 2 (ad hoc routing).
- Beyond dynamic communication systems.

What have you learnt?

- Graph Grammars: powerful specification formalism for highly dynamic systems
  - → Motives: Reiko Heckel, Andy Schürr.
  - → `http://gratra.org/`
  - → Handbook of Graph Grammars and Computing by Graph Transformation. World Scientific 1997 - 99, published in three volumes.
- Platoon case study
  - → `http://www.path.berkeley.edu/`

What have you learnt?

- Abstract interpretation: powerful verification formalism
  - → Patrick & Radhia Cousot. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. ACM POPL 1977.
  - → Patrick & Radhia Cousot. Systematic design of program analysis frameworks. ACM POPL, 1979.
  - → Hanne Nielson, Flemming Nielson, and Chris Hankin. Principles of Program Analysis. Springer 1999.
  - → Motives: Sylvie Putot, Reinhard Wilhelm, Hanne Riis Nielson.

What have you learnt?

- Graph grammar verification
  - → Barbara König, Vitali Kozioura. Counterexample-Guided Abstraction Refinement for the Analysis of Graph Transformation Systems., TACAS 2006.
  - → Arend Rensink, Dino Distefano. Abstract Graph Transformation. Electr. Notes Theor. Comput. Sci. 157(1): 39-59 (2006).
  - → Reiko Heckel. Compositional Verification of Reactive Systems Specified by Graph Transformation. FASE 1998.
  - → `http://www2.imm.dtu.dk/~joba/phd.pdf`