



TECHNISCHE UNIVERSITÄT
CAROLO-WILHELMINA
ZU BRAUNSCHWEIG



SymTA/S

Symbolic Timing Analysis for Systems

ARTIST2 PhD Course, June 12, DTU Copenhagen, Denmark

Razvan Racu

Arne Hamann



Day schedule

0900 – 0945 Introduction to system performance verification

1000 – 1045 Compositional performance analysis

1100 – 1200 Hands-on tutorial 1: Basics SymTA/S

1330 – 1415 Sensitivity analysis

1430 – 1515 Design space exploration and robustness optimization

1530 – 1630 Hand-on tutorial 2: Advanced SymTA/S features

1630 – 1700 Discussion





System design challenges

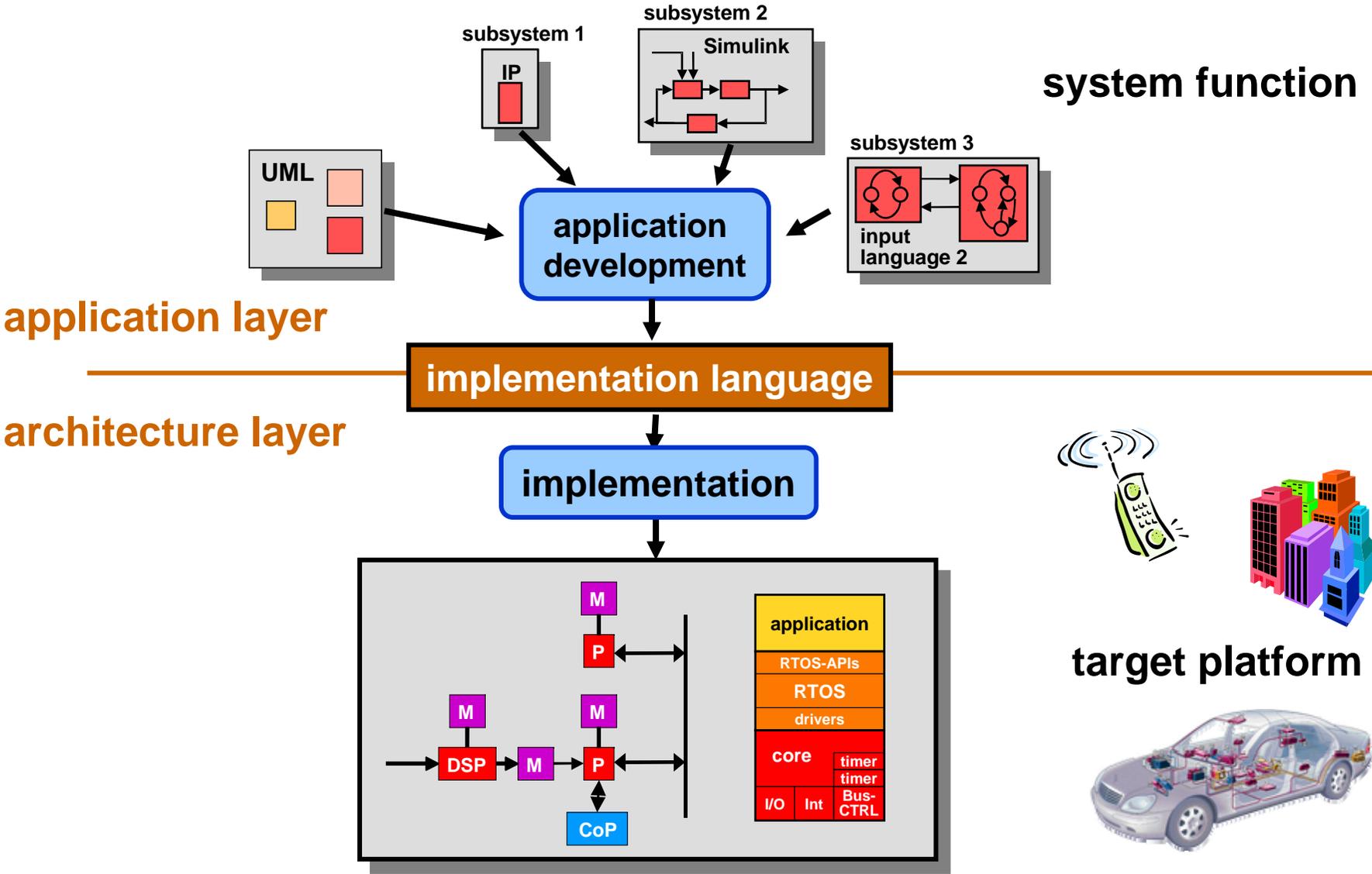
Functional vs. performance verification

- **Separate function verification from performance verification**
 - **functional** verification/test determines functional correctness **independent of the target architecture**
 - **performance** verification/test determines platform adherence to
 - load conditions and response times (deadlines)
 - jitter bounds
 - buffer sizes

This presentation is about **performance verification !!**



Introduction

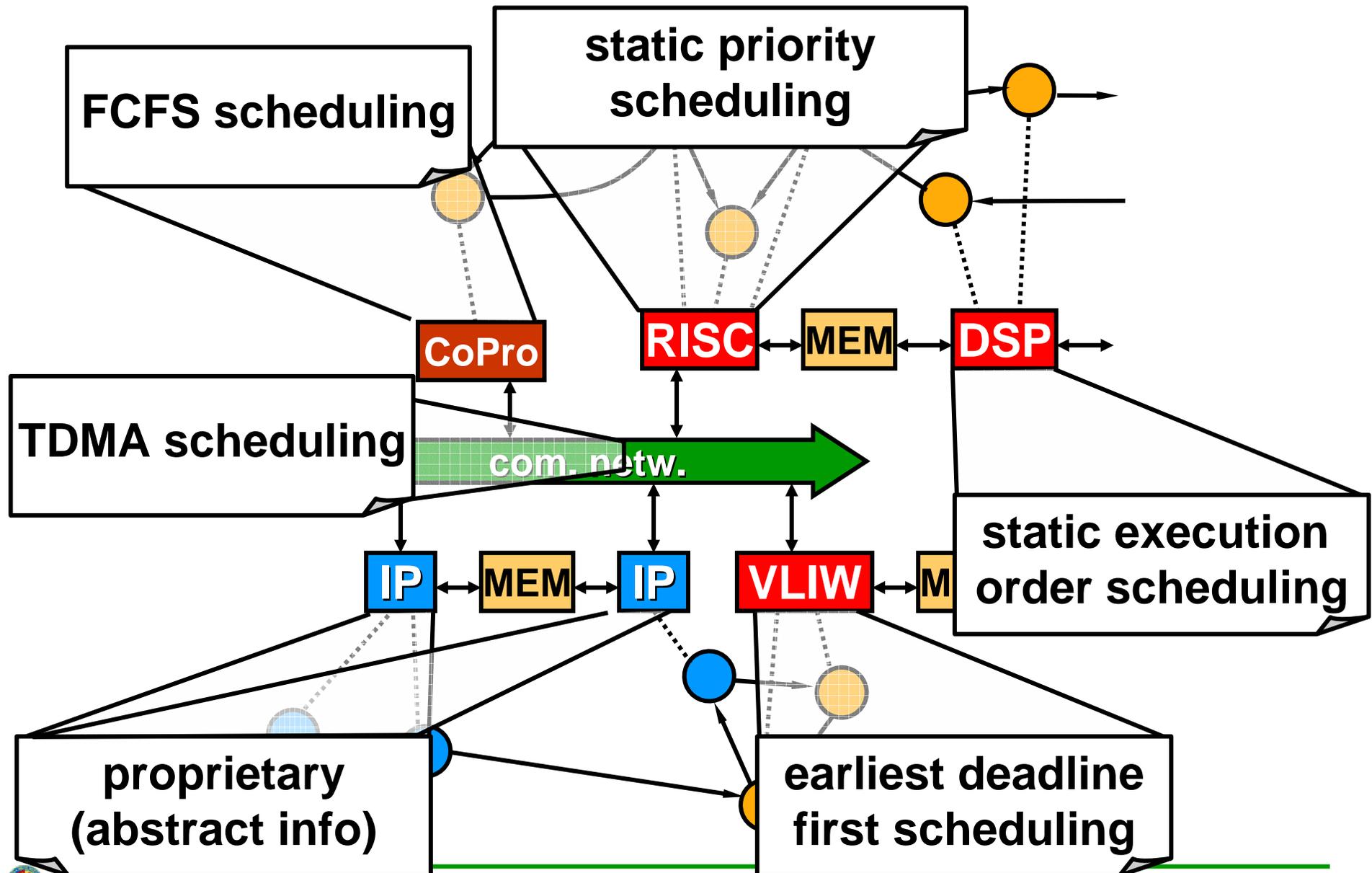


Embedded system platform properties

- ES platforms are **heterogeneous**
 - components
 - networks
 - communication
 - **scheduling (static, dynamic, event-, time-driven, ...)**
 - ...
- Heterogeneity results from
 - hardware and software component specialization (cost, power, dependability)
 - HW/SW reuse



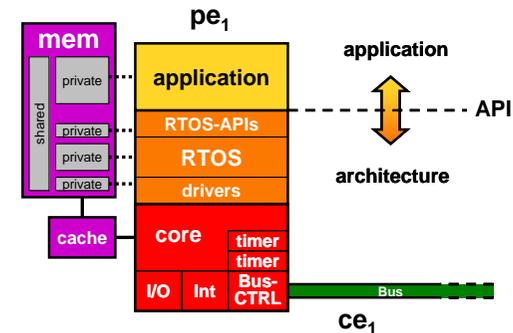
Heterogeneous resource sharing



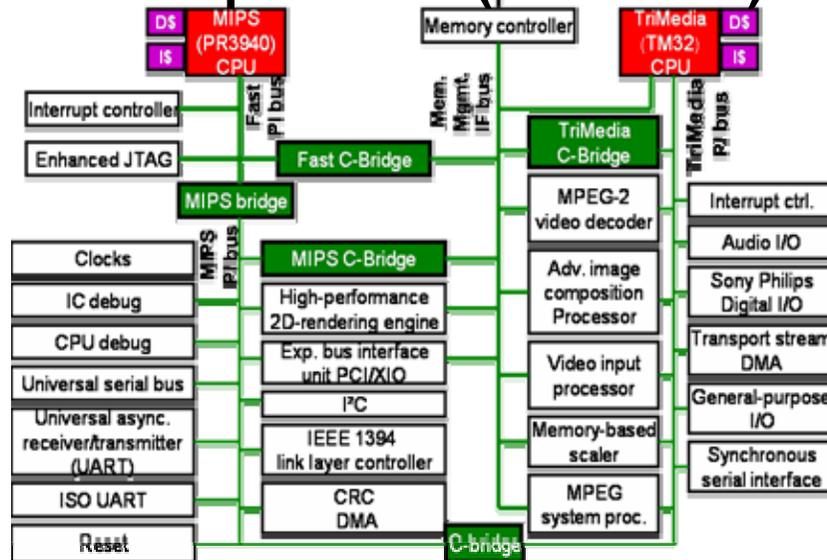
Exemple 1 : MPSOC

- Heterogeneity resulting from
 - hardware and software component specialization
 - reuse

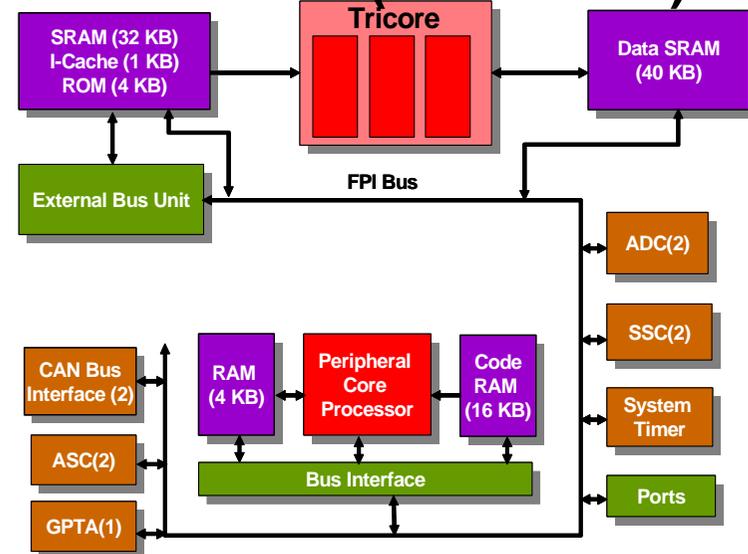
multilayered SW



Philips VIPER (consumer)

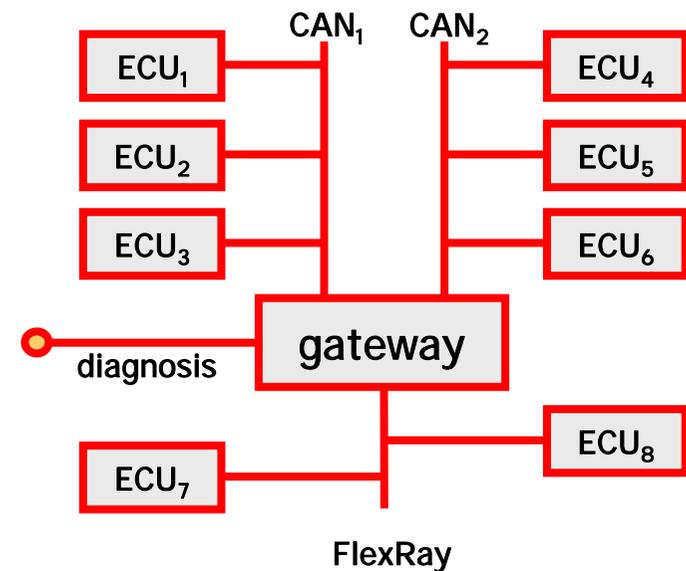
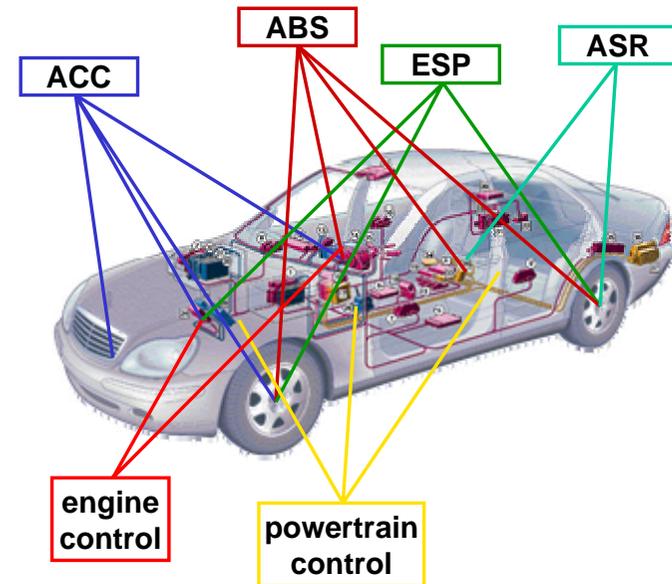


TriCore 1775 (automotive)



Example 2: Automotive Platform

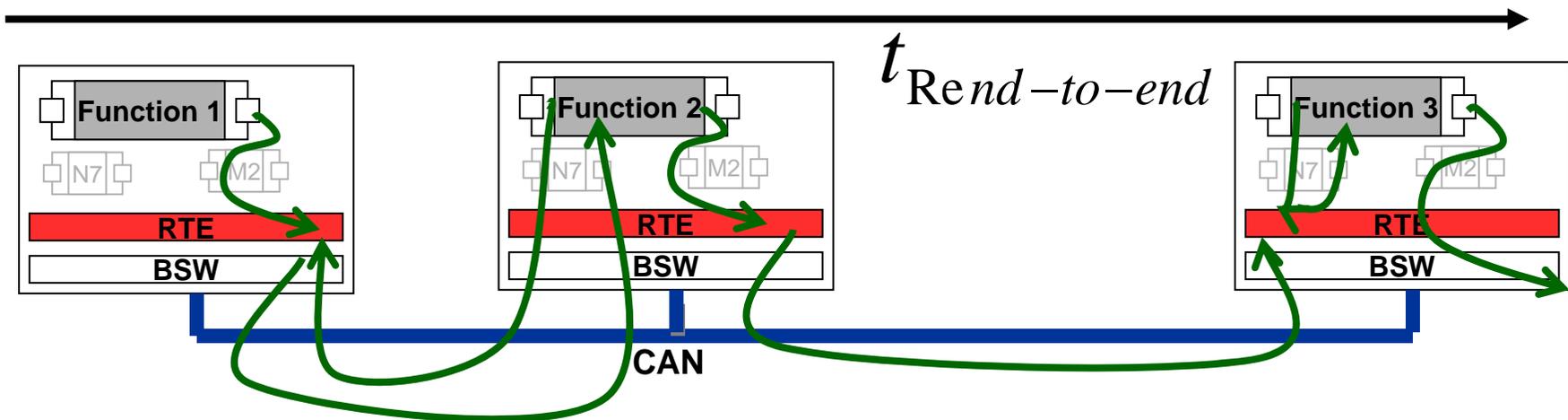
- **Heterogeneous**
 - 50+ ECUs
 - many suppliers
 - several RTOSes and protocols
 - strongly networked
- **Complex**
 - end-to-end deadlines
 - hidden dependencies
 - global memories



End-to-end times do not easily compose

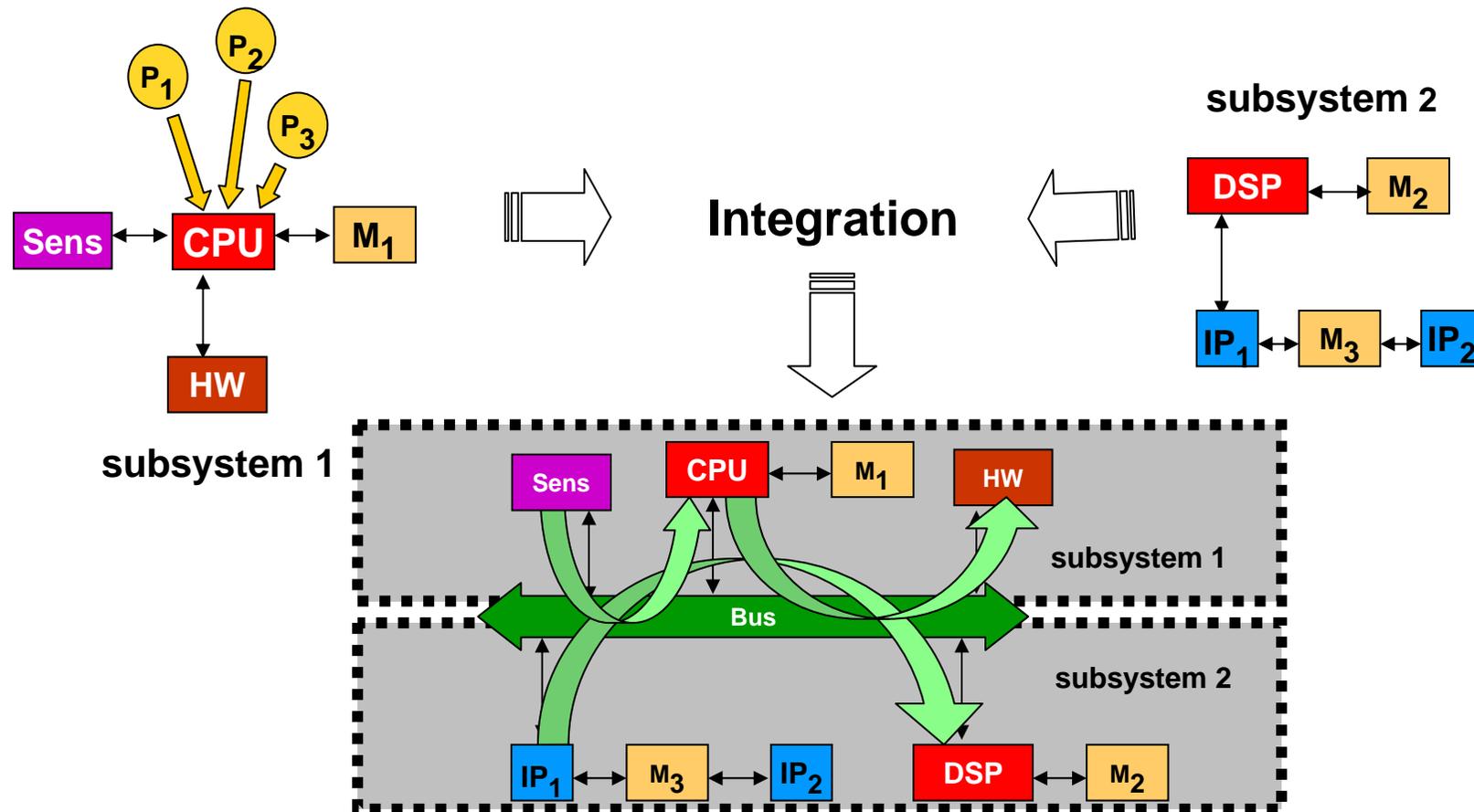


$$t_{RFunction1} + t_{RFunction2} + t_{RFunction3} \neq t_{Rend-to-end}$$



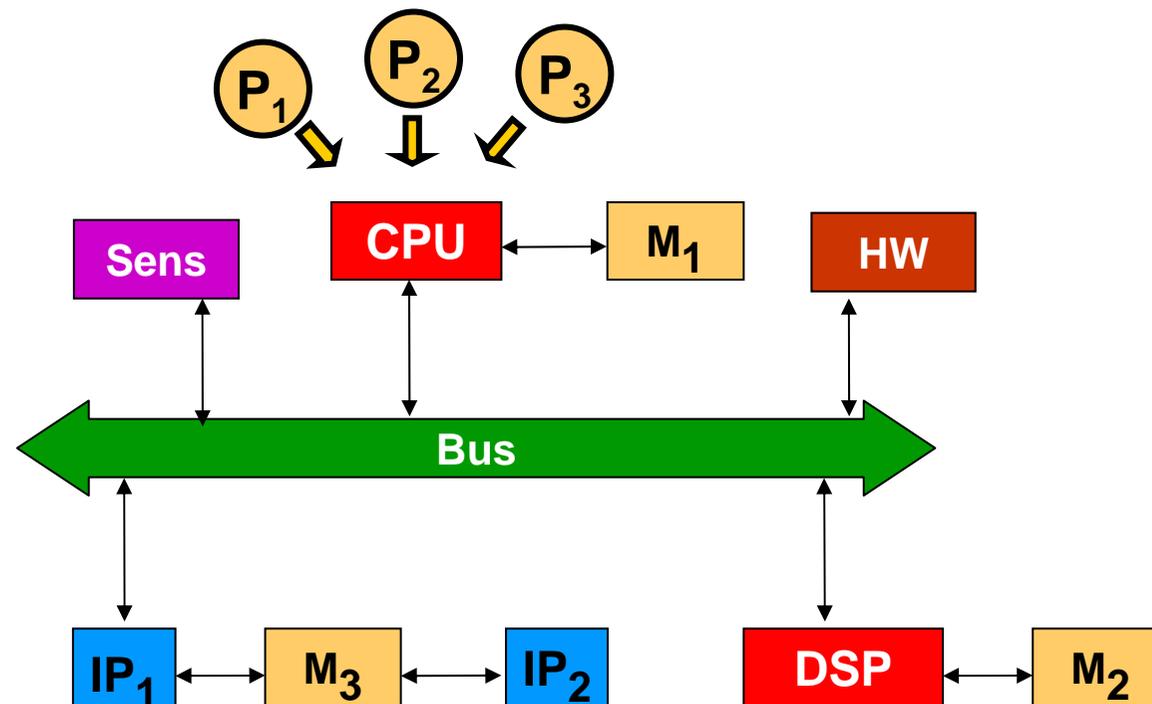
Design as integration problem

- System design is to a large extent an integration problem

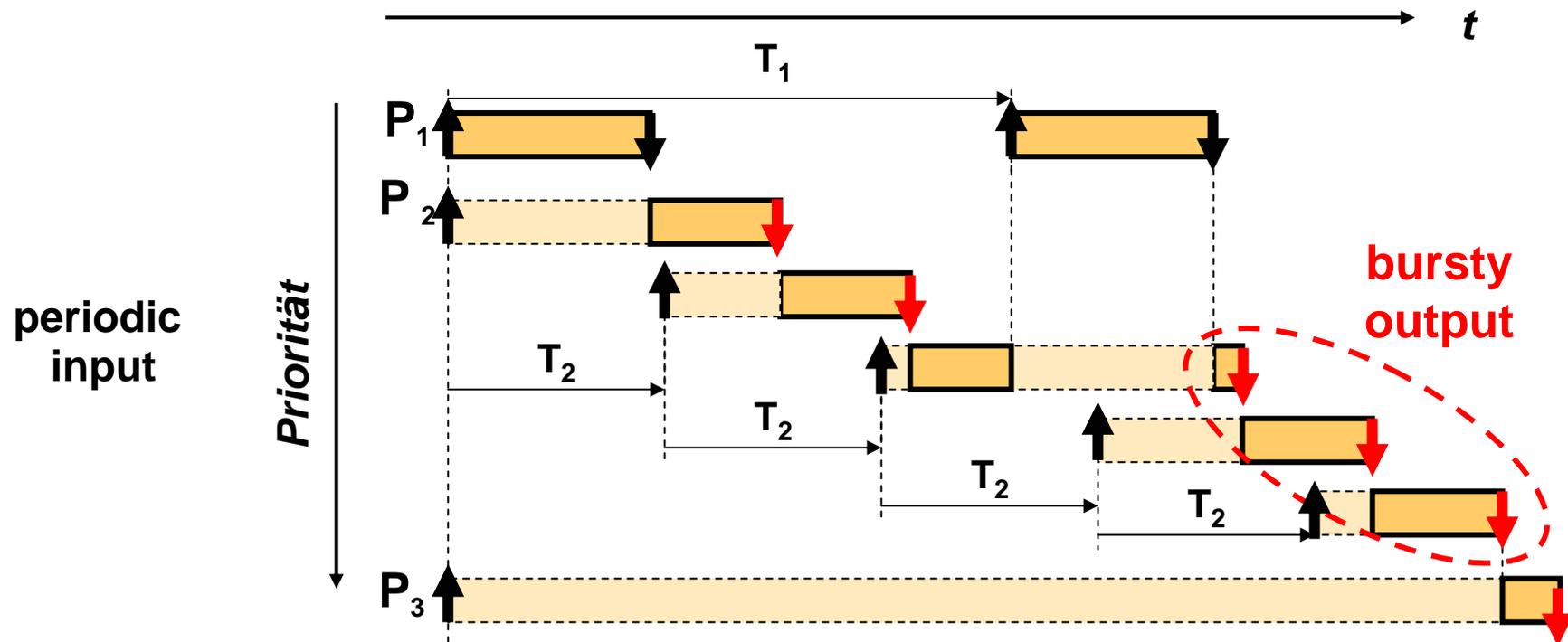


Coupling effects – a closer look

- Example: 3 periodic tasks on CPU send data over the bus
- Static priority scheduling on CPU: $P_1 > P_2 > P_3$



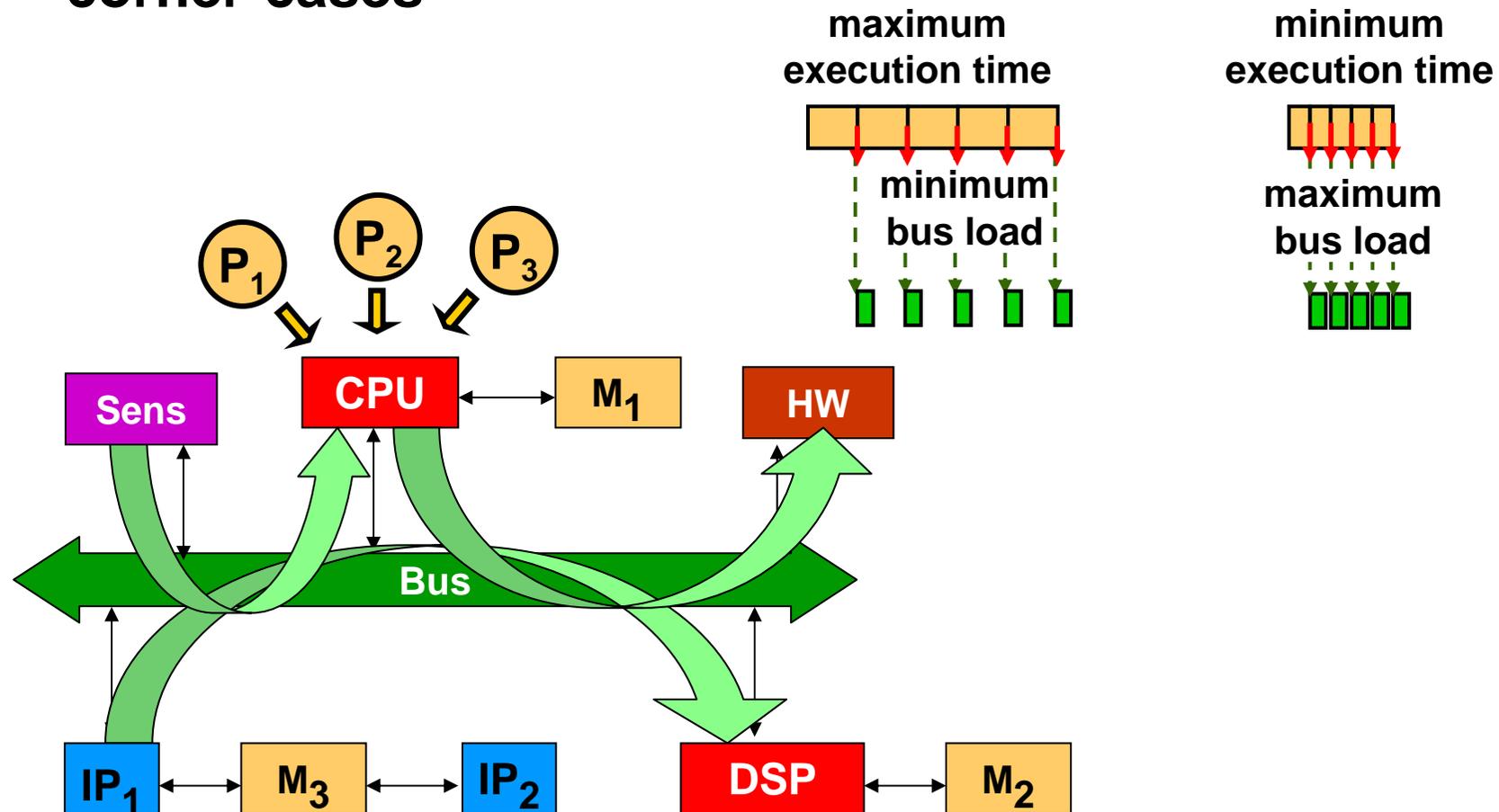
Coupling effects – creation of bursts



- Complex execution traces with dynamic behavior
- Burst events at the output
- Consequences: transient overload, missed deadlines, data loss, ...

Scheduling anomalies

- System corner-cases different of component corner-cases

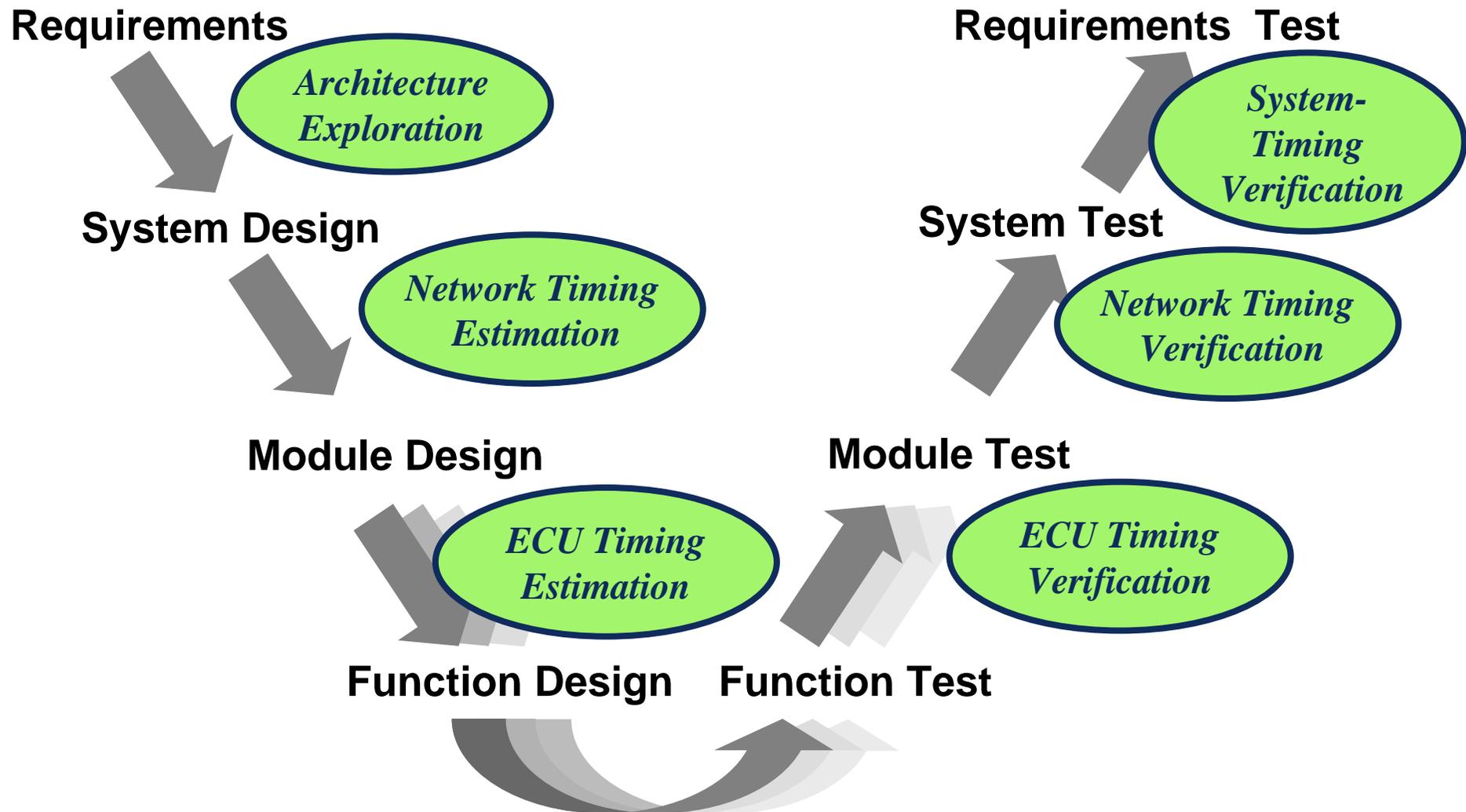


Key platform design challenges

- **Increasing system complexity**
 - from single processor to multi-processor (MpSoC)
 - from buses to networks (NoC)
- **Complex dependencies and modifications threaten design robustness**
- **Global end-to-end constraints added for control applications**
- **Integration under optimization requirements**
 - cost (memory, power, ...)
 - robustness
 - extendibility – consider upcoming features, SW updates, platform updates in product lines
- **Reliable system integration is key requirement**
 - **Performance verification** required at every design stage



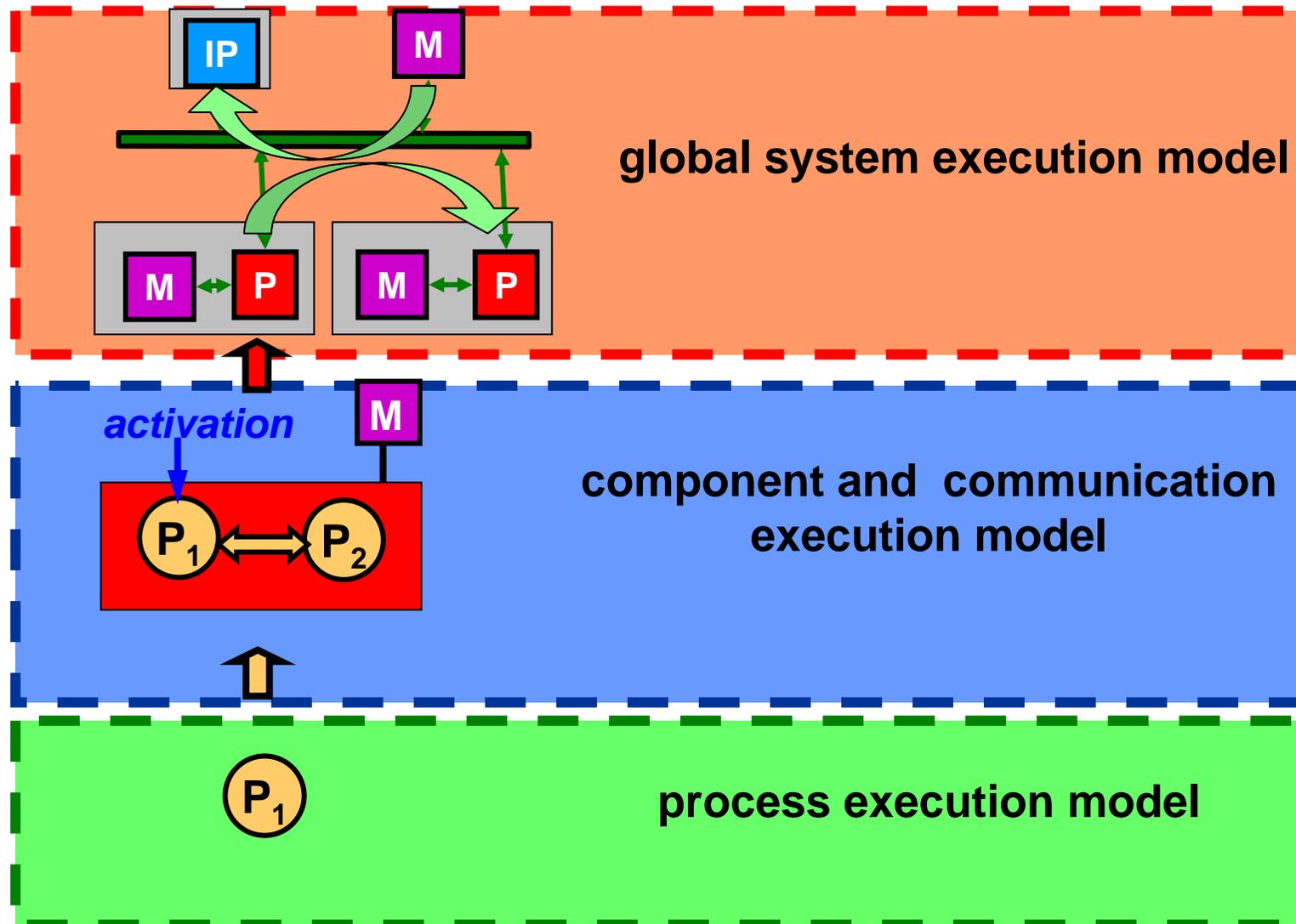
Timing is everywhere



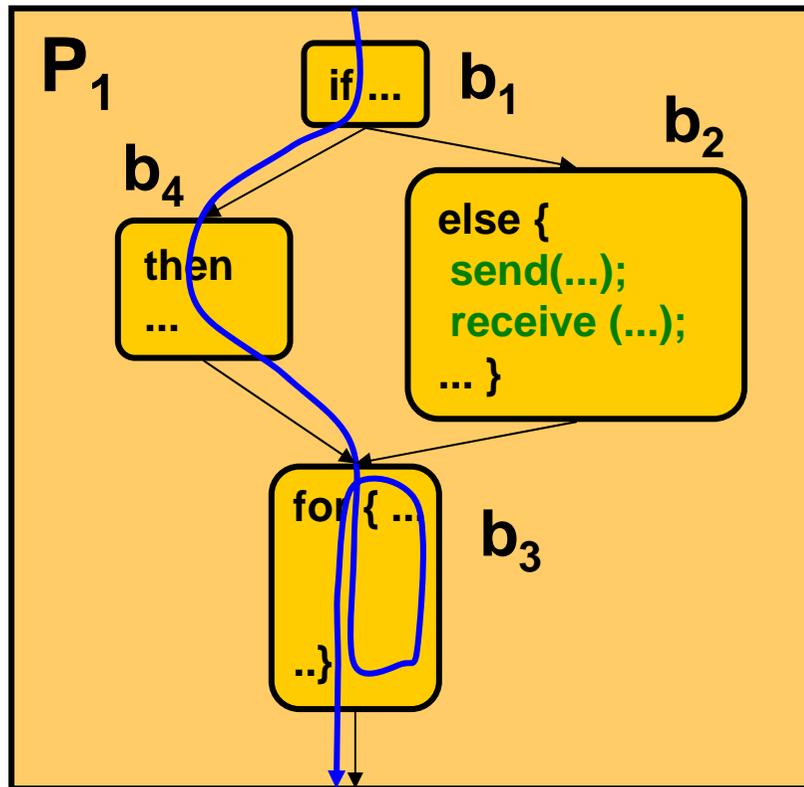


Performance verification flow

Target architecture performance – general view



Process execution model



- Influenced by
 - execution path
 - data dependent
 - execution path *timing*
 - target architecture dependent
 - process communication (here: message passing)
 - execution path dependent
 - communication volume
 - data and type dependent



Process timing and communication

- **State of industrial practice - simulation/performance monitoring**
 - trigger points at process beginning and end
 - data dependent execution → upper and lower timing bounds
- **simulation challenges**
 - coverage?
 - cache and context switch overhead due to run-time scheduling with process preemptions
- **Alternative - formal analysis of individual process timing**
 - provides conservative bounds
 - serious progress in recent years

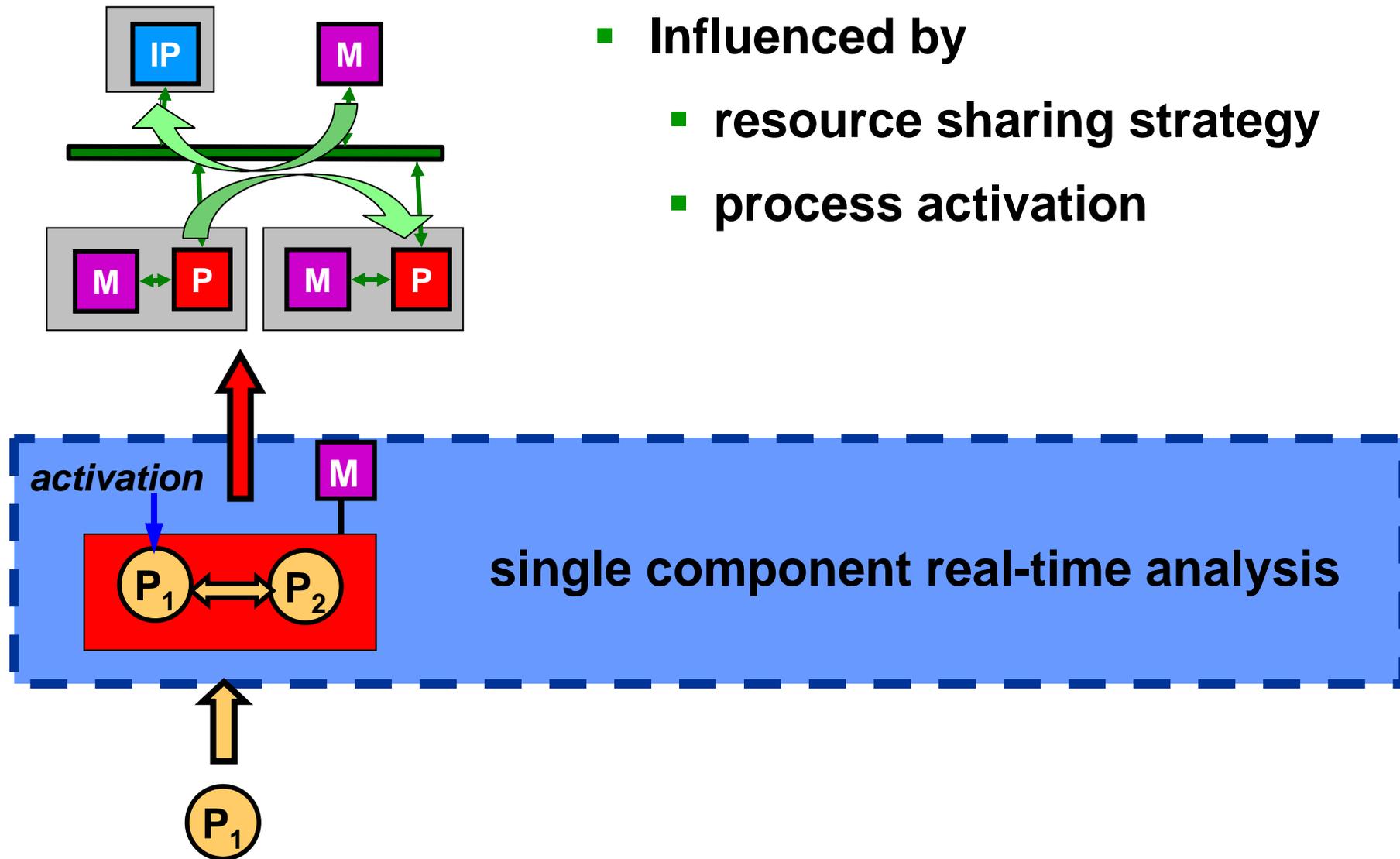


Formal process execution time analysis

- Active research area with dedicated events (e.g. Euromicro WS)
- Formal analysis using simple processor models
 - Li/Malik (Princeton) (95): Cinderella
- Detailed execution models with abstract interpretation
 - Wilhelm/Ferdinand (97 ff.): commercial tool AbsInt
- Combinations with simulation/measurement of program segments
 - Wolf/Ernst (99): SymTA/P
- All tools provide (conservative) upper execution time bounds (WCET) or time intervals (WCET/BCET)



Component and communication execution model



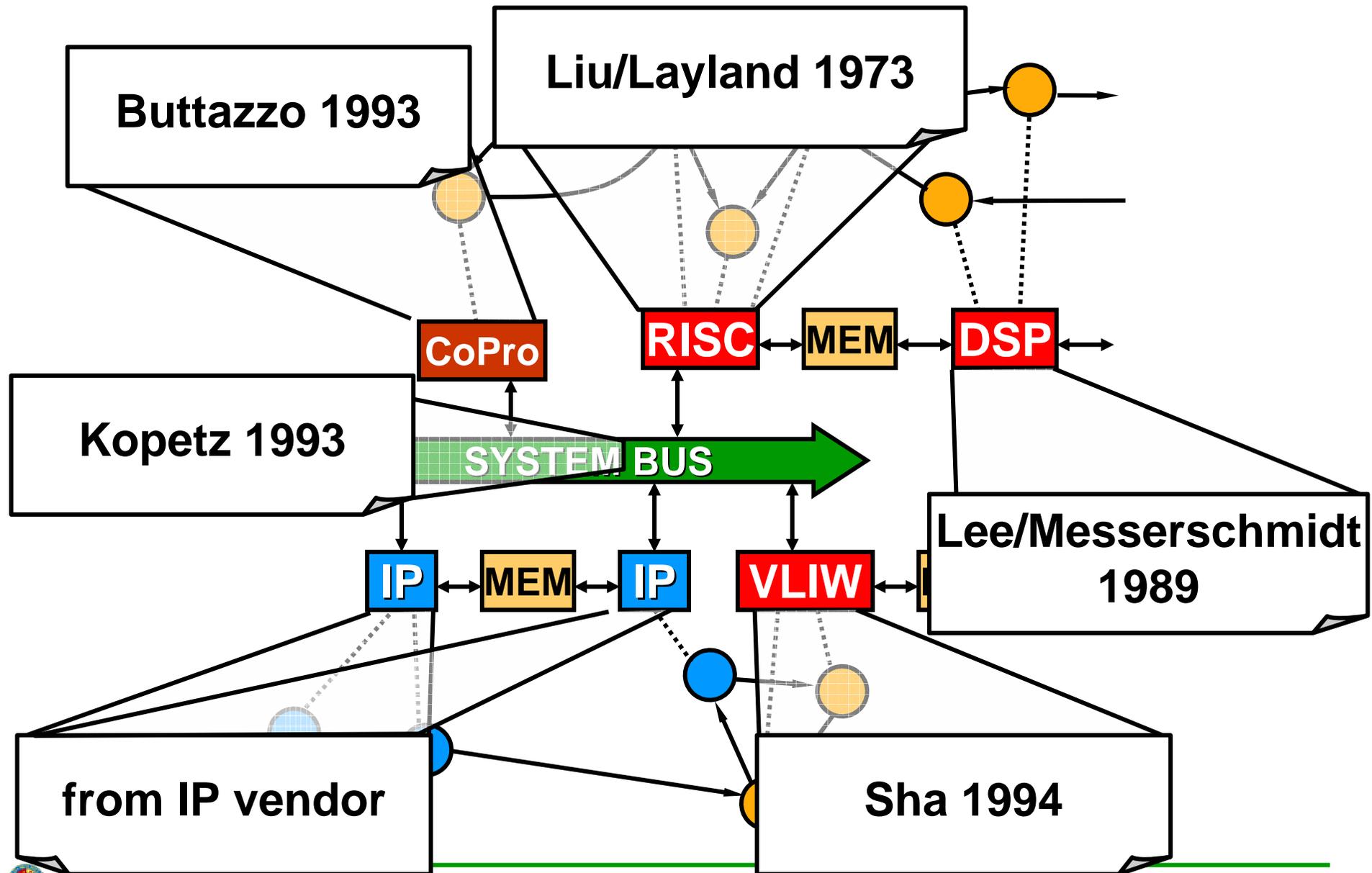
- Influenced by
 - resource sharing strategy
 - process activation

Component and communication execution model

- Resource sharing strategy
 - process and communication scheduling
 - static execution order
 - time driven scheduling
 - fixed: TDMA
 - dynamic: Round-Robin
 - priority driven scheduling
 - static priority assignment: RMS, SPP
 - dynamic priority assignment: EDF
- Timing depends on environment model
 - determines frequency of process activations or communication



Scheduling Analysis Techniques



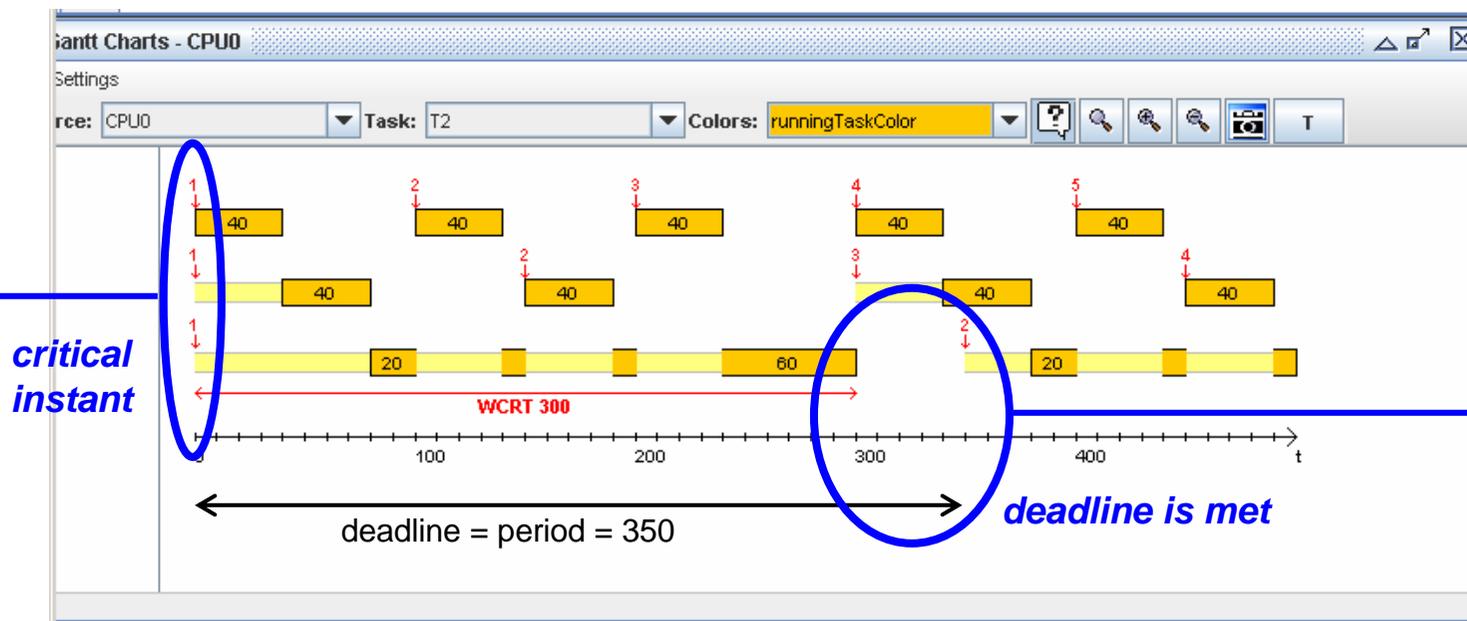
Example: Rate Monotonic Scheduling (RMS)

- **Very simple system model**
 - **periodic tasks with deadlines equal to periods**
 - **fixed priorities according to task periods**
 - **no communication between tasks**
 - **(theoretically) optimal solution for single processors**
 - **several practical limitations but good starting point**
- **Schedulability tests for RMS guarantee correct timing behavior**
 - **processor utilization (load) approach**
 - **response time approach (basis for many extensions)**



RMS Theory – The response time approach

- **Critical instant:**
all tasks start at $t=0$ („synchronous assumption“ to ensure maximum interference in the beginning of task execution)
- when each task meets its first deadline, it will meet all other future deadlines (proof exists!)
- test by „unrolling the schedule“ (symbolic simulation)

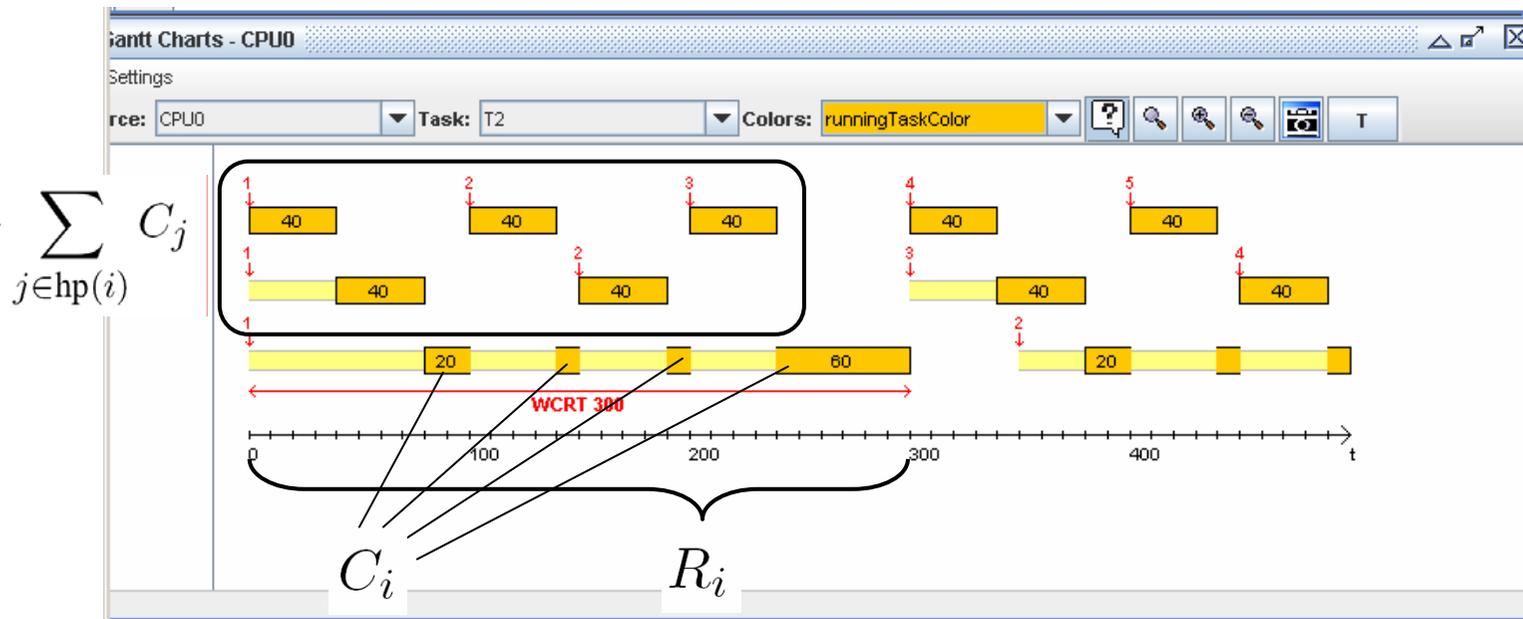


RMS Theory – The response time formula

fix-point problem

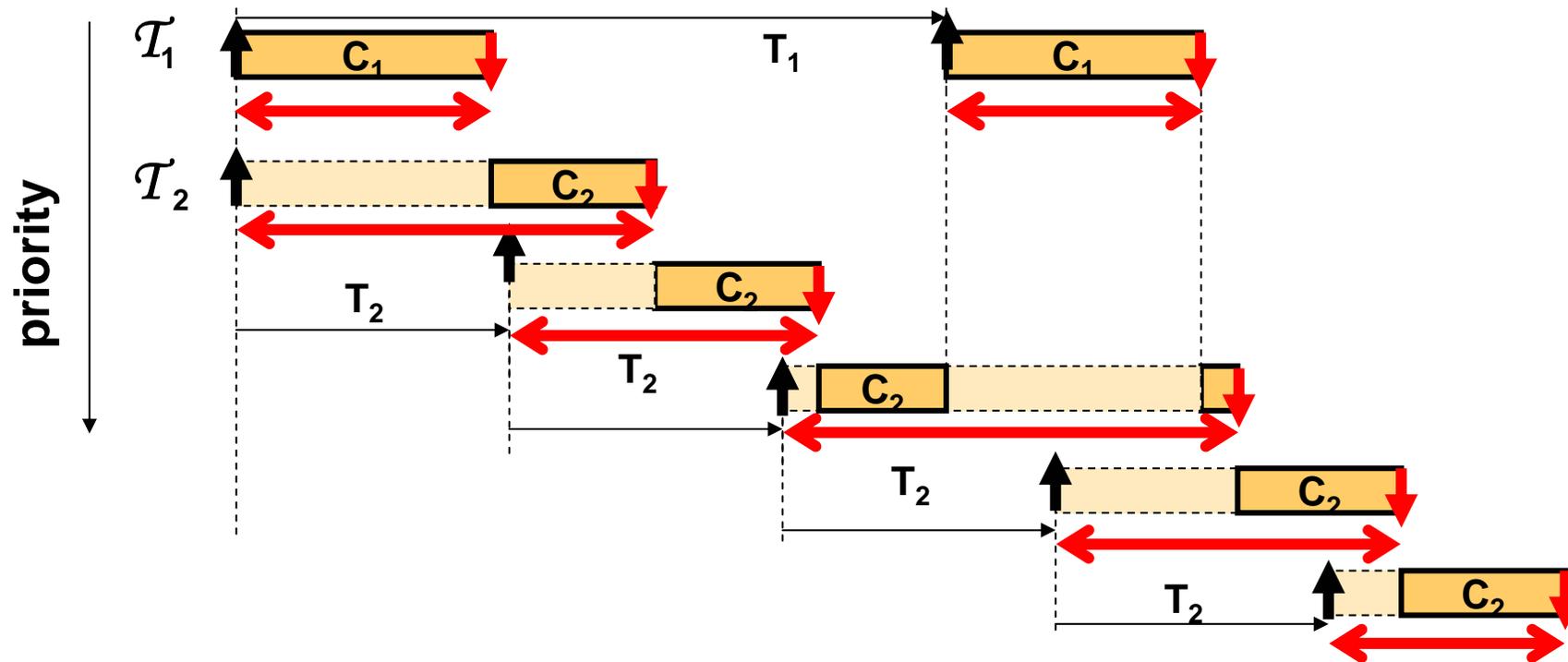
$$R_i = C_i + \underbrace{\sum_{j \in \text{hp}(i)} C_j \left\lfloor \frac{R_i}{T_j} \right\rfloor}_{\text{interference term } I_i} \leq D_i = T_i$$

↑ response time
↑ core execution time
of preemptions

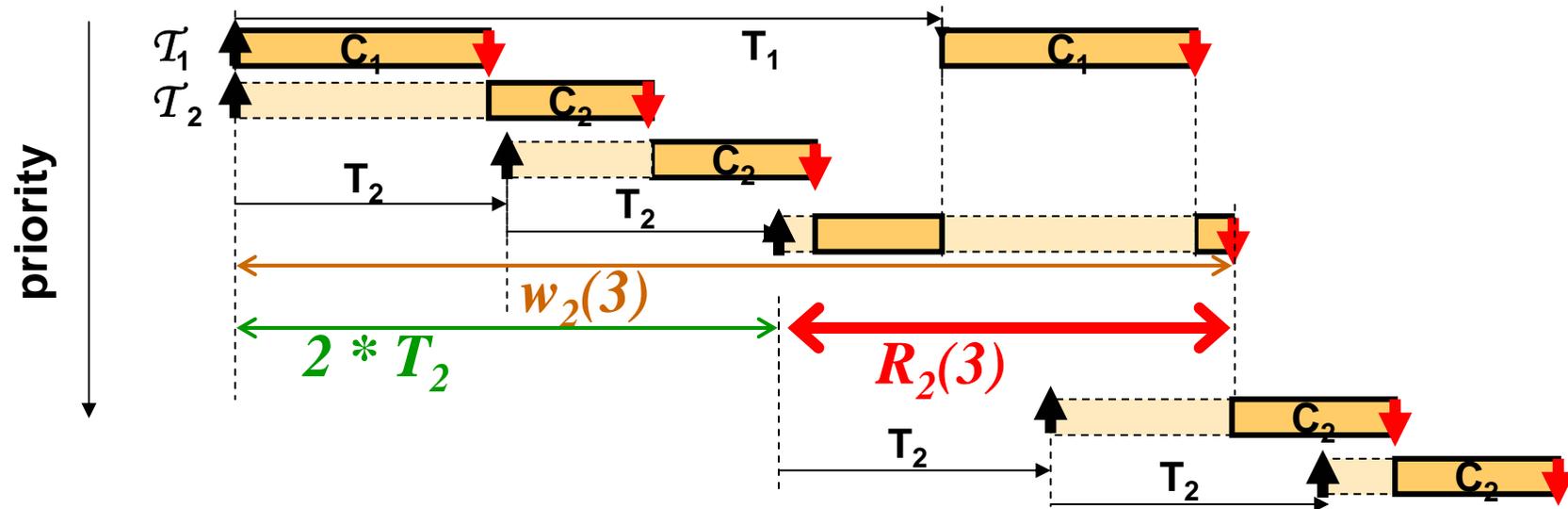


Example: Static priority w/ arbitrary deadlines

- Assume:
 - tasks with periods T , worst-case execution times C
 - static priorities
 - deadlines (**arbitrary**) larger than the period



Analysis uses “Busy Window” approach (Lehoczky)



$$w_i(q) =$$

$$q C_i + \sum_{j \in \text{hp}(i)} C_j \left\lceil \frac{w_i(q)}{T_j} \right\rceil$$

$$R_i(q) =$$

$$w_i(q) - (q - 1) T_i$$

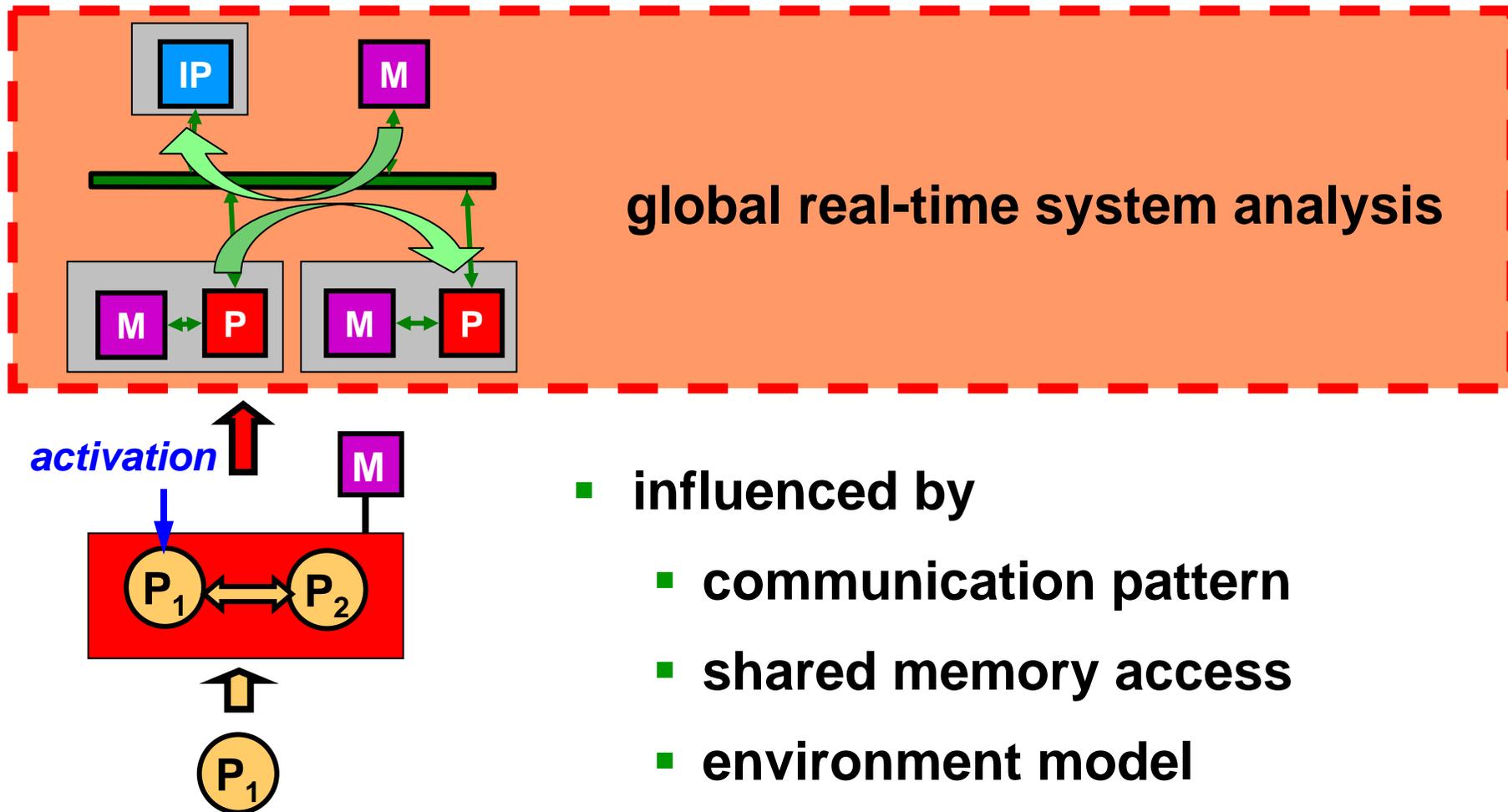
find fix point
where
equations
hold!

Other Extensions in Literature

- **Jitter and burst activation**
- **Static and dynamic offsets between task activations**
- **Different task modes**
- **Execution scenarios**
- **Blocking and non-preemptiveness**
- **Scheduling overhead → context switch time**
- **etc...**



Global system execution model



- influenced by
 - communication pattern
 - shared memory access
 - environment model



System performance analysis

System performance analysis - state of the art 1/2

- **Current approach: target architecture co-simulation, performance simulation**
- **Simulation challenges**
 - **identification of **system** performance corner cases**
 - different from **component** performance corner cases
 - complex phase and data dependent “transient” run-time effects w. scheduling anomalies
 - target architecture behavior unknown to the application function developer
 - **test case definition and selection?**
 - **simulation of incomplete application specifications ?**
 - how to do design space exploration before code implementation is available?



System performance analysis - state of the art 2/2

- Load analysis
 - Example: “all deadlines are met if the resource load is below 69%”
 - Consider only average scenarios (no transient load)
 - No performance metrics → **no constraint validation**



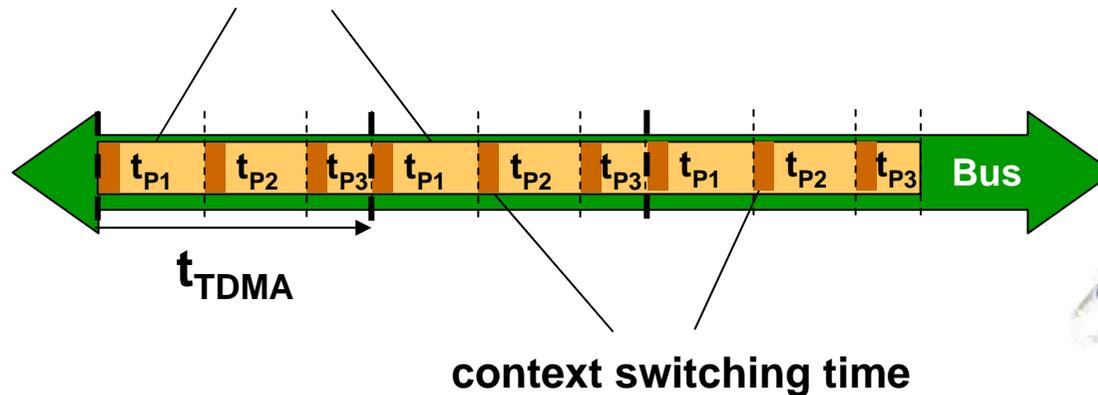
Conservative design

- Popular as a system level technique for safety critical systems design
- Strict separation of subsystems
 - fixed allocation of memory
 - fixed allocation of communication resources
 - fixed allocation of computation resources
- Spatial and temporal decoupling of resources
 - not-in-use allocated parts are locked
 - no coupling effects
- Requires system synchronization ...
- ... paid by timing overhead



TDMA 1/2

time slot assigned to sender P1



Time Triggered System (TDMA)

- periodic assignment of fixed time slots for communication and processing
- unused slots remain empty
- requires system synchronization
- no coupling effects

TDMA 2/2

- Predictable, independent system capacity

$$R_i = C_i + (t_{TDMA} - t_{P_i}) \times \left[\frac{C_i}{t_{P_i}} \right]$$

R_i response time P_i , C_i core execution time P_i

- Used in avionics and automotive (TTP, FlexRay)
- Can be used at system level (Giotto - Berkeley)

Conservative design - Summary

- **Limitations**
 - **low resource utilization**
 - **extended response times (problem for adaptive control engineering)**
 - **requires general time base (scalability?)**
 - **little flexibility (fixed time slots)**
 - **not a general solution**
 - **inefficiency (performance, bandwidth, costs, power) increases with system size**
- **Time-triggered systems are a good example for systematic integration, but...**
- **... reliable integration does not necessarily require conservative design style**



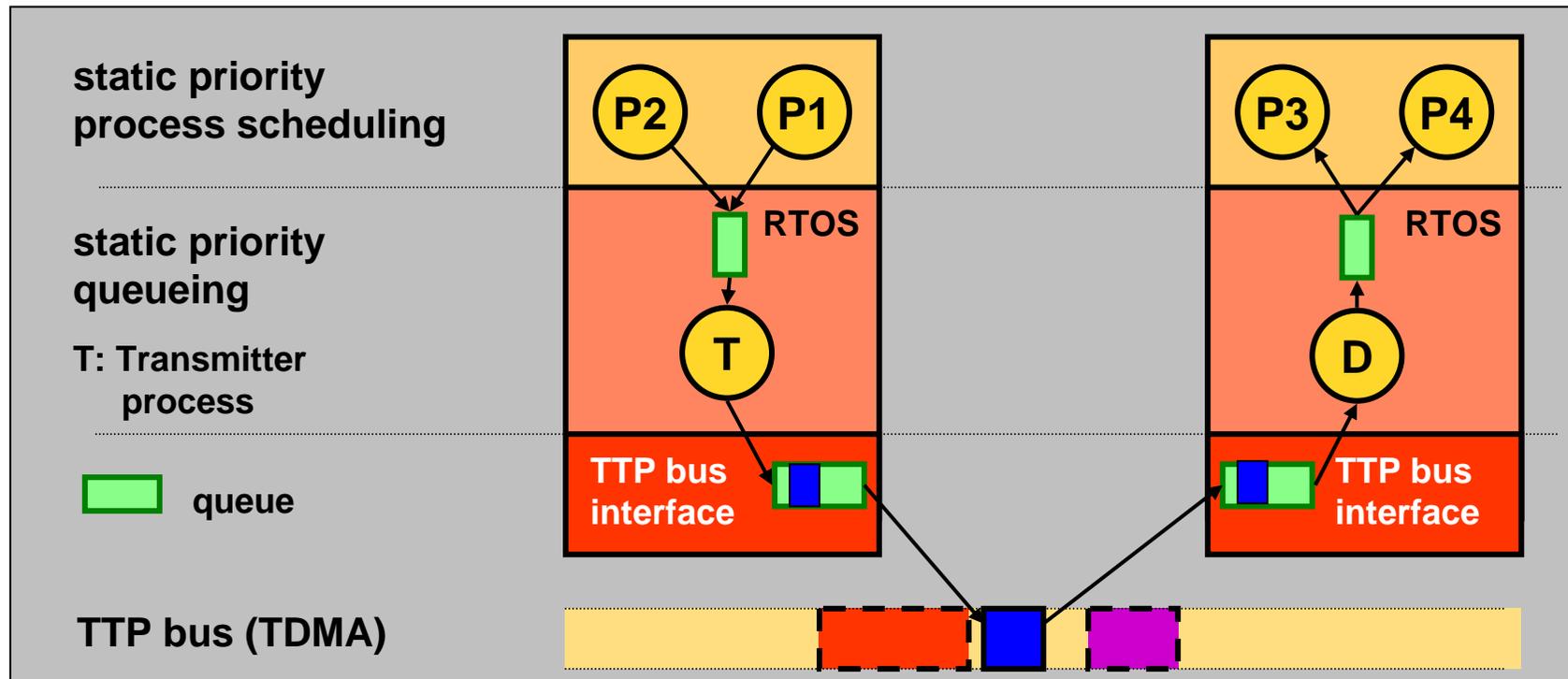
System level performance analysis

- **Global approach („Holistic“)**
 - **local analysis scope extension to several subsystems**
- **Compositional approach**
 - **global flow analysis combined with local scheduling analysis**



Analysis scope extension – „Holistic“

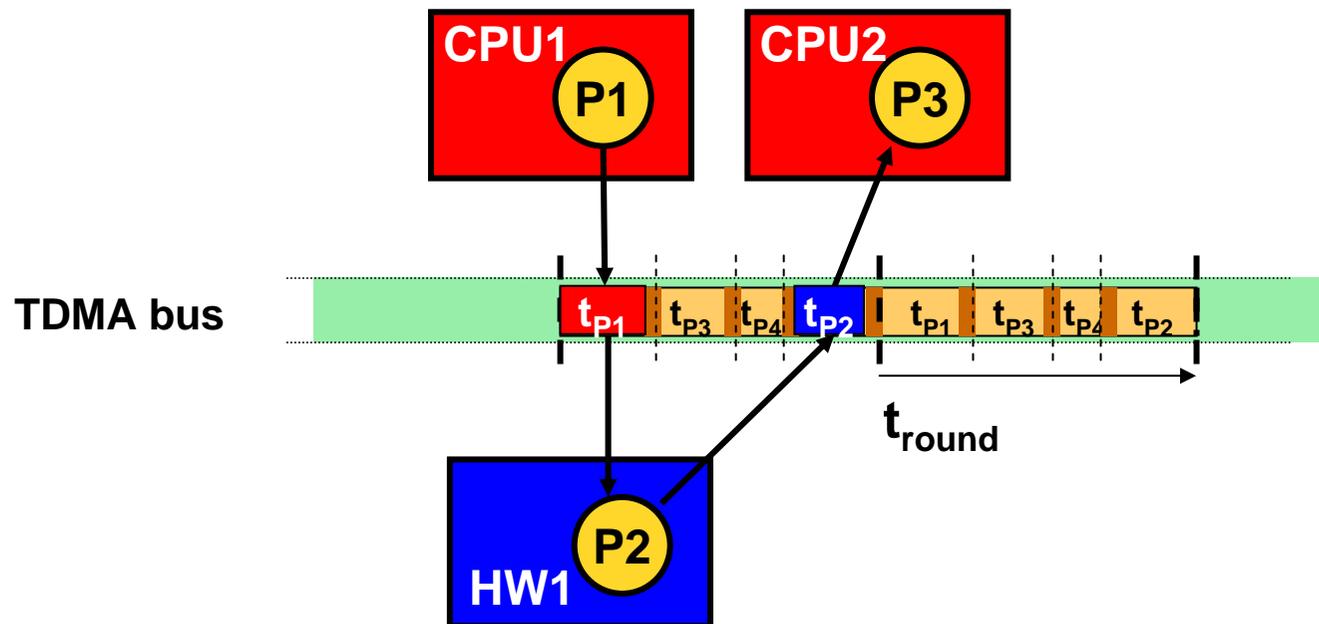
- Coherent analysis („holistic“ approach)
- Example: Tindell 94, Palencia/Harbour 98, Pop/Eles (DATE 2000, DAC 2002): TDMA + static priority – automotive applications



- **Problem: scalability**

Analysis scope extension (cont'd)

- Benefit: scope extension can take global system knowledge into account
- Example: using dependency information to detect that P2 can send in the same TDMA round as P1, if $R_{P2} < t_{P3} + t_{P4}$, where R_{P2} is the worst-case response time of P2





Compositional performance analysis

After the break!



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Compositional performance analysis

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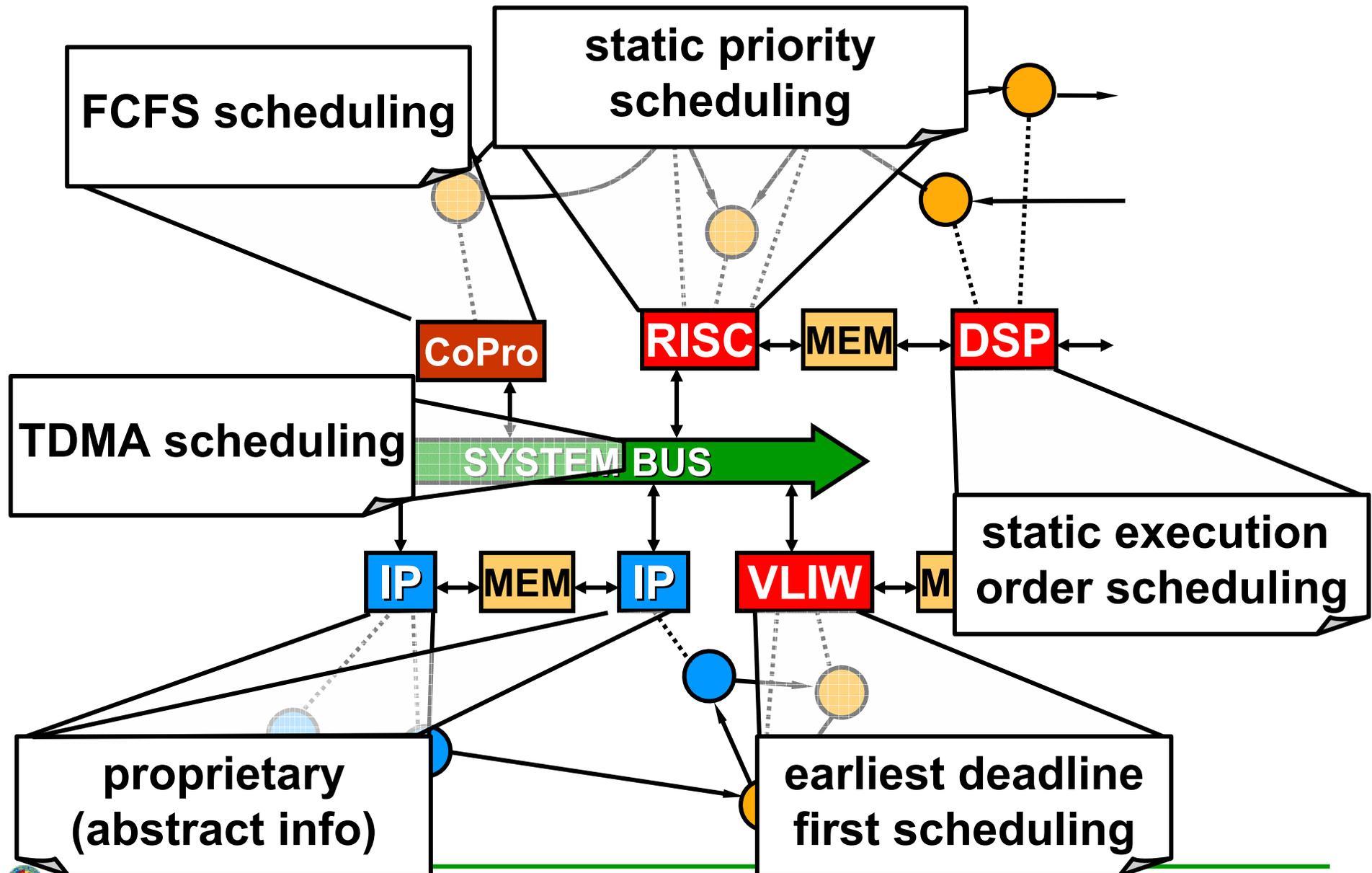
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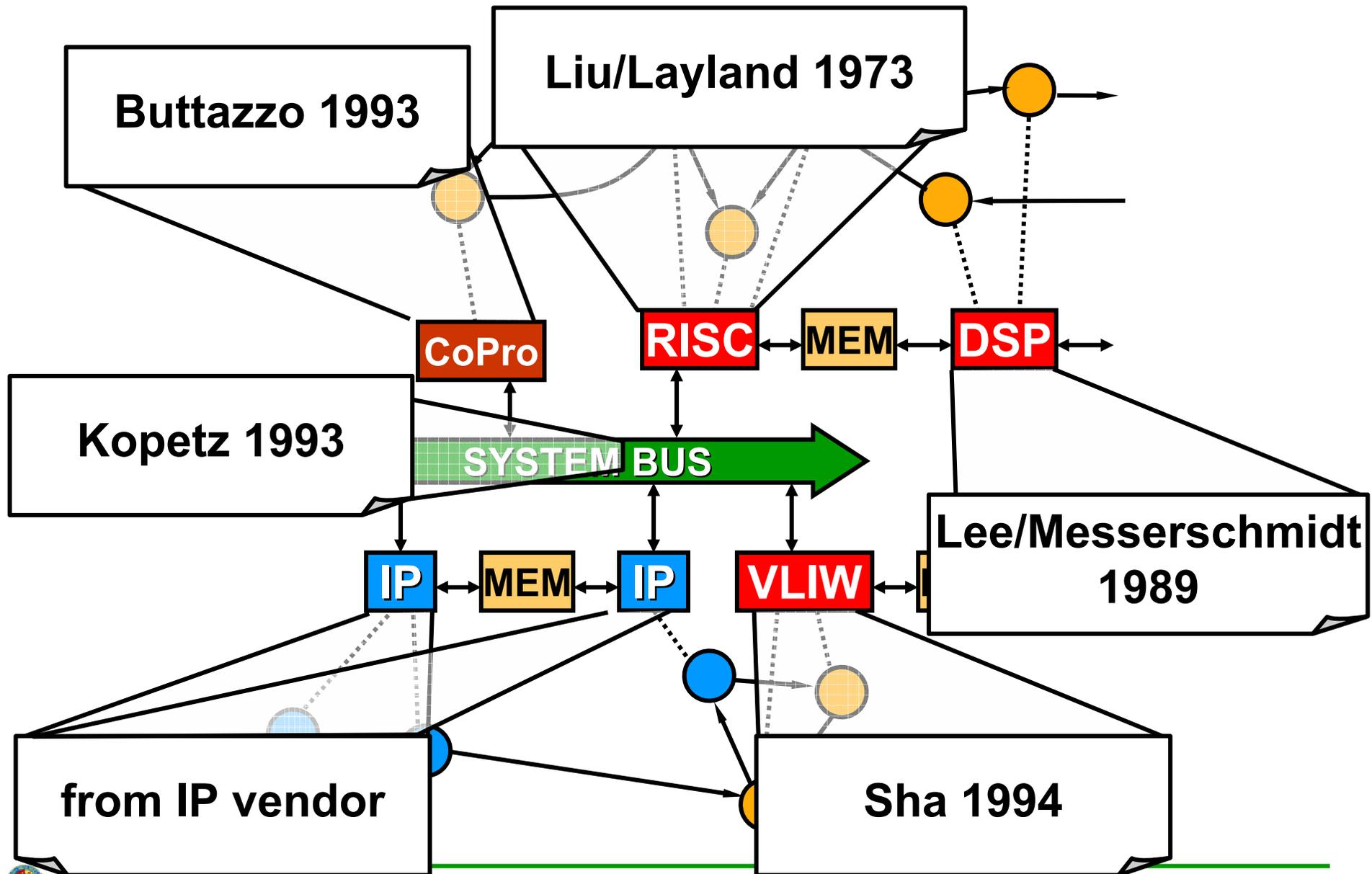


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NETWORK ENGINEERING

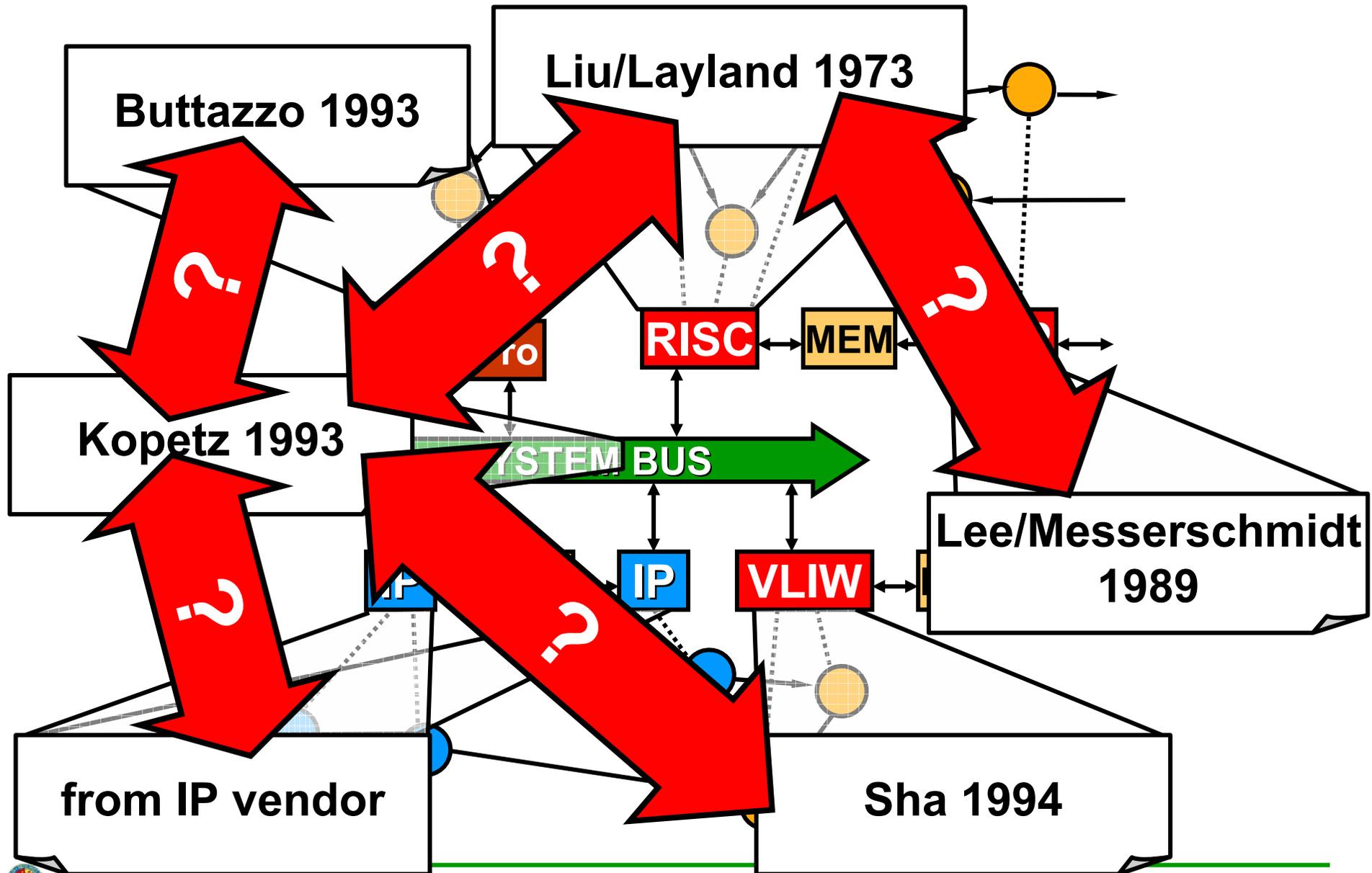
Multiple Scheduling Strategies



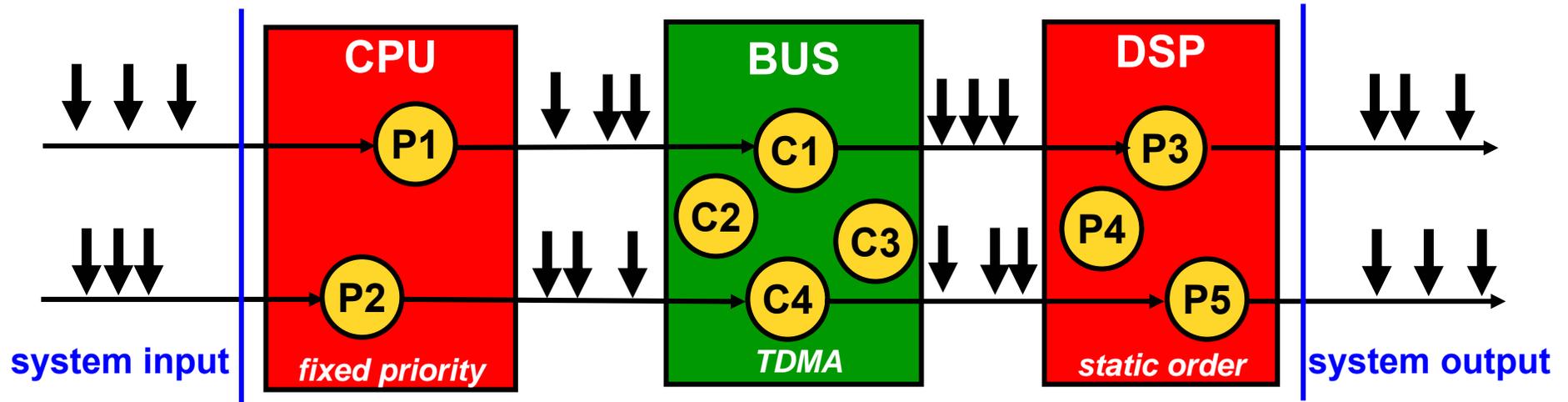
Corresponding Analysis Techniques



Integration ???



Compositional approach



- Tasks are coupled by event sequences
- Composition by means of event stream propagation
 - apply local scheduling techniques at resource level
 - determine the behavior of the output stream
 - propagate to the next component

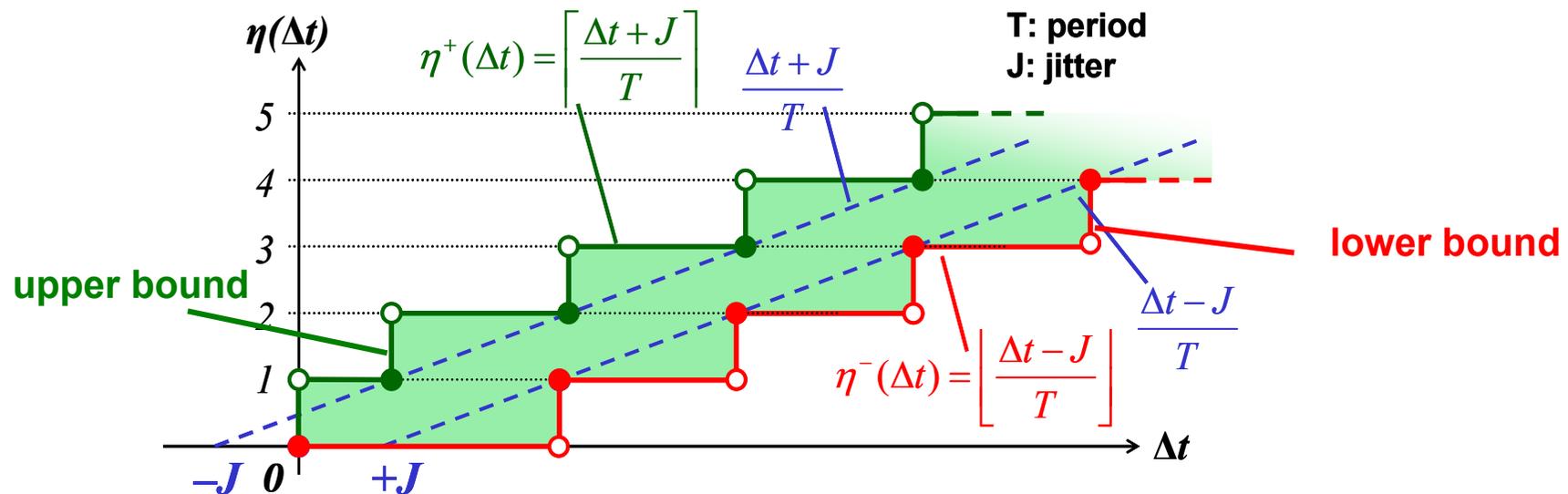
Idea

- Use network calculus + additional information as intermediate mathematical formalism
- Arrival curve functions of network calculus
 - $\eta^+(\Delta t)$ maximum number of activating events occurring in time window Δt
 - $\eta^-(\Delta t)$ minimum number of activating events occurring in time window Δt
 - d^- minimum event distance - limits burst density



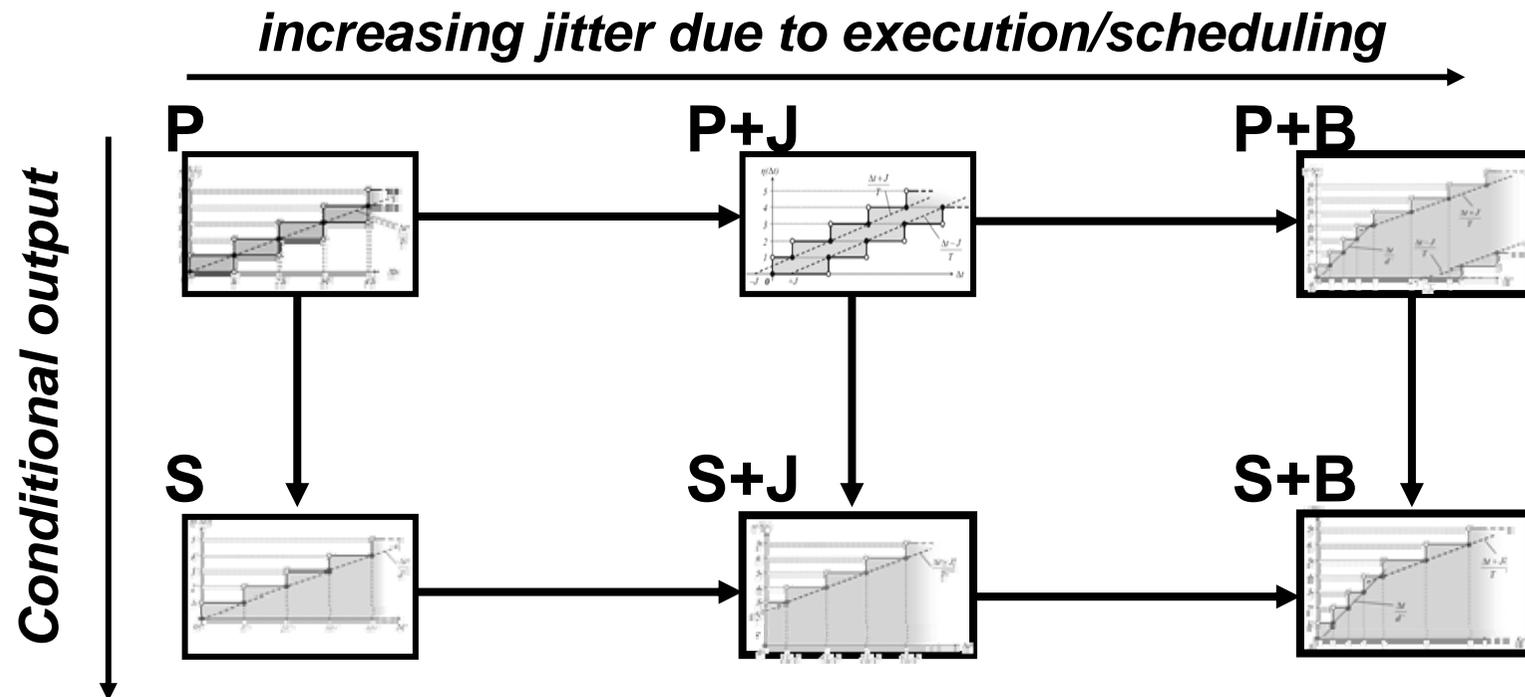
Event specification

- Derive event stream models with parameters
 - individual events replaced by stream variables (vectors) with stream parameters period, jitter, min. distance, ...
 - derive arrival curve functions from model parameters



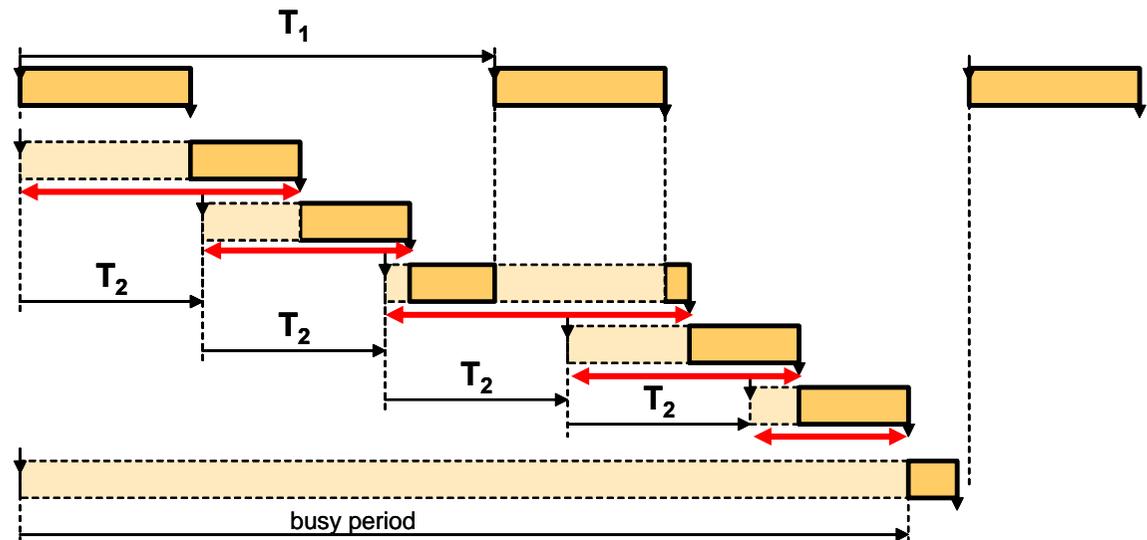
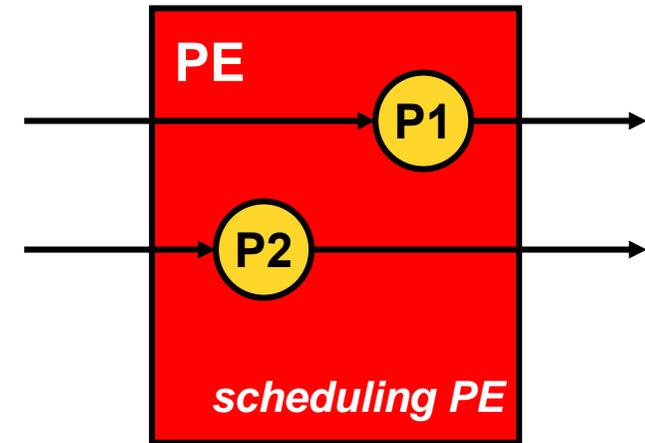
SymTA/S standard event models

- Required by RTA
 - Periodic/sporadic
 - Periodic/sporadic with jitter
 - Periodic/sporadic with burst

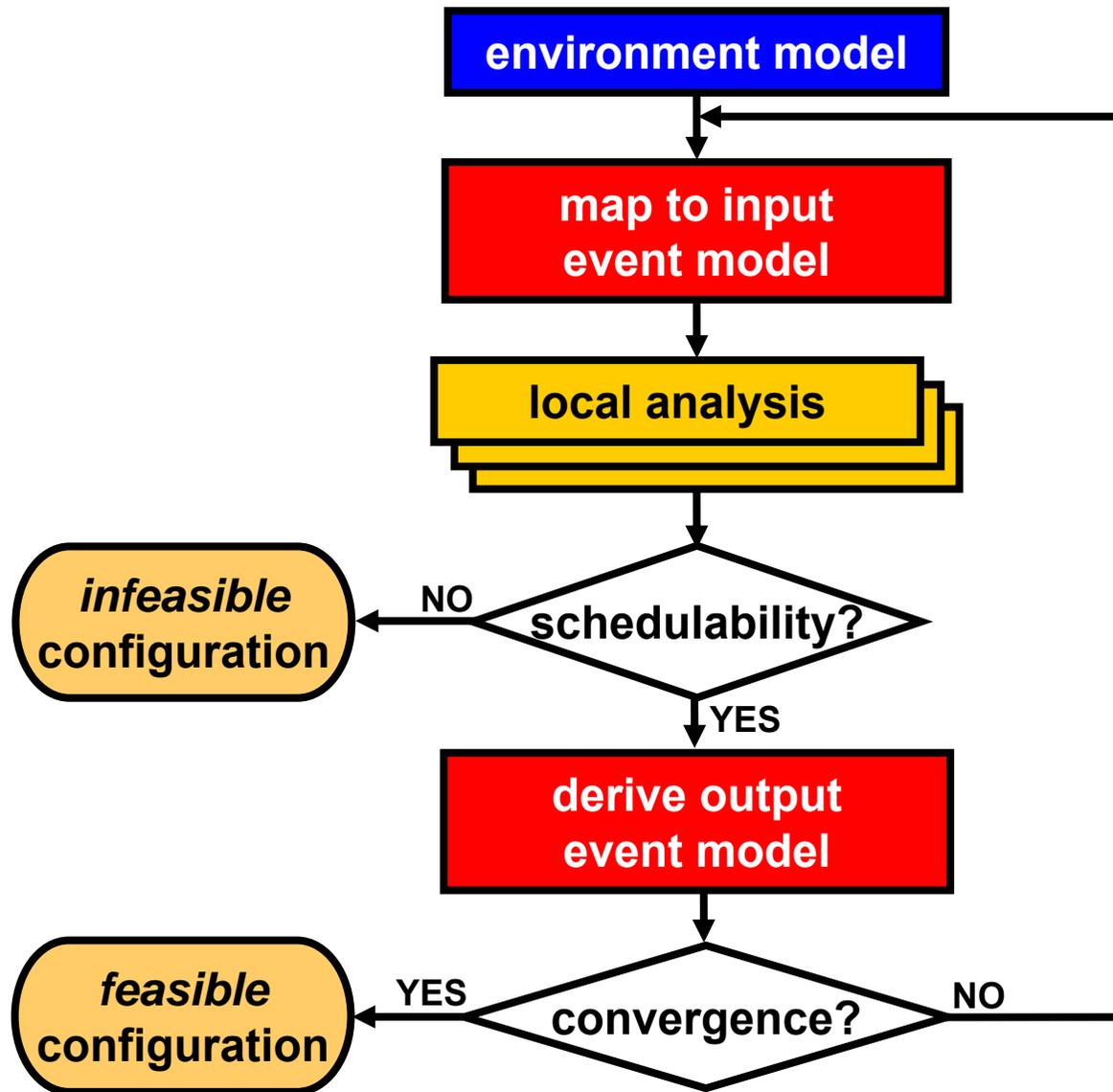


Input – output event model relation

- Any scheduling increases jitter
- Jitter grows along functional path
- Increasing jitter leads to
 - burst and transient overloads
 - higher memory requirements
 - power peaks

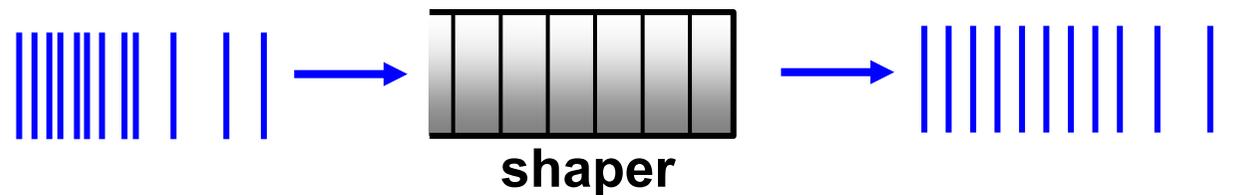


System analysis loop

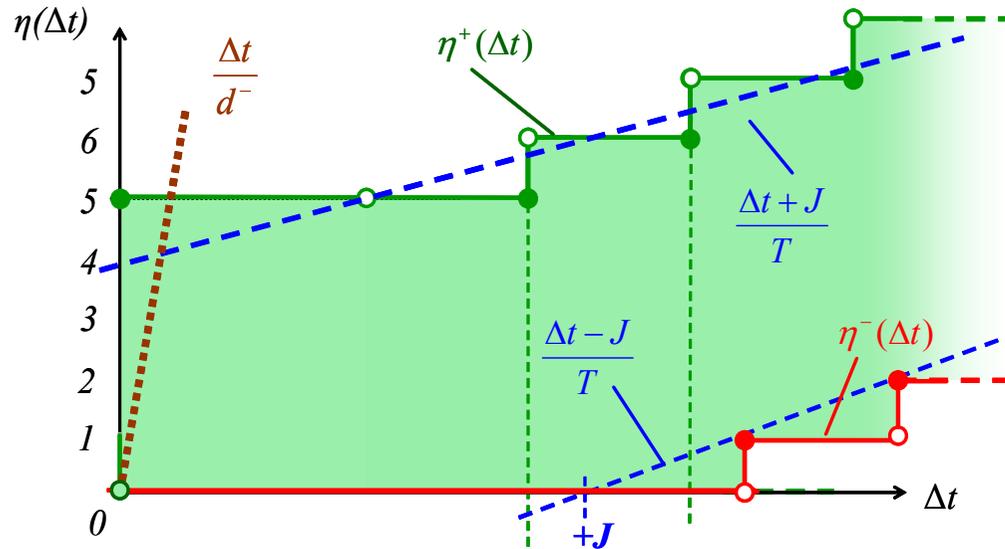


Reducing transient load in design

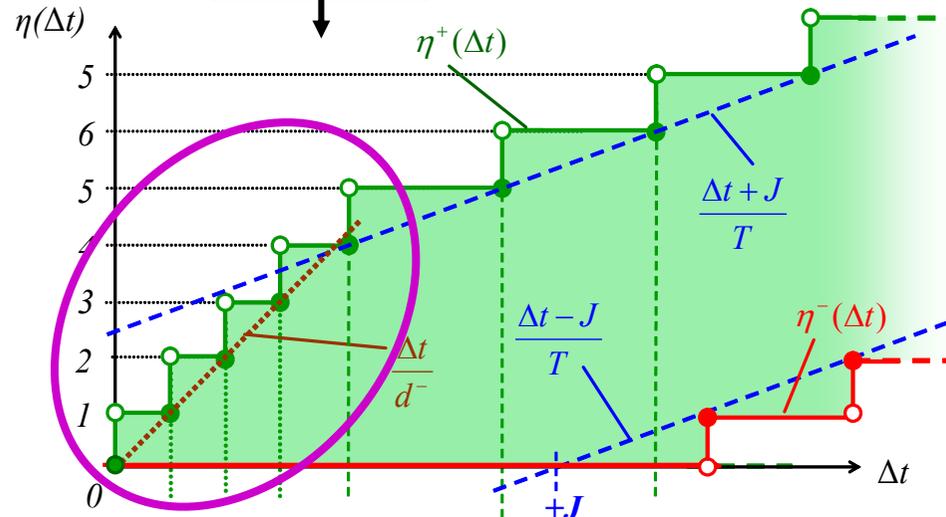
- Re-synchronization
- Minimum event separation using „traffic shaping“
- Requires memory and possibly increases latency



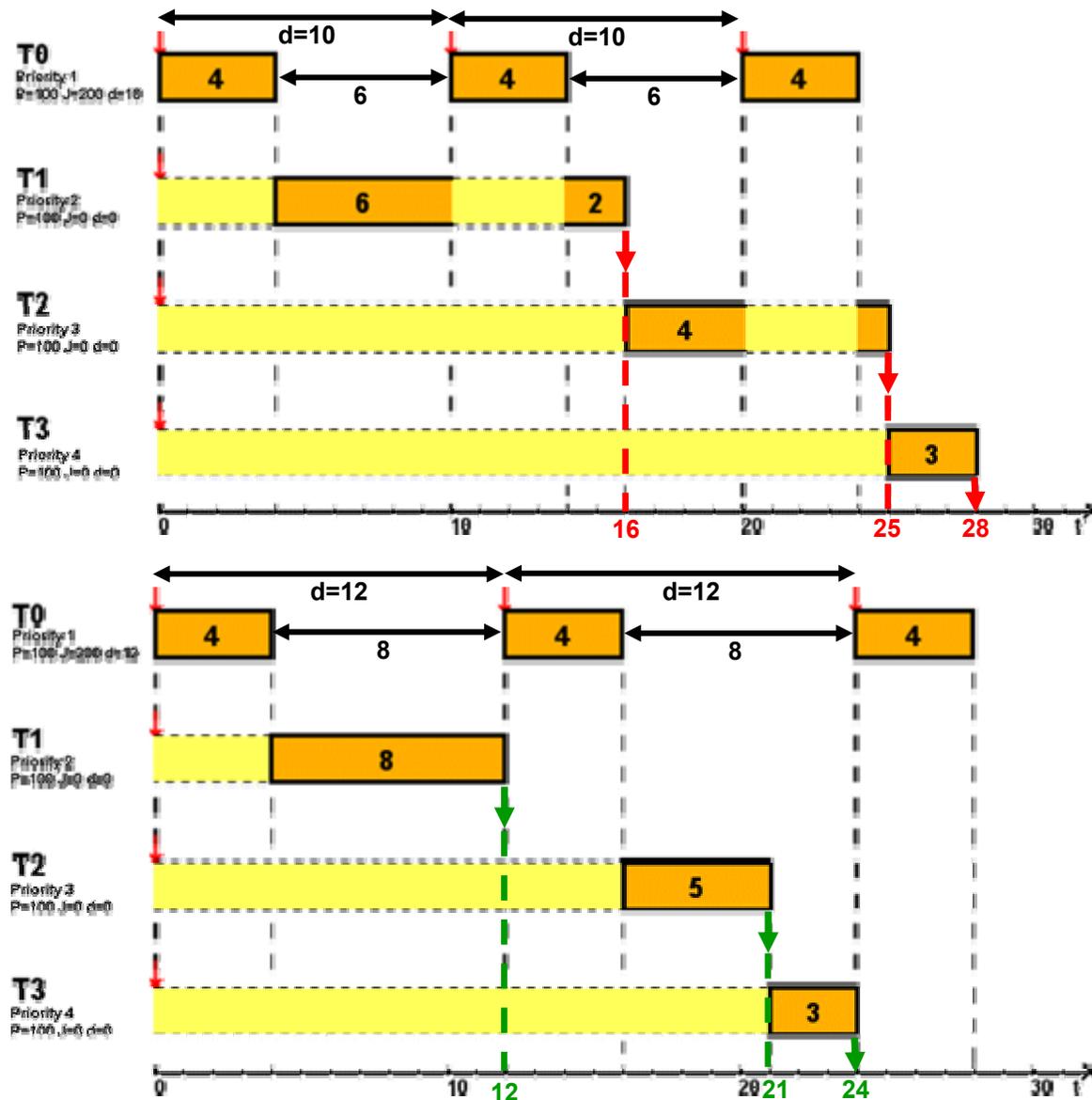
Traffic shaping - example



**d-
shaping**

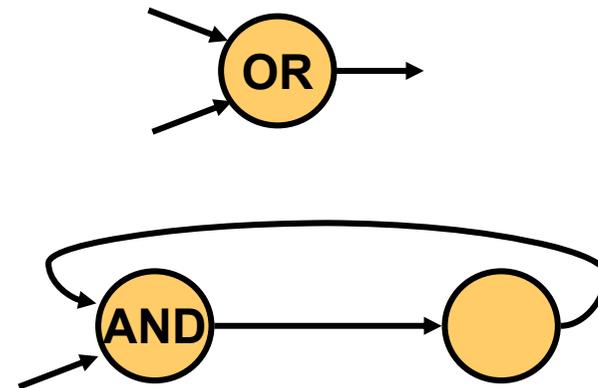


Optimization potential of Traffic Shaping



RTA event models are not sufficient

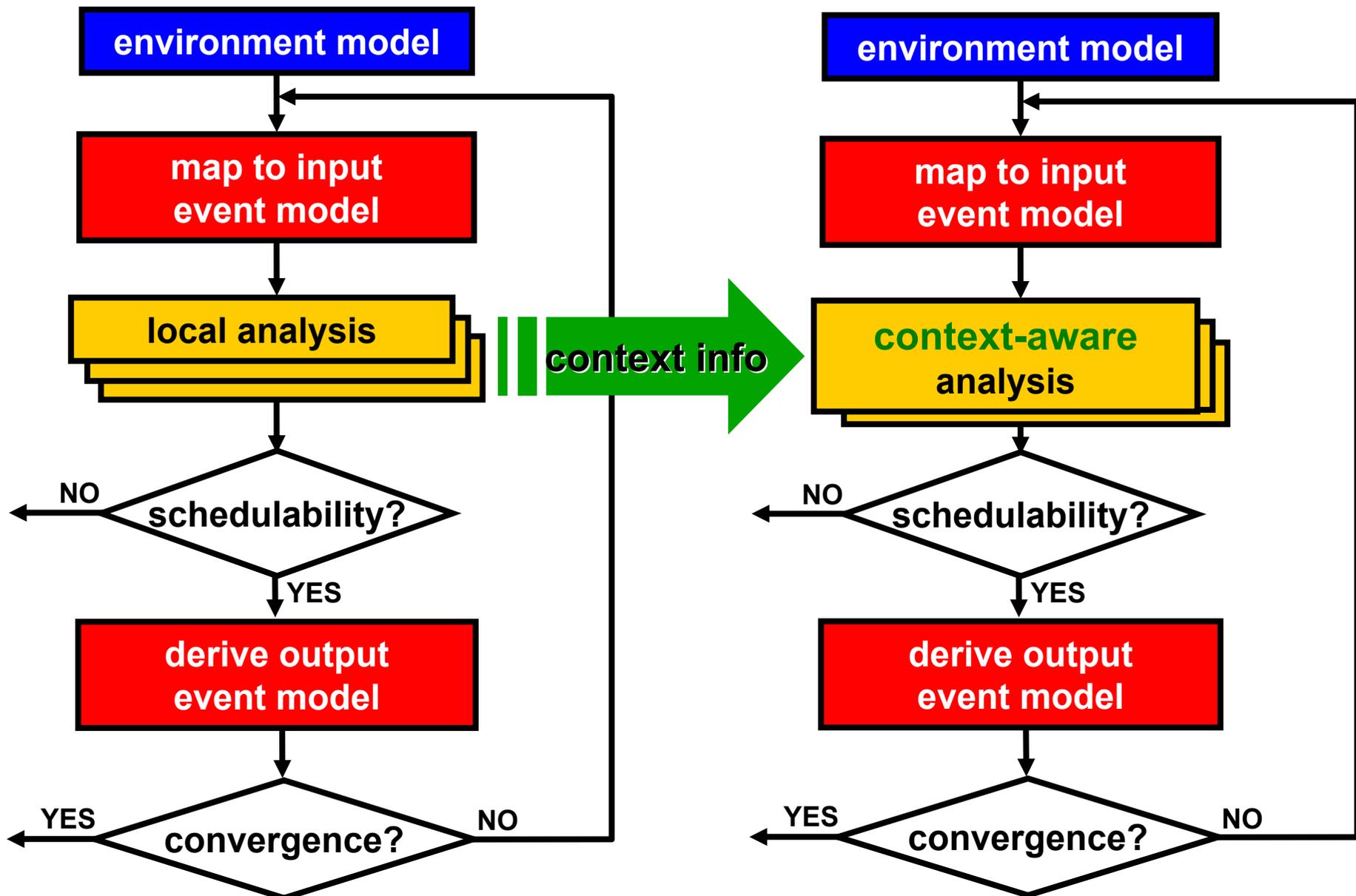
- Event model transitions needed to couple different subsystems and scheduling domains
- More complex activation models needed
 - OR activation
 - typical in event driven systems
 - AND activation and loops
 - typical for signal processing





Analysis extensions

System analysis loop

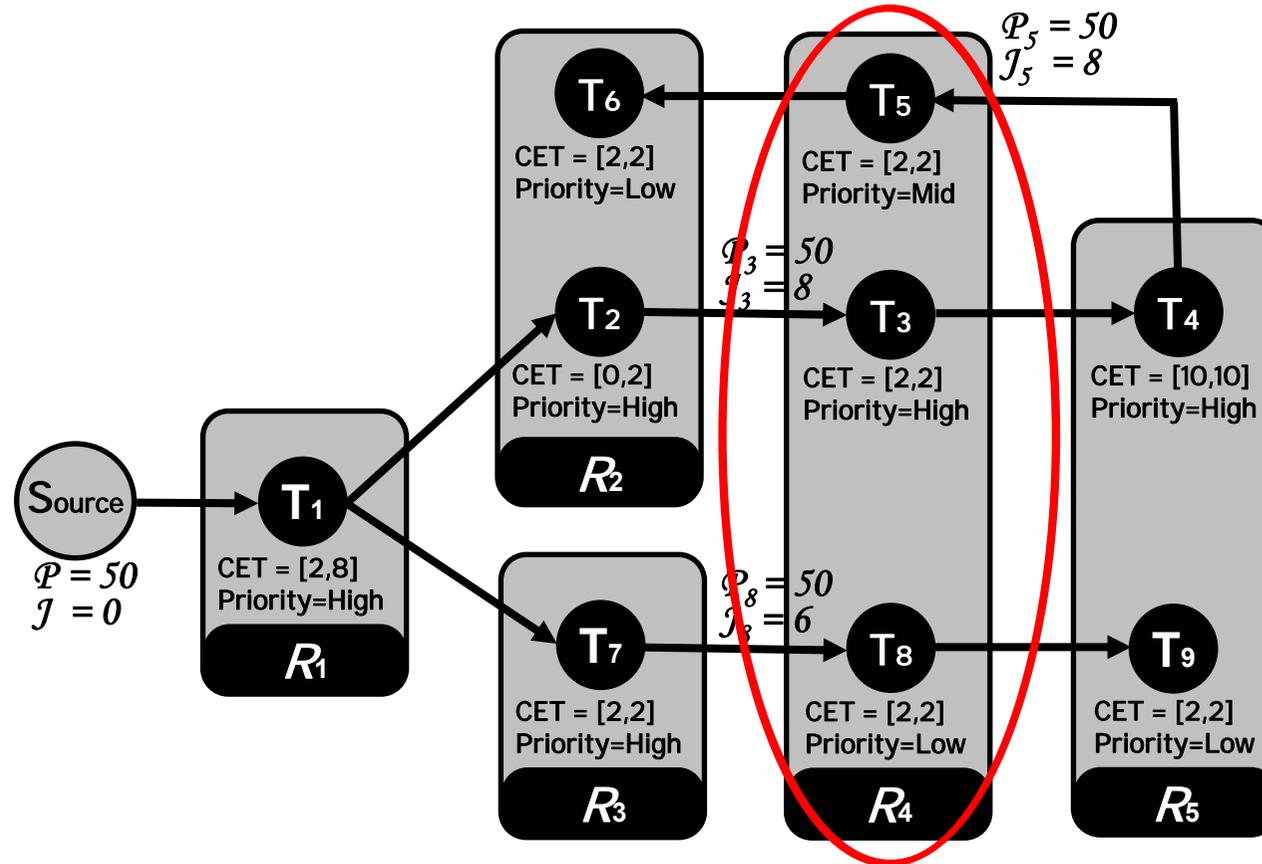


Taking global dependencies into account

- **„intra-context“ dependencies**
 - **different events in a single event stream often activate different task behaviors with different execution times or communication loads**
- **„inter-context“ dependencies**
 - **activating events in different event streams are often time-correlated which rules out the simultaneous activation of all tasks**
- **can be combined leading overall to less conservative analysis results**

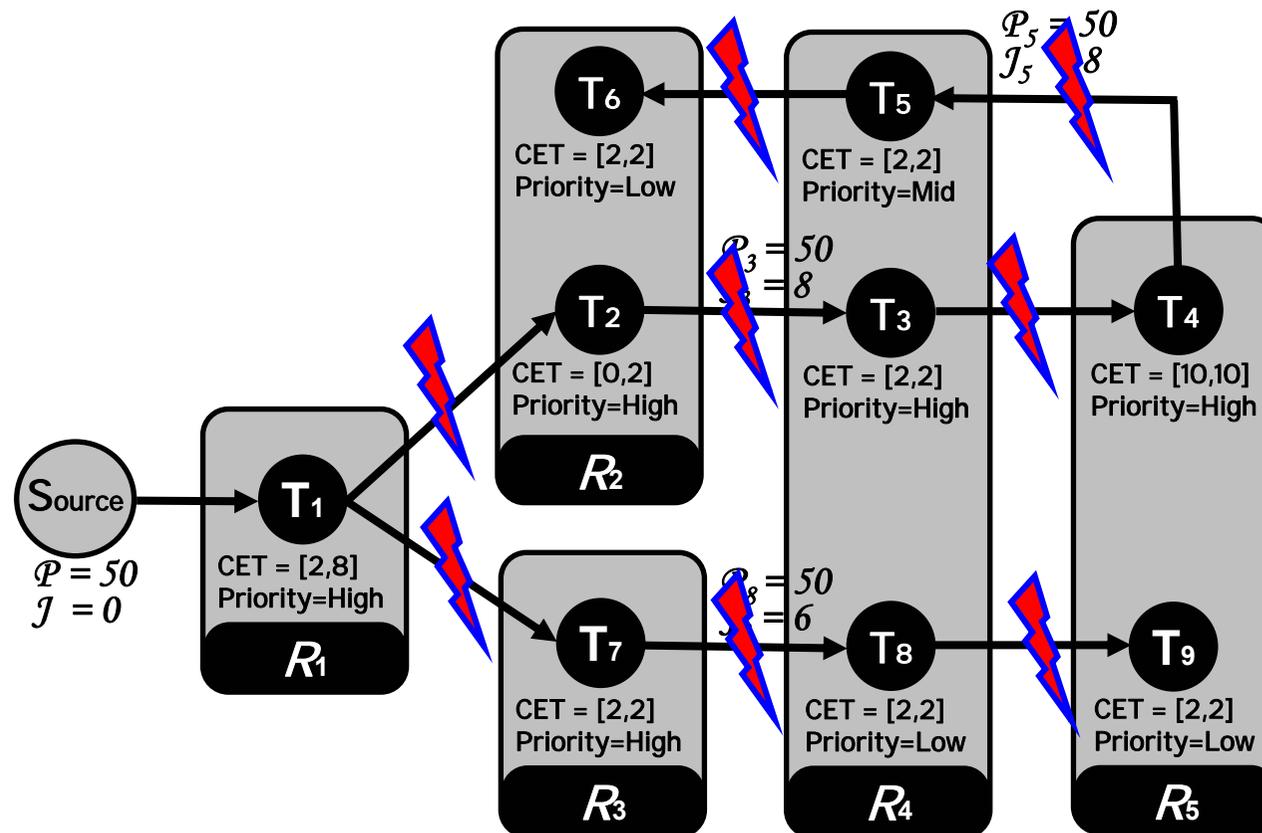


Motivating Example



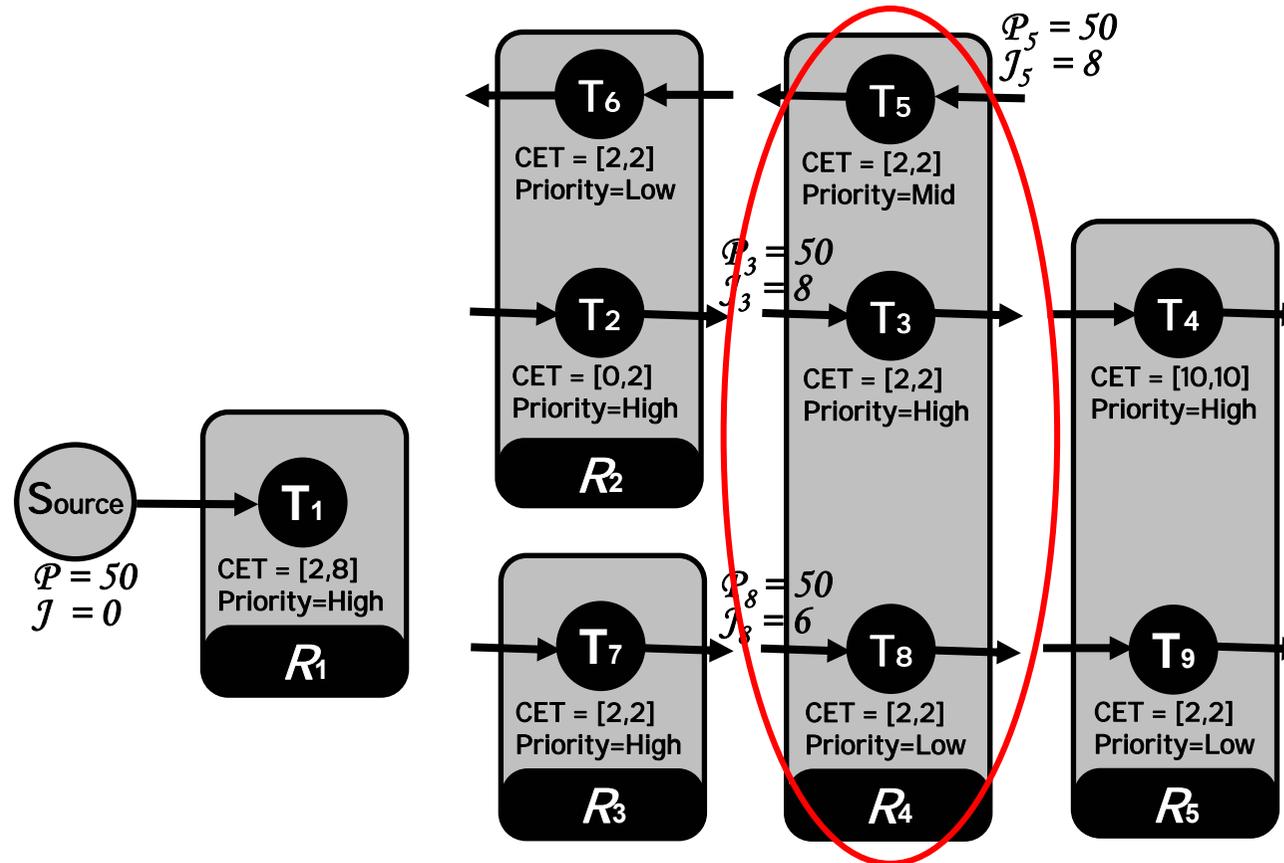
- **Static priority preemptive scheduling on all resources**
- **Compositional performance analysis approach (Richter)**

Lehoczky (1990)



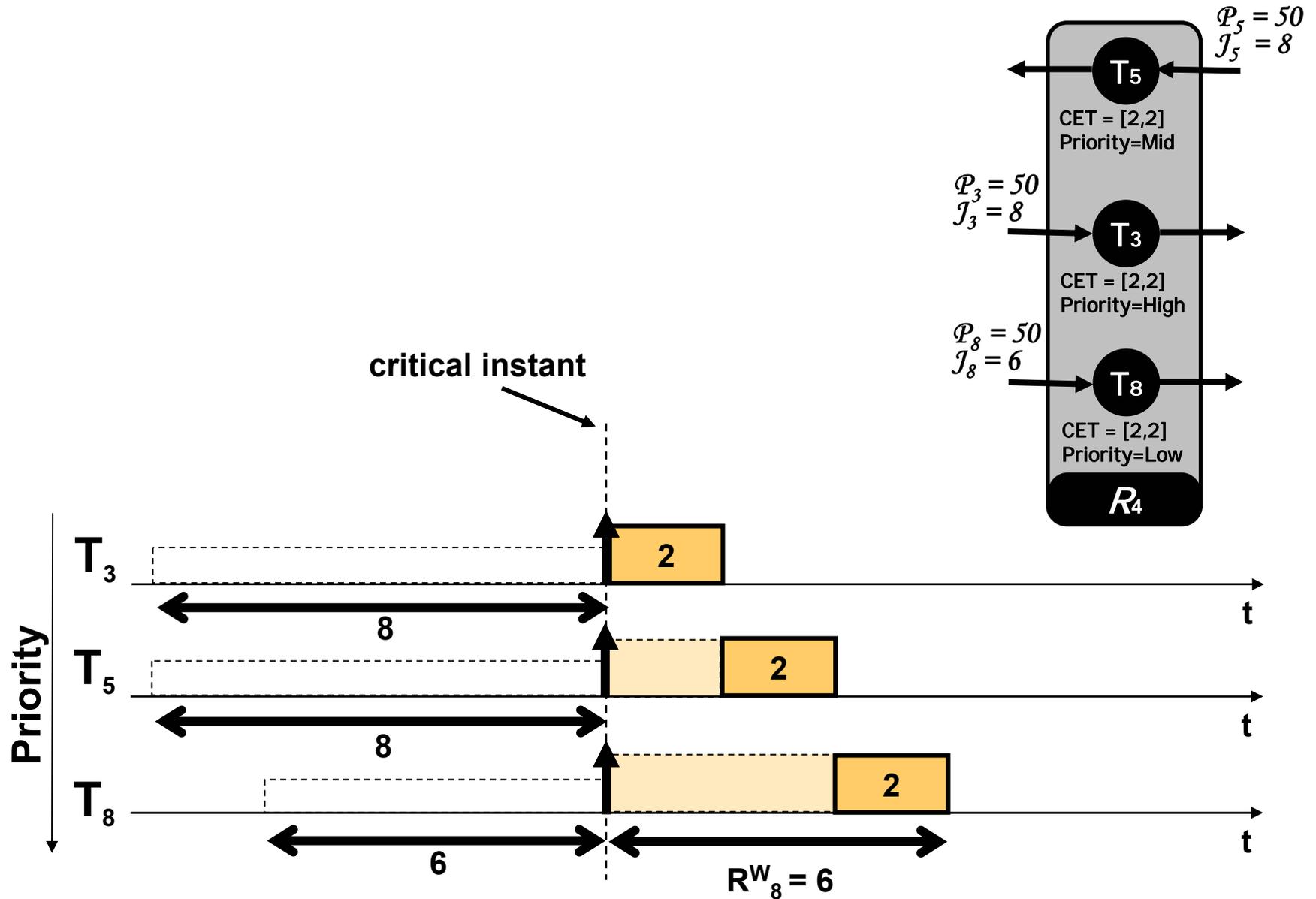
•Ignore correlation between tasks!

Lehoczky (1990)

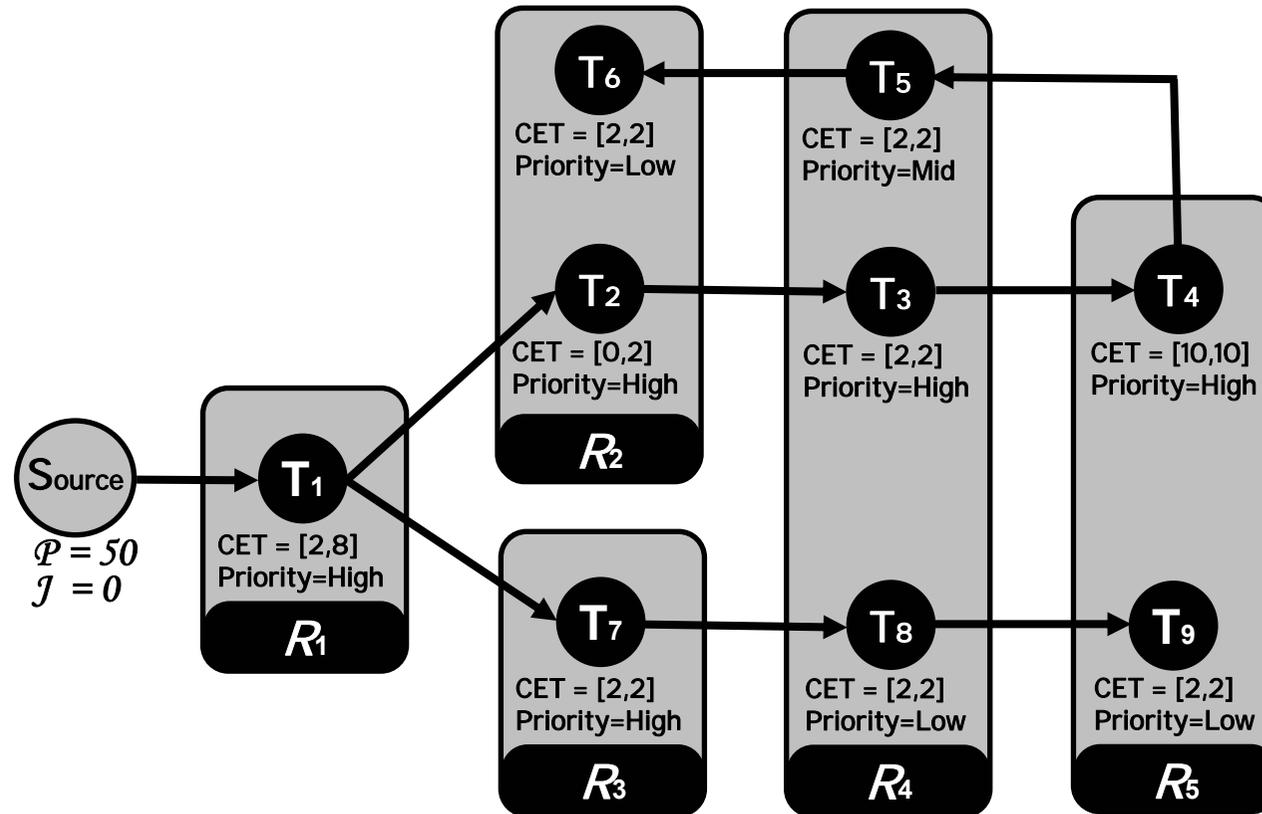


•Ignore correlation between tasks!

Lehoczky (1990)

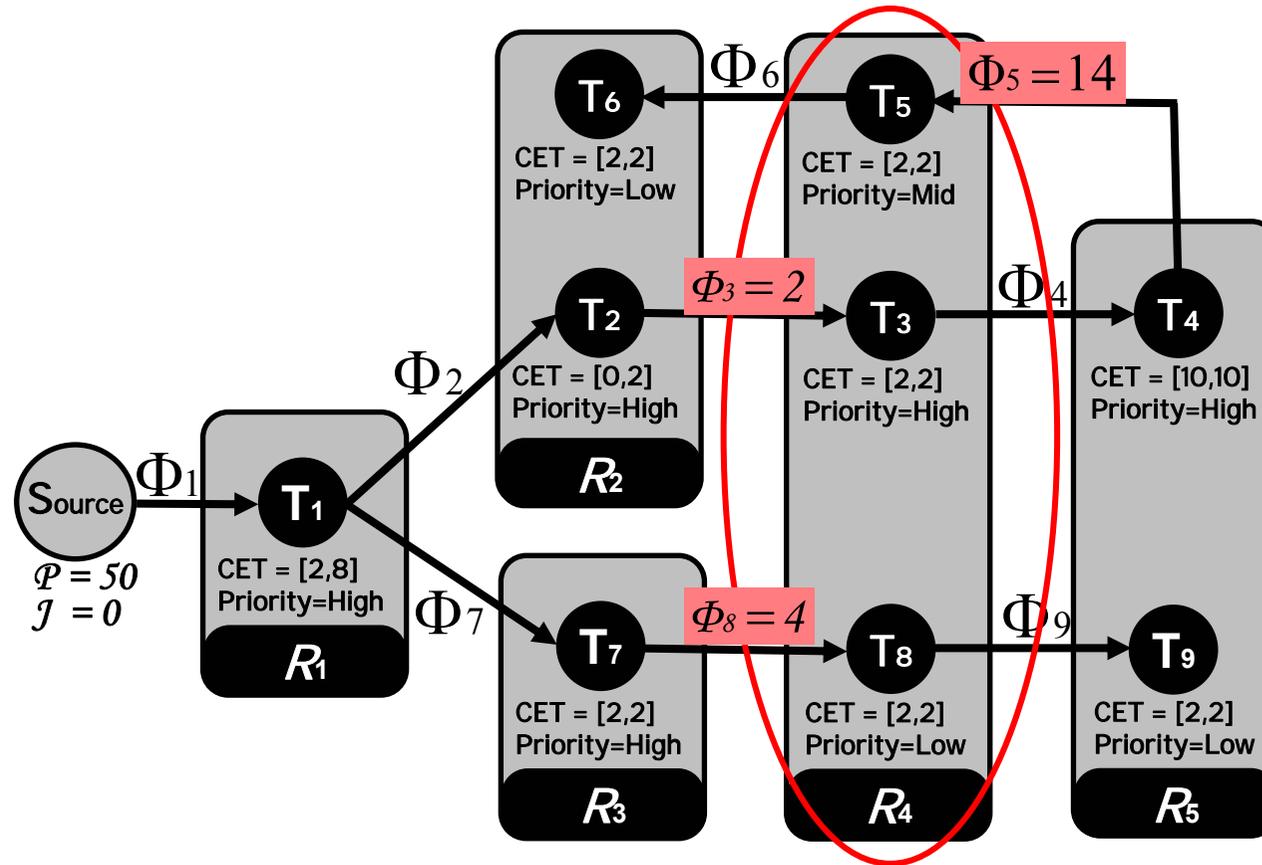


Tindell (1994)



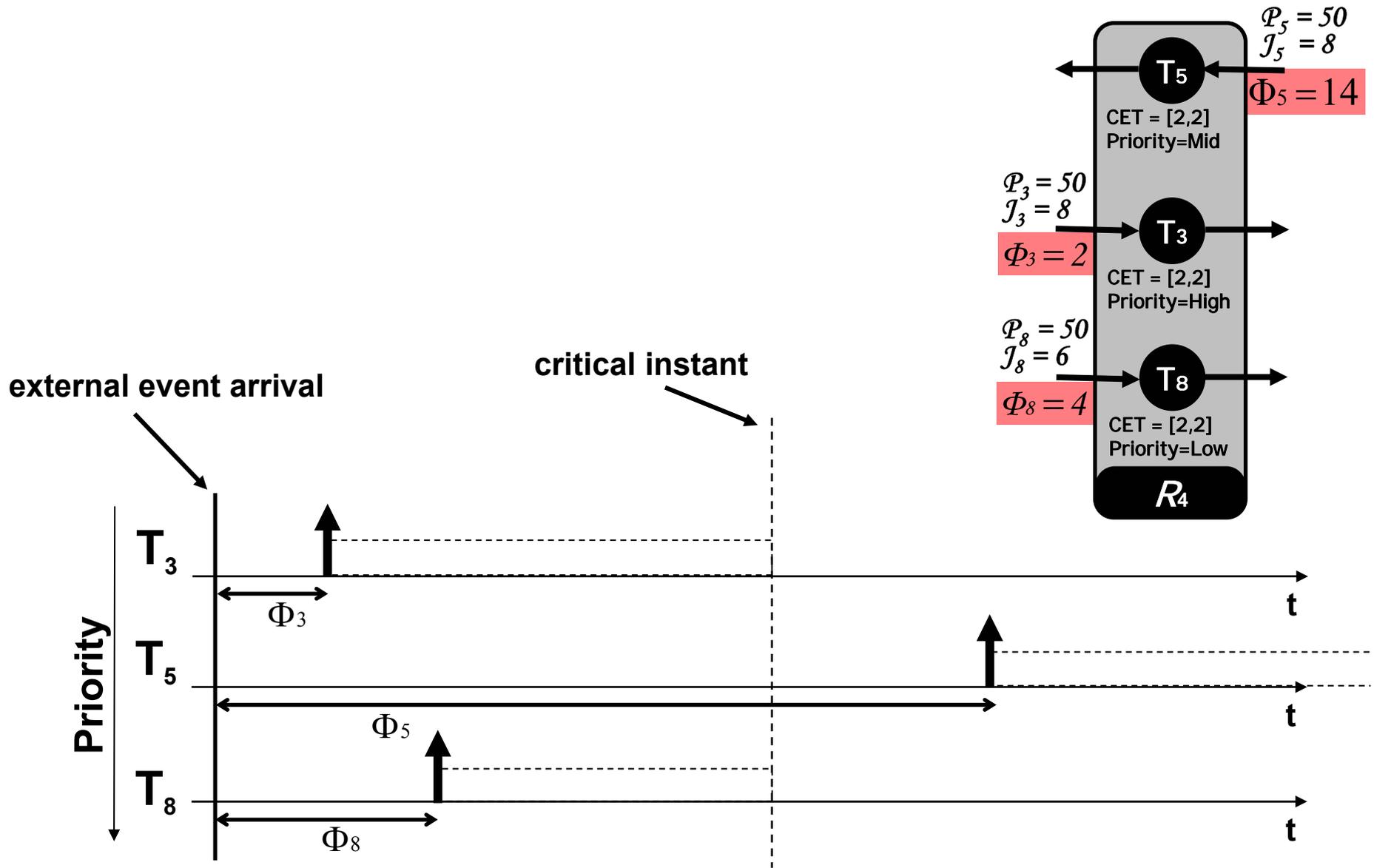
- Periodic arrival of events at system inputs as timing-reference

Tindell (1994)

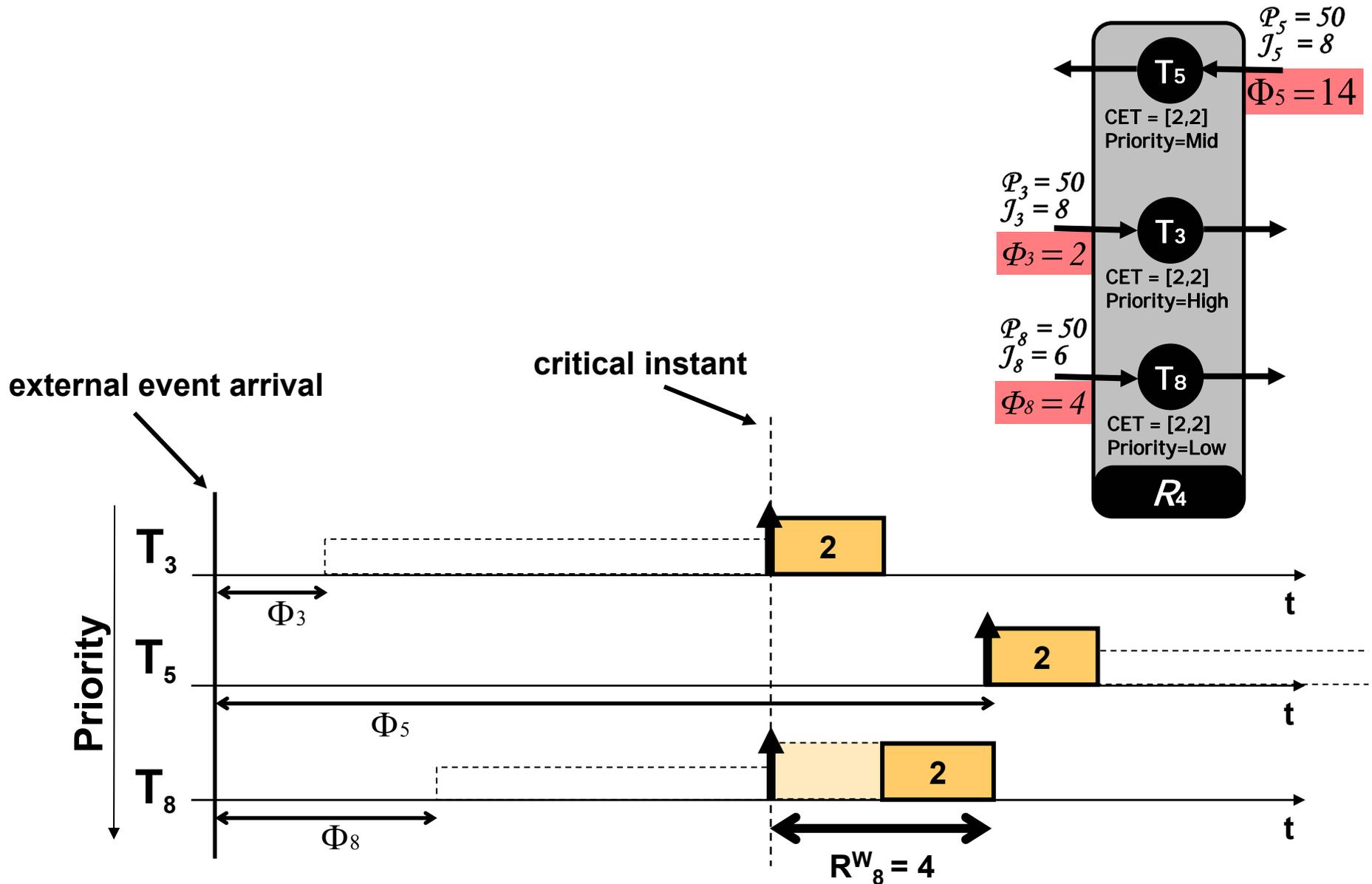


Global Offset $\Phi_i =$ earliest activation time of T_i relative to the periodical arrival of an external event at the system input

Tindell (1994)



Tindell (1994)

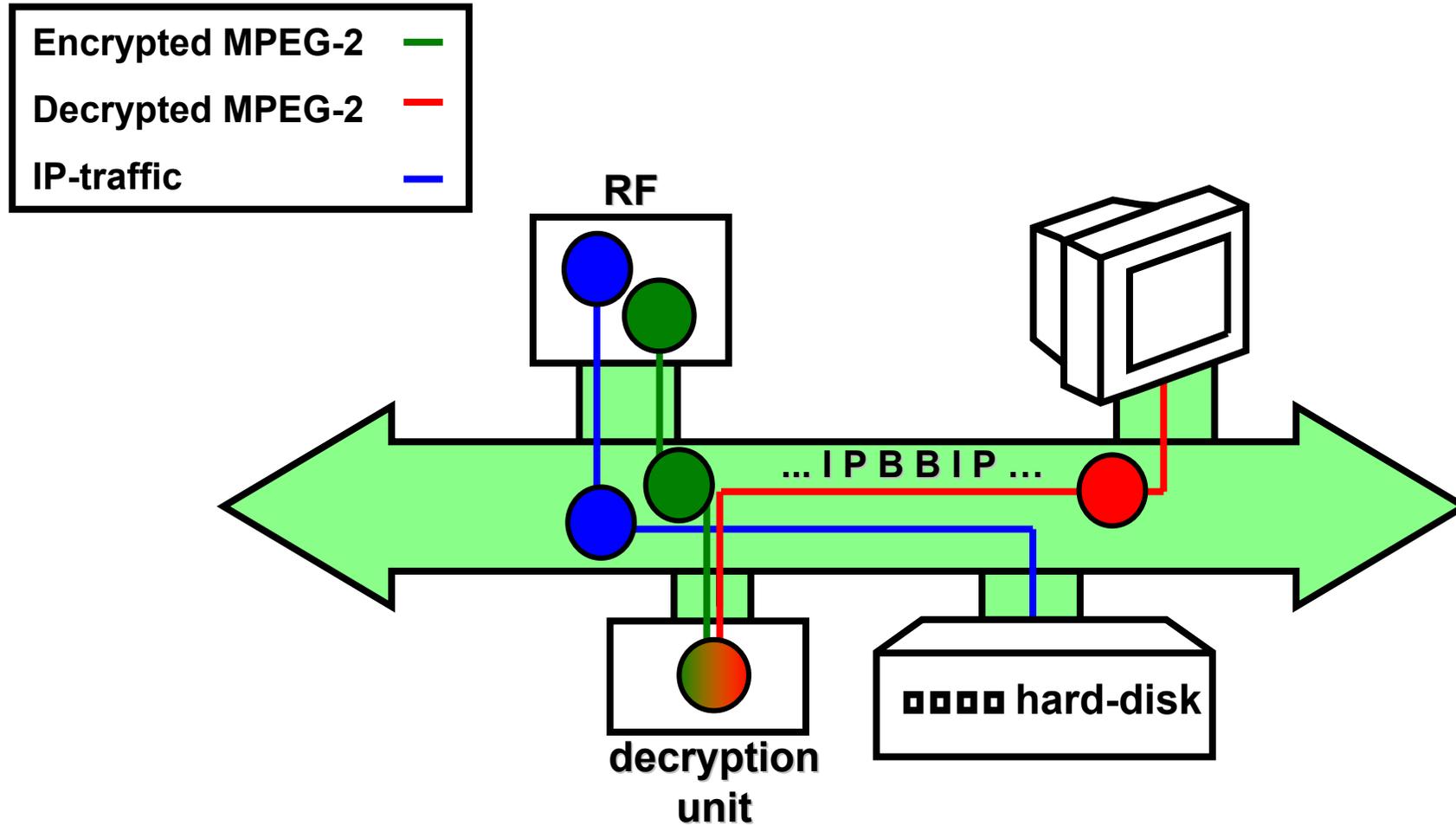


Further Techniques

- **Relative offsets and relative jitter**
 - **Extends idea of global offsets**
 - **Describes the earliest activation time of a task relative to a timing-reference *ref***
 - **Reference is not necessarily a periodic external event**
 - **Enables tighter response time calculation**
- **Precedence relations**
 - **Explicitly considers precedence relations between tasks (i.e. task *i* cannot start until task *j* has finished execution)**
 - **Orthogonal to offset based techniques**

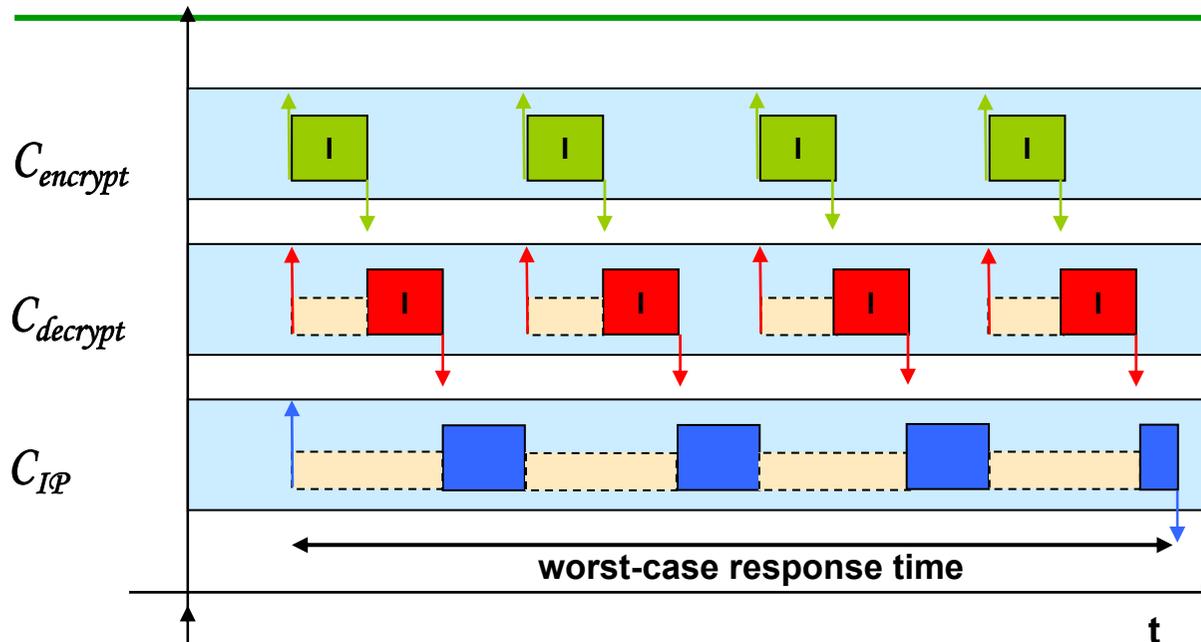


Set-top box



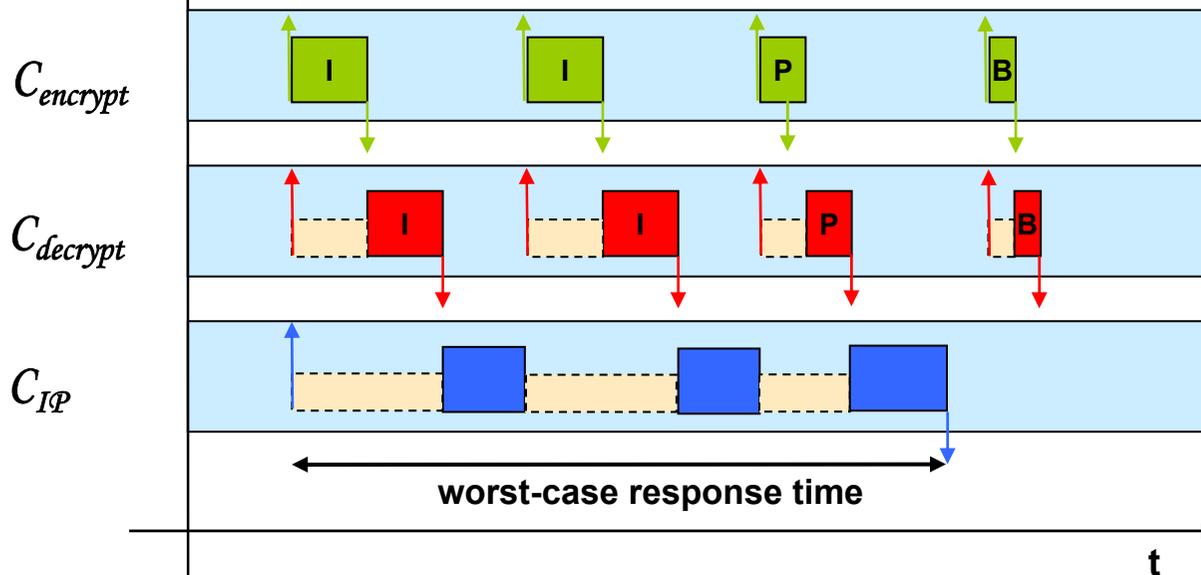
- set top box: **decrypt video** + **download file via IP**

Intra context dependencies



classical analysis:

each frame is assumed to be an I-Frame



intra context analysis:

information about frame types allows accurate load calculation





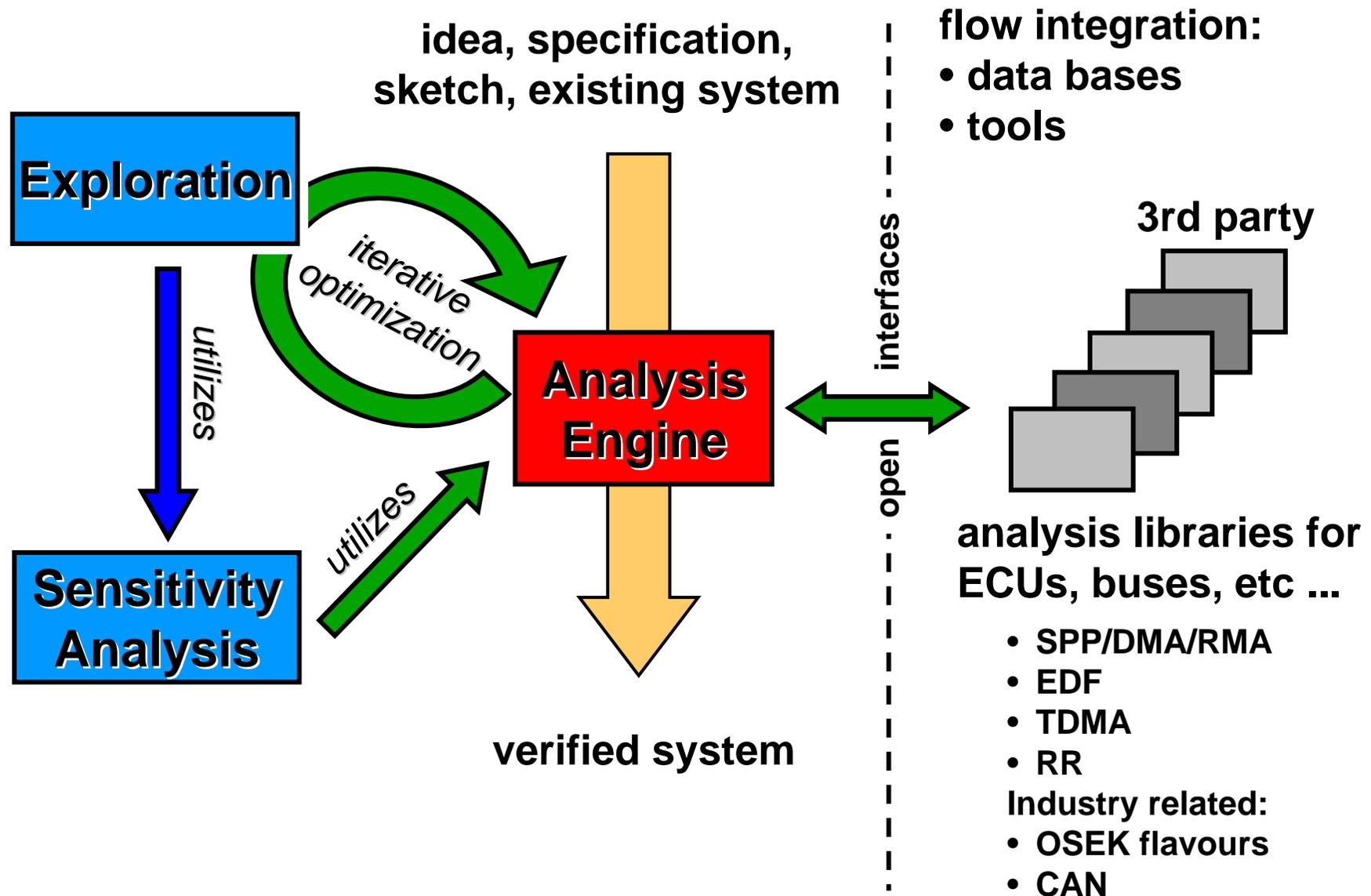
That's all !
Hands-on Session



SymTA/S Tool



SymTA/S Tool Suite

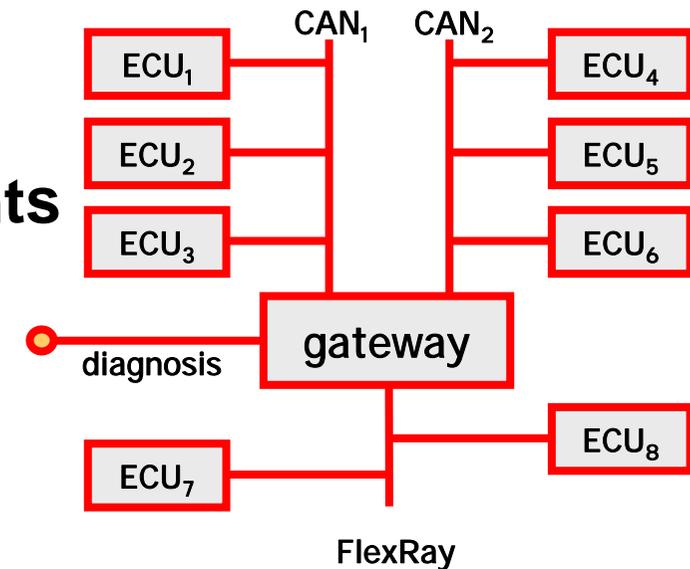
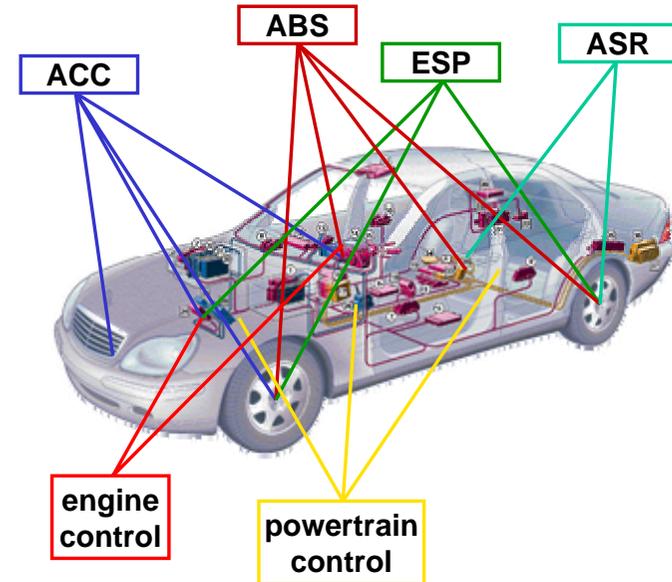




Sensitivity Analysis

Challenges

- **Heterogeneous**
 - Hundreds of functions
 - 50+ ECUs
 - Several RTOSes and protocols
 - Strongly networked
 - Many suppliers
- **Complex performance requirements**
 - End-to-end deadlines
 - Hidden timing dependencies

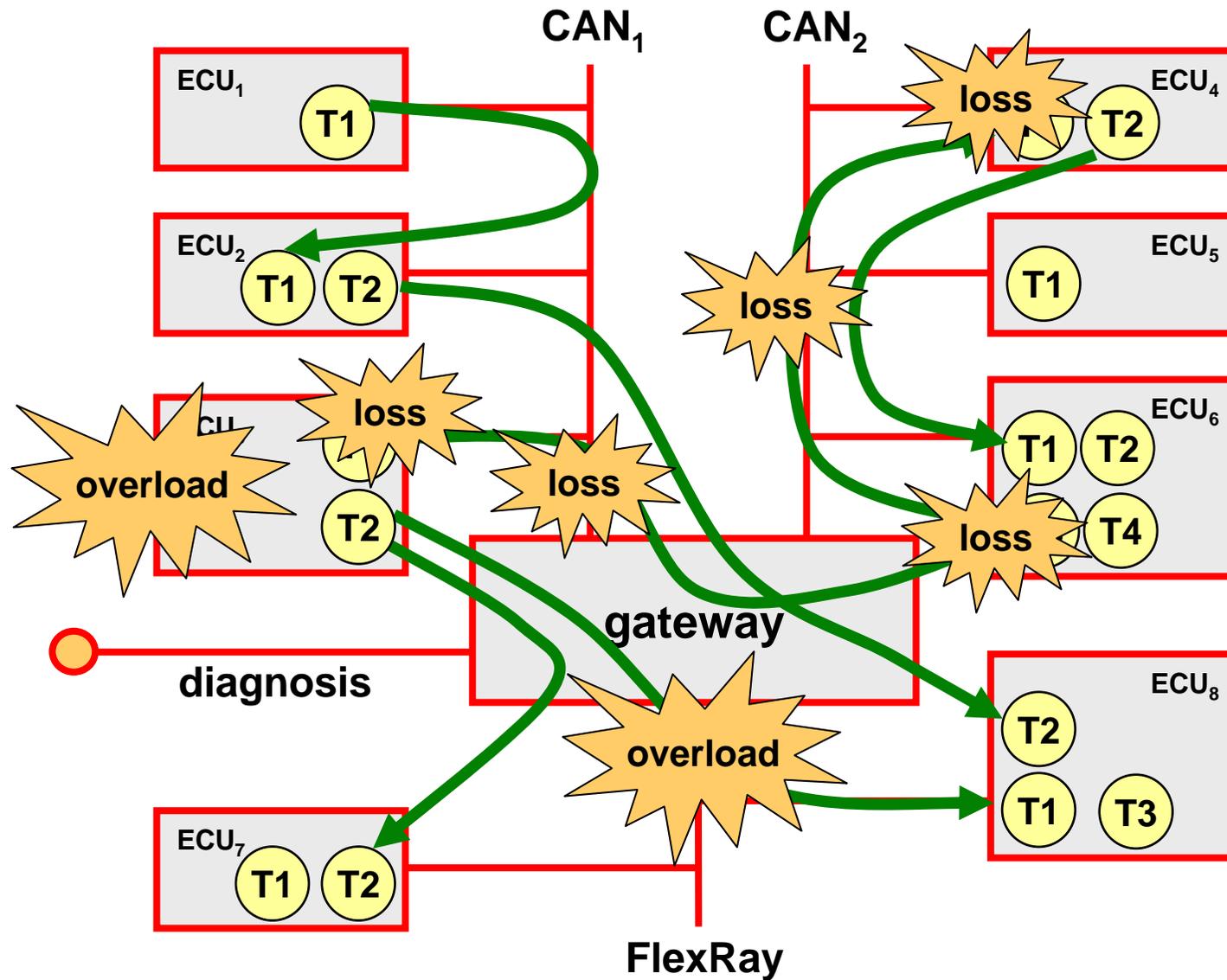


Motivation

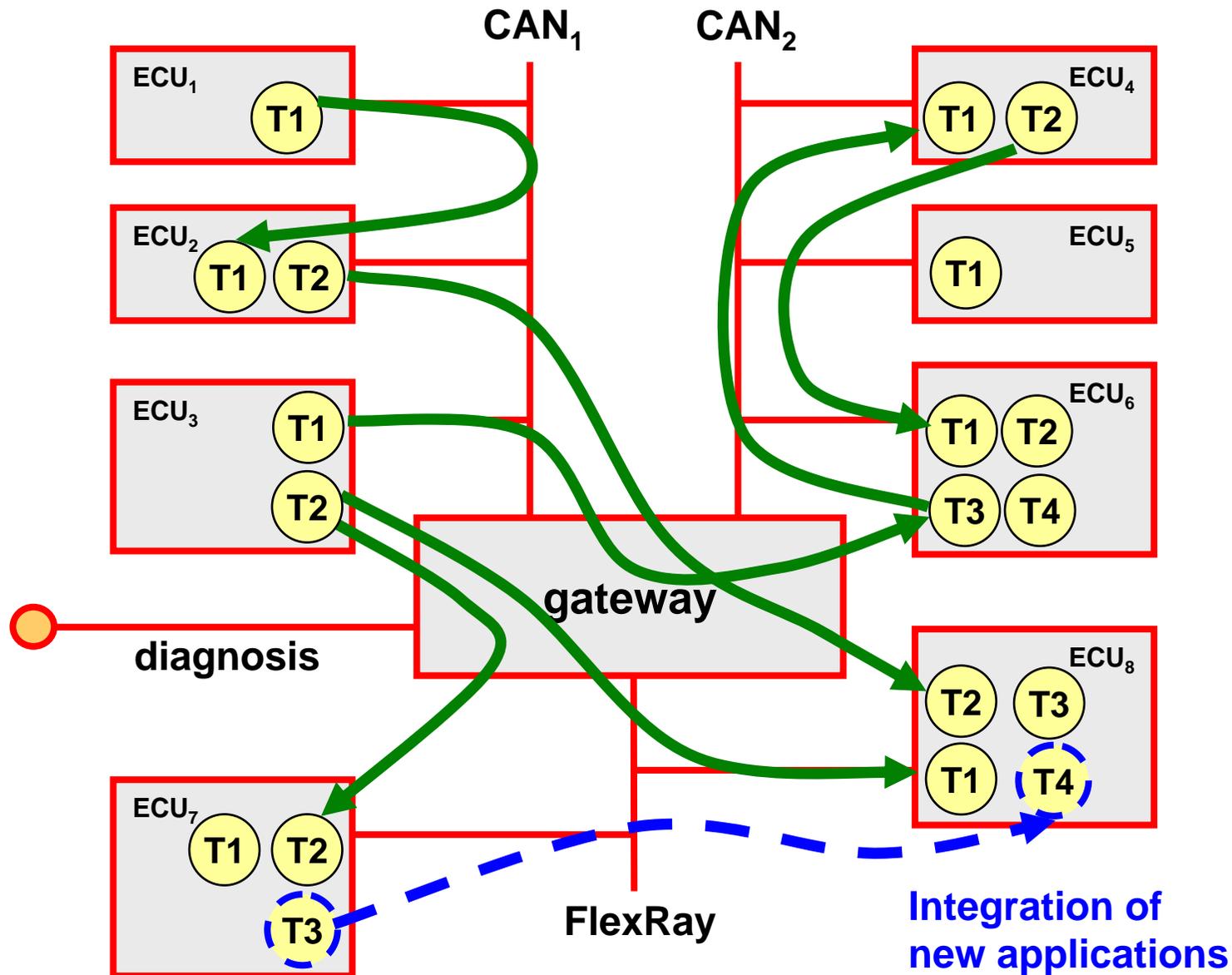
- **Modifications of design properties**
 - **During the design process**
 - Refinement of early design data estimations
 - Refinement and changes of specification
 - Exchange of platform components: replace CPU or memory type
 - **In the product lifecycle**
 - Product updates (HW, firmware and SW)
 - Integration of new components or subsystems
 - Change in the environment: applications (smart phone), technical system (motor speed)
 - **In the field**
 - Dynamic systems
 - Unplanned environment situations (resilience)
- **Such changes introduce uncertainties and **increase design risk****



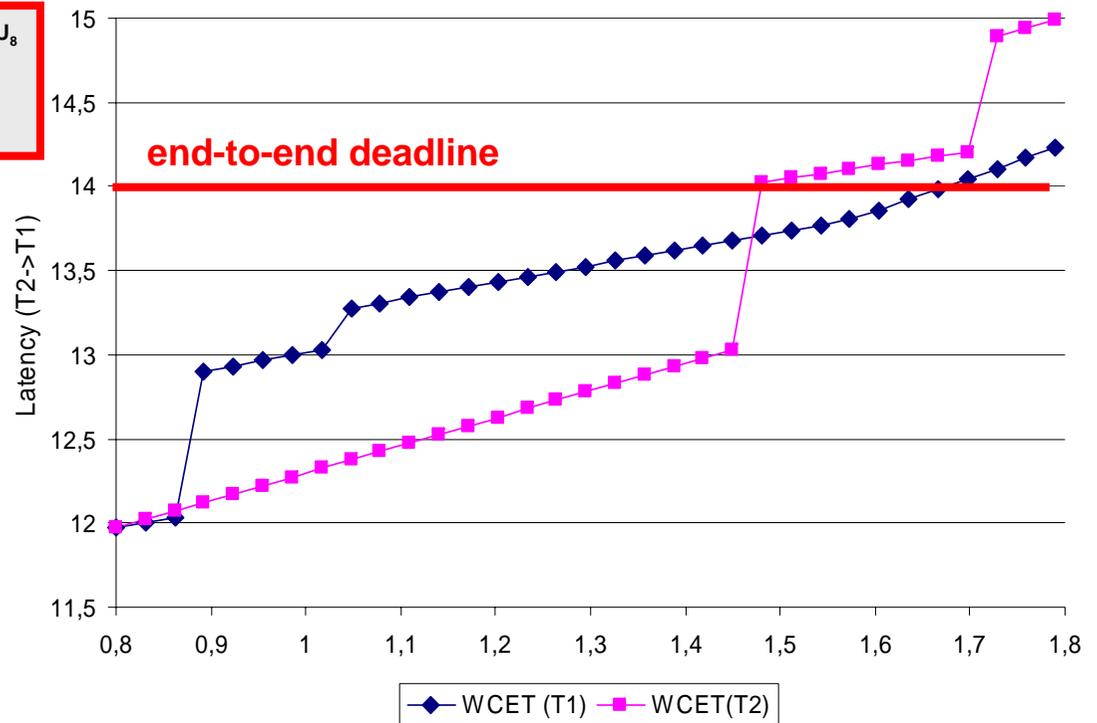
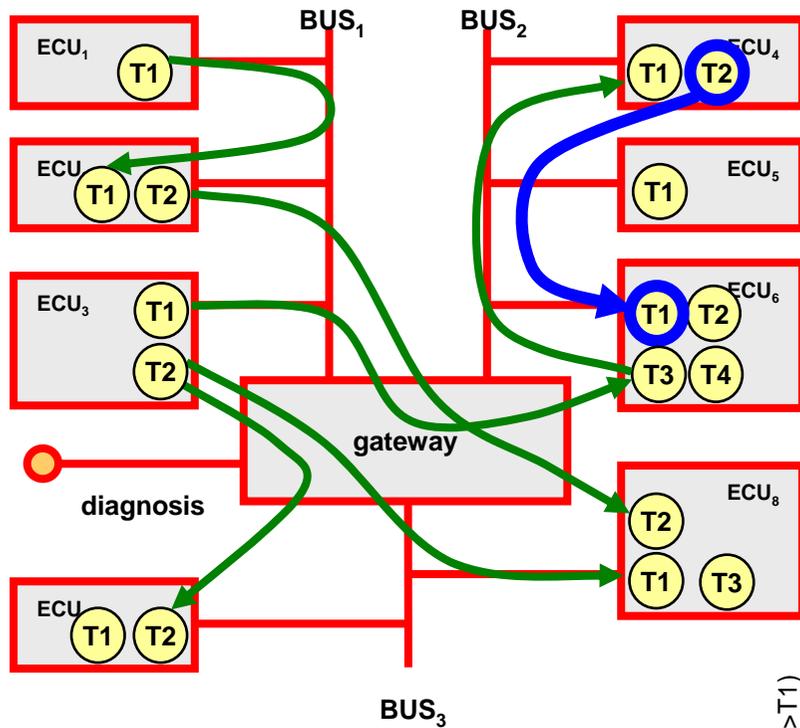
Domino effects due to parameter changes



Multi-dimensional sensitivity analysis



Example: WCET variation



Sensitivity analysis

- **Sensitivity analysis identifies limits of feasible design**
 - **How far can system properties be changed before the system fails → slack ?**
 - **What is the impact of property changes on the performance metrics?**



Sensitivity analysis key features

- **Evaluates design risk linked with a specific component**
 - **helps to controls parameter changes**
 - **captures „domino“- effects**
 - **metric for design robustness**
- **Assistance for system dimensioning/configuration**
 - **choose optimal bus bandwidth, CPU clock speed**

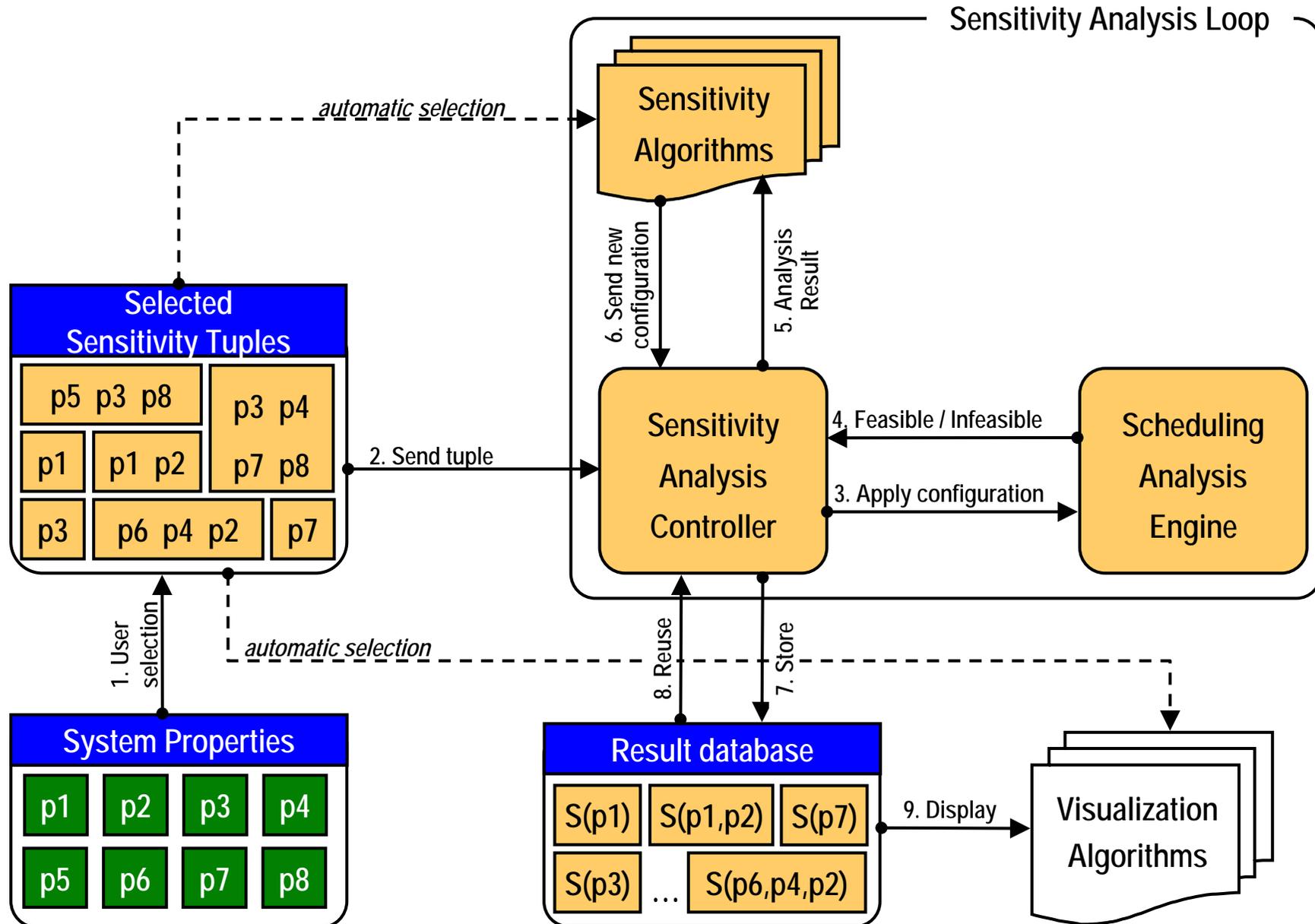


Design properties considered

- All design data can be subject to changes → **complex issue**
- Here we assume
 - Fixed architecture
 - Fixed mapping of functions to components
- **Modification** of performance related SW and HW component properties
 - Platform component performance (processor and communication links)
 - Execution times of individual processes
 - Process communication volumes
- Considered **performance metrics**
 - Predictable design → worst case data
 - Response times
 - End-to-end latencies



Sensitivity analysis framework in SymTA/S



Sensitivity Analysis Framework

- **Based on SymTA/S analysis engine**
- **Formally derived search space boundaries**
 - **based on load conditions**
 - **finds discontinuity points (scheduling anomalies)**
- **Binary search technique**
 - **optimal → minimum number of search steps**
 - **bidirectional search space**
 - **feasible → infeasible**
 - **Infeasible → feasible**
 - **transparent with respect to scheduling algorithms**
 - **applicable only on monotonic search spaces**
 - **if non-monotonic behavior, then split search space in monotonic sub-spaces**



Sensitivity Analysis - Algorithms

- **One dimensional analysis**
 - **Formally derived search space boundaries**
 - **Binary search like search**
- **Two dimensional analysis**
 - **Divide-and-conquer like search algorithm**
 - **Parameter specific heuristics for search space reduction**





SymTA/S

Design space exploration and System Robustness Optimization

ARTIST2 PhD Course, June 12, DTU Copenhagen, Denmark

Razvan Racu

Arne Hamann



Design Space Exploration Framework

Outline

- **SymTA/S design space exploration framework**
- **Problem independent selector algorithms**
- **Example application: Timing optimization in SymTA/S**

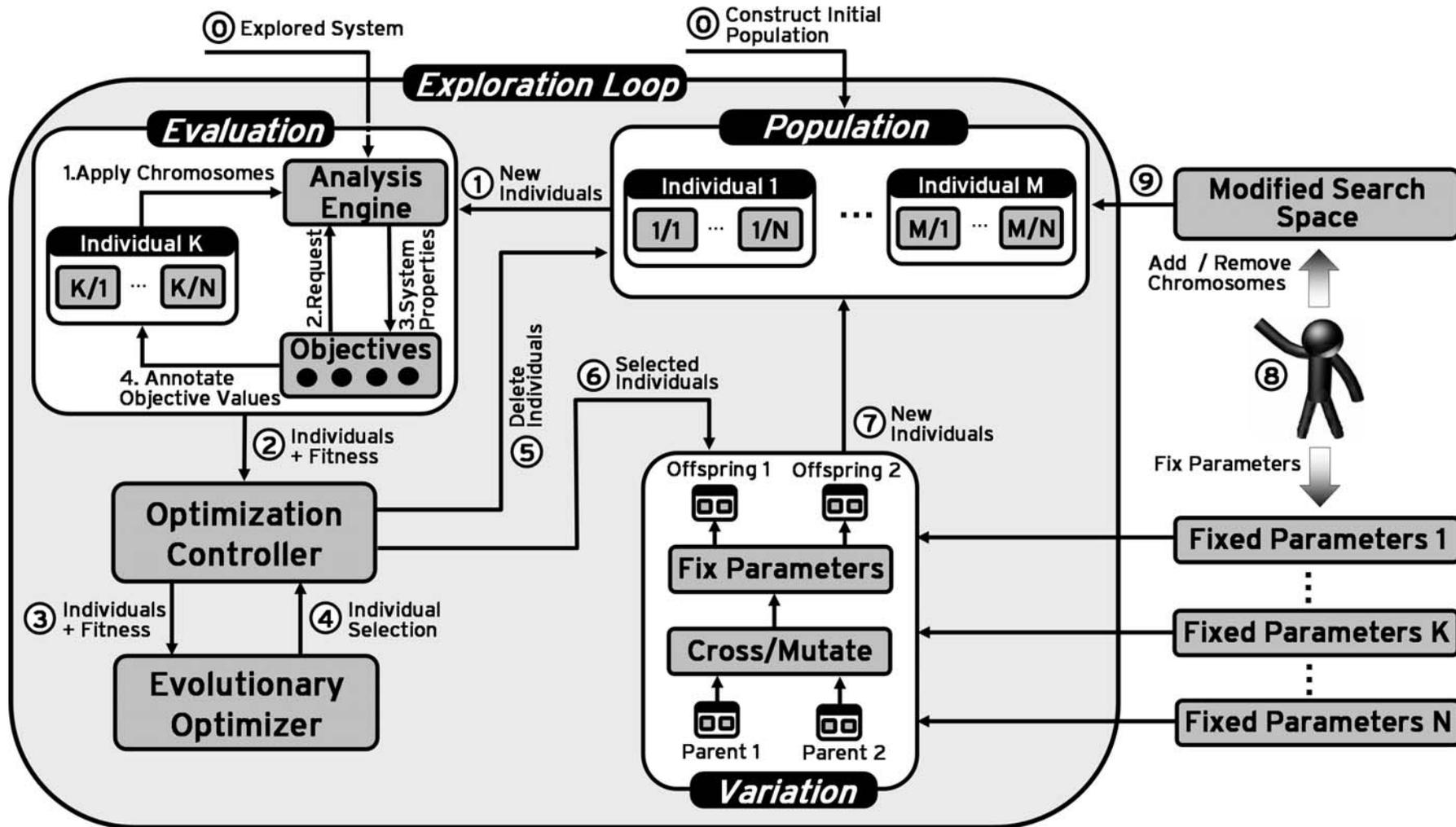


Design Space Exploration Framework

- **Compositional search space encoding scheme**
- **Dynamic search space modification**
 - **user-controlled exploration**
 - **automatic search space adaptation**
- **High flexibility and extensibility**
- **Pareto-optimization of arbitrary optimization objectives**
 - **Evolutionary search techniques, PISA, ETH Zurich**
- **Exploration speed-up through meta-heuristics**
 - **problem independent**
 - **problem dependent**



Exploration loop



Selector example: FEMO

- **Fair Evolutionary Multi-objective Optimizer (FEMO)**
- **Developed by Zitzler and Thiele (~2002), ETH Zürich**
- **Idea: Offspring based selection**
 - **Count for each individual the number of his offsprings**
 - **Select individuals with equal rate for procreation → Fairness**
- **Remove all dominated (i.e. not Pareto-optimal individuals) after each generation → variable population size**
 - **All individuals in population are Pareto-optimal, none is “better” than another**
 - **Possible problem: search space coverage**



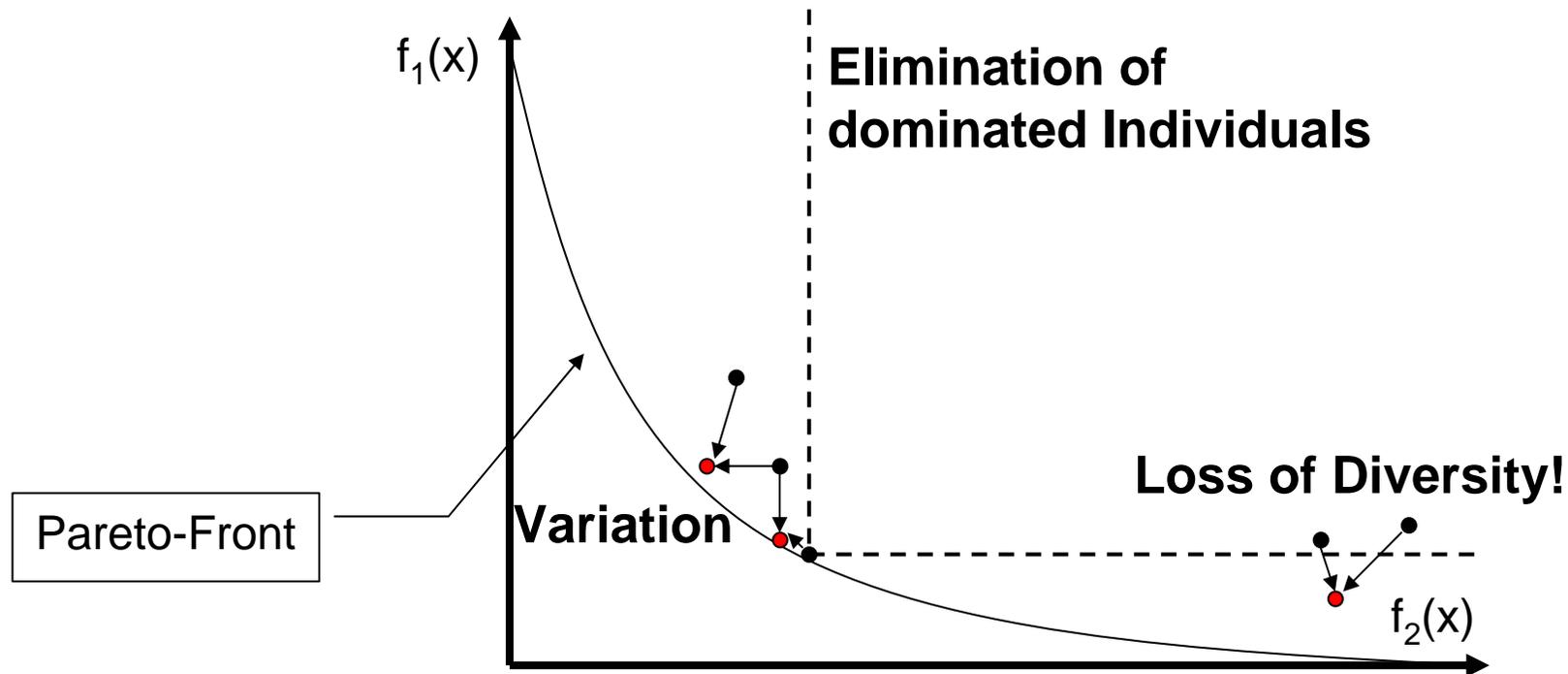
FEMO Algorithm

- **Add random initial individuals to the population**
- **Repeat until stop condition:**
 - **Select individual i with the least offsprings**
 - **Create offspring i' through crossover and mutation**
 - **Remove all individuals from population that are Pareto-dominated by i'**
 - **Add i' to the population if it is not Pareto-dominated by any other individual**



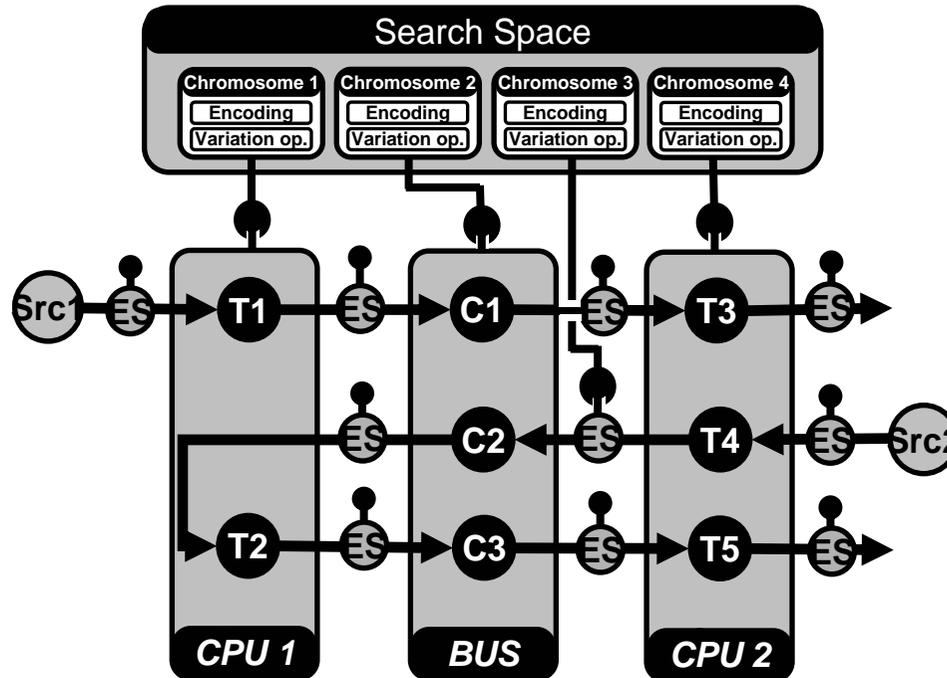
FEMO: Evolutionary Search Strategy

- Diversity vs. Convergence speed

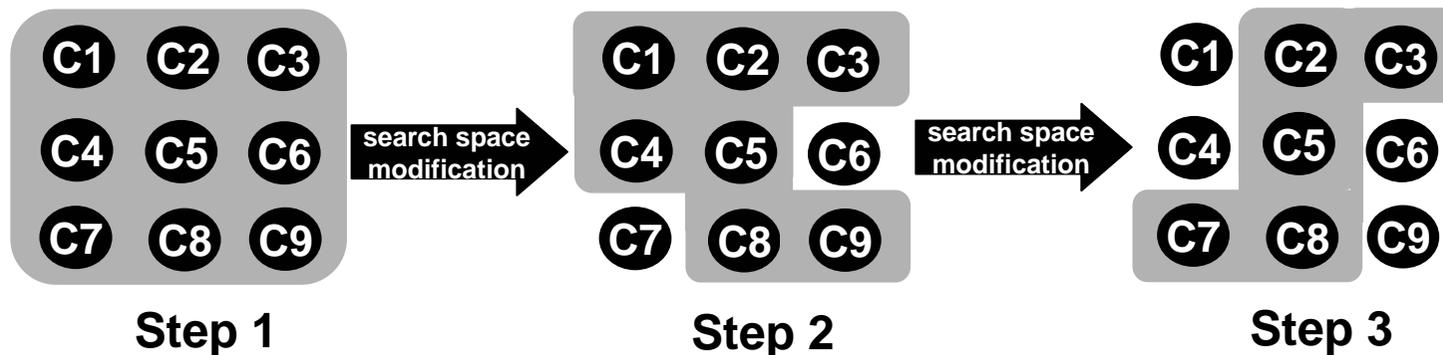


Exploration control

- Compositional encoding



- Search space adaptation



Application domains

- **System optimization**
 - **timing (jitter, end-to-end deadlines)**
 - **buffer sizes**
 - **power dissipation**
 - **mapping**
- **Robustness optimization**
- **Multi-dimensional sensitivity analysis**
- **System generation**
- **...**

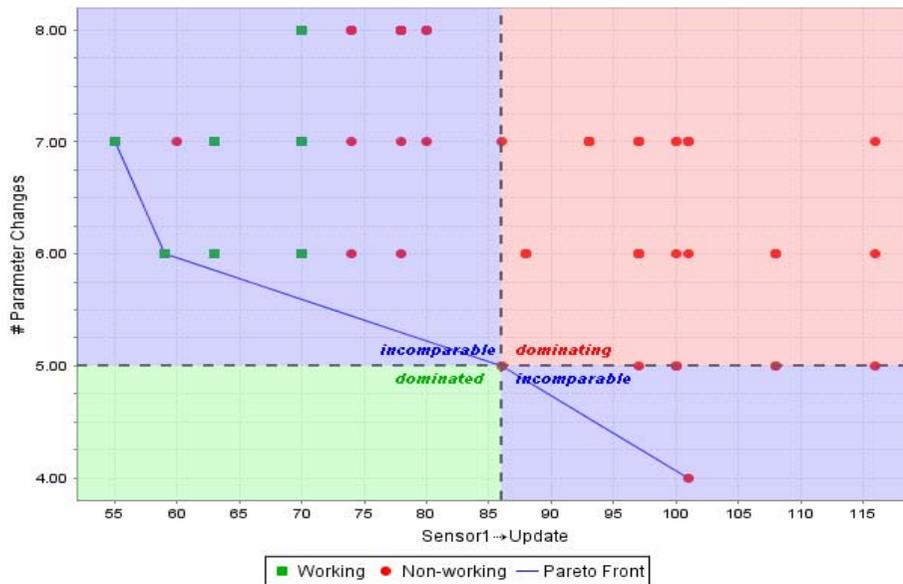


Example application: Timing optimization 1/2

- **Search space**
 - **scheduling parameter for various policies: SPP, TDMA ,RR, EDF**
 - **optimization of parameters for real world RTOSes and bus protocols: ERCOSEK, CAN**
 - **optimization through traffic shaping**
 - **mapping optimization**
 - **...**
- **Optimization Objectives**
 - **end-to-end latencies, worst-case response times**
 - **buffer sizes**
 - **power consumption**
 - **system cost**
 - **# parameter changes**
 - **...**

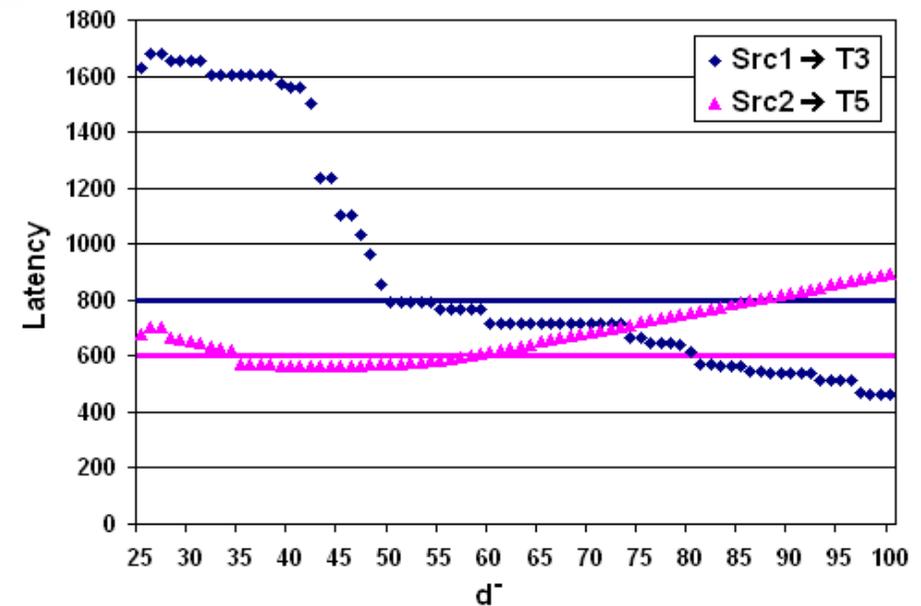


Example application: Timing optimization 2/2



Pareto-front: end-to-end deadline vs. # parameter changes

Influence of Traffic Shaping on System Performance





Robustness Optimization

Outline

- **System property variations**
- **Sensitivity Analysis**
- **Stochastic Multi-dimensional Sensitivity Analysis**
- **Robustness Metrics**
 - **Hypervolume calculation**
 - **Minimum Guaranteed Robustness (MGR)**
 - **Maximum Possible Robustness (MPR)**
- **Experiments**



System Property Variations (1)

- **Two types of system property variations**
 - **Variations influencing the system load**
 - **Software execution path length**
 - **Communication volumes**
 - **Input data rates**
 - **Variations influencing the system service capacity**
 - **Processor clock-rate**
 - **Communication link performance**



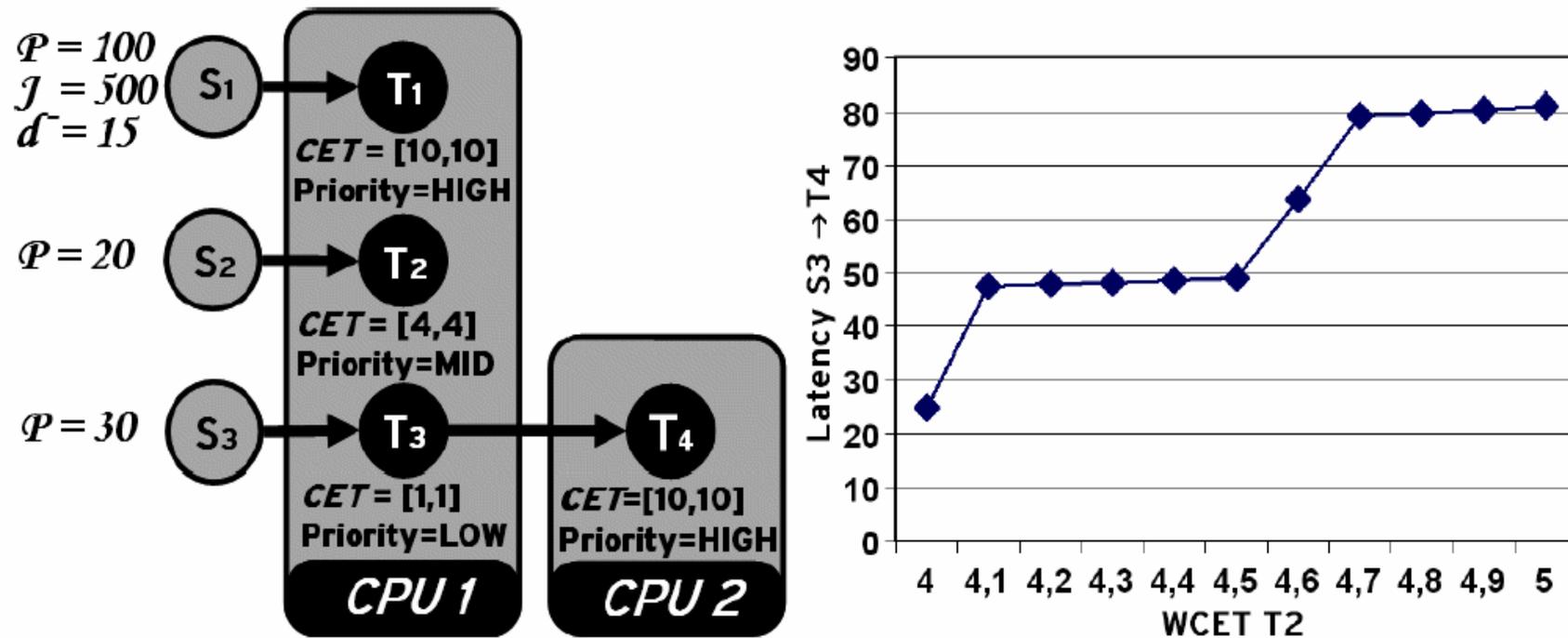
System Property Variations (2)

- **Why do system property variations occur?**
 - **Specification changes, late feature requests, product variants, software updates, bug-fixes**
- **Robustness to property variations**
 - **decreases design risk, and**
 - **increases system maintainability and extensibility**
- **Property variations can have severe unintuitive effects on system performance**



Example: WCET Variation

- End-to-End latency $S3 \rightarrow T4$ as a function of execution demand of T2



System Property Variations (3)

- **Property variations invalidate the assumption under which the system was dimensioned and configured**
- **→ Correct function and performance of the system is put at risk**
- **How can we increase the robustness of the system to property variations ?**
 - **Adaptivity: feedback-based scheduling, self-organizing systems,...**
 - **Sensitivity analysis: achieve robustness without on-line parameter adaptation**



Problem Formulation

- Find parameter configuration that ...
- ... maximizes the robustness of the given system w.r.t. changes of several properties
- Robustness = the system can sustain a certain degree of property variations without severe performance degradation
- → Multi-dimensional optimization problem
- Not included: dynamic parameter adaptations as a reaction to property variations

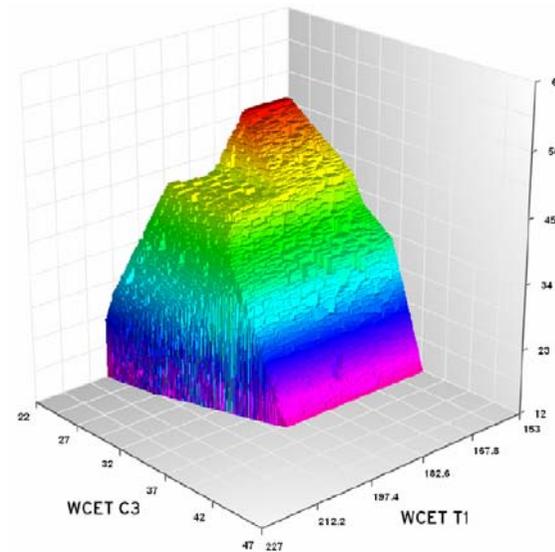
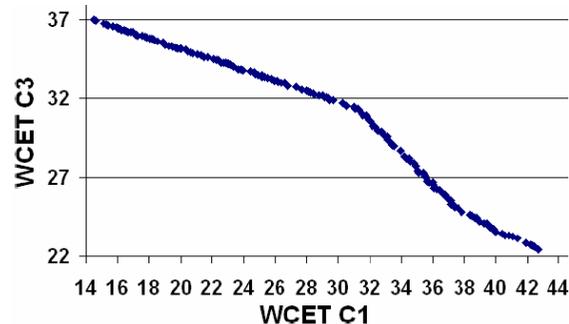
Sensitivity Analysis (1)

- **Calculates maximum/minimum admissible values for given system properties**
- **Supported system properties**
 - **WCETs / BCETs**
 - **Communication volume**
 - **CPU clock rate**
 - **Bus throughput, ...**



Sensitivity Analysis (2)

- One-dimensional case
 - maximum/minimum feasible property value
- Multi-dimensional case
 - front separating feasible and non-feasible system property combinations: sensitivity front



Sensitivity Analysis (3)

- **Recent results:**
 - **One-dimensional sensitivity analysis**
 - **Calculates slack for a single system property**
 - **Vestal: Trans. on Software Engineering 1994**
 - **Racu: RTAS 2005**
 - **Multi-dimensional sensitivity analysis**
 - **Considers interdependencies between multiple system properties**
 - **Racu / Hamann: ECRTS 2006**
- **Problem: computational effort grows exponentially with problem dimension**



Stochastic Sensitivity Analysis (1)

- **Solution: scalable stochastic analysis to bound system sensitivity**
 - **Sensitivity analysis formulated as multi-objective optimization problem**
 - **Search space: System properties including WCETs, Periods, Jitters, ...**
 - **Optimization objectives: maximization / minimization of considered system properties**
 - **Pareto-optimization**
- **Pareto-front of optimization task corresponds to sought-after sensitivity front**



Stochastic Sensitivity Analysis (2)

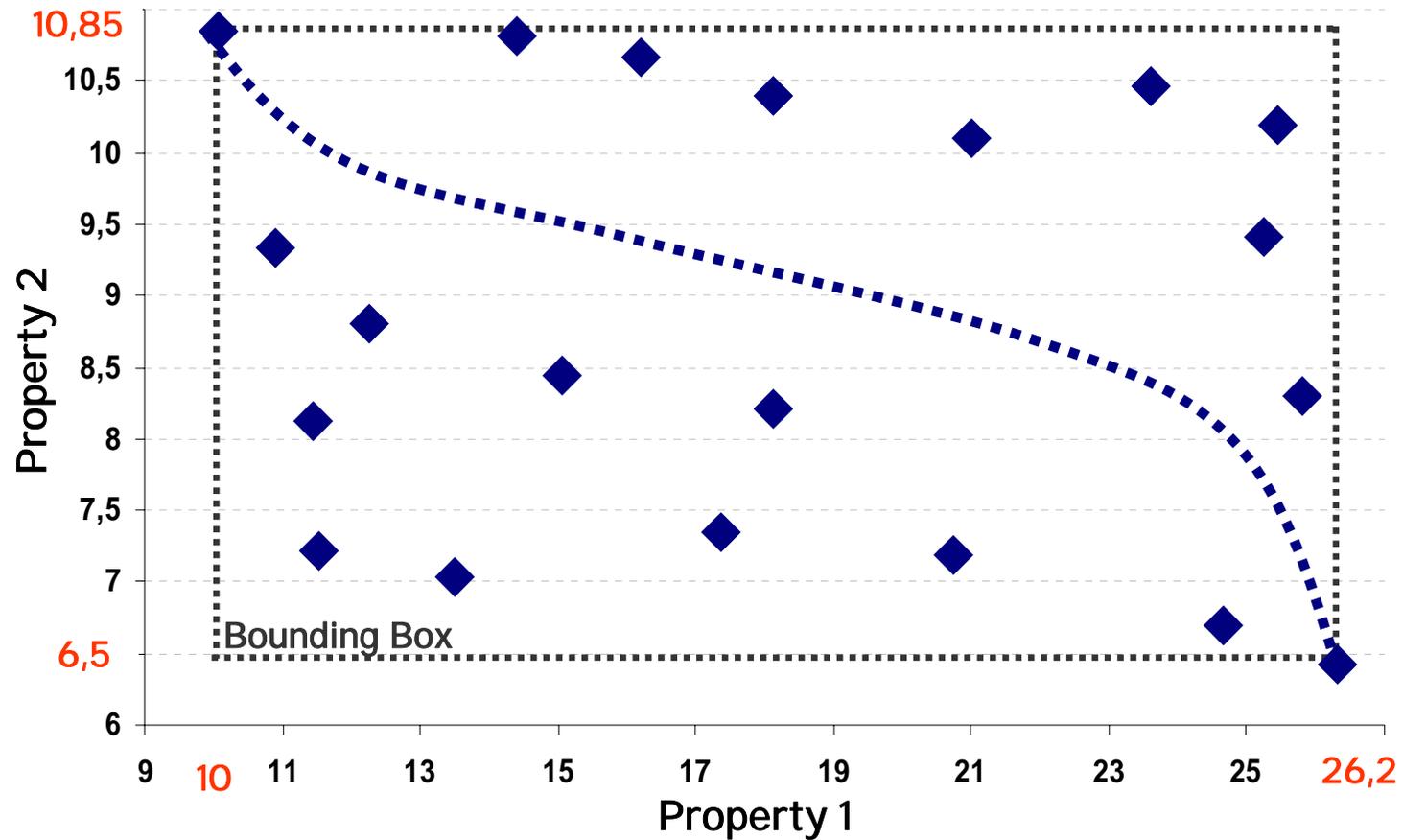
- **Uses multi-criteria evolutionary algorithms to approximate sensitivity front**
 - **responsible for sensitivity front coverage**
 - **Currently used SPEA2 (ETH Zurich): diversified sensitivity front approximation through Pareto-dominance based selection and density approximation**
- **Can be used for system properties subject to maximization (e.g. WCETs) and minimization (e.g. Periods)**
- **In the following: properties are subject to maximization**



Creation of the Initial Population

- **Creates a certain number of points representing a first approximation of sensitivity front**
- **Uses 1-dim sensitivity analysis**
 - **to bound the search space in each dimension (bounding hypercube)**
 - **to generate points representing the extrema of the sought-after sensitivity front**
- **Randomly place the rest of the initial points in bounding hypercube**

Initial Population - Example



Bounding the Search Space (1)

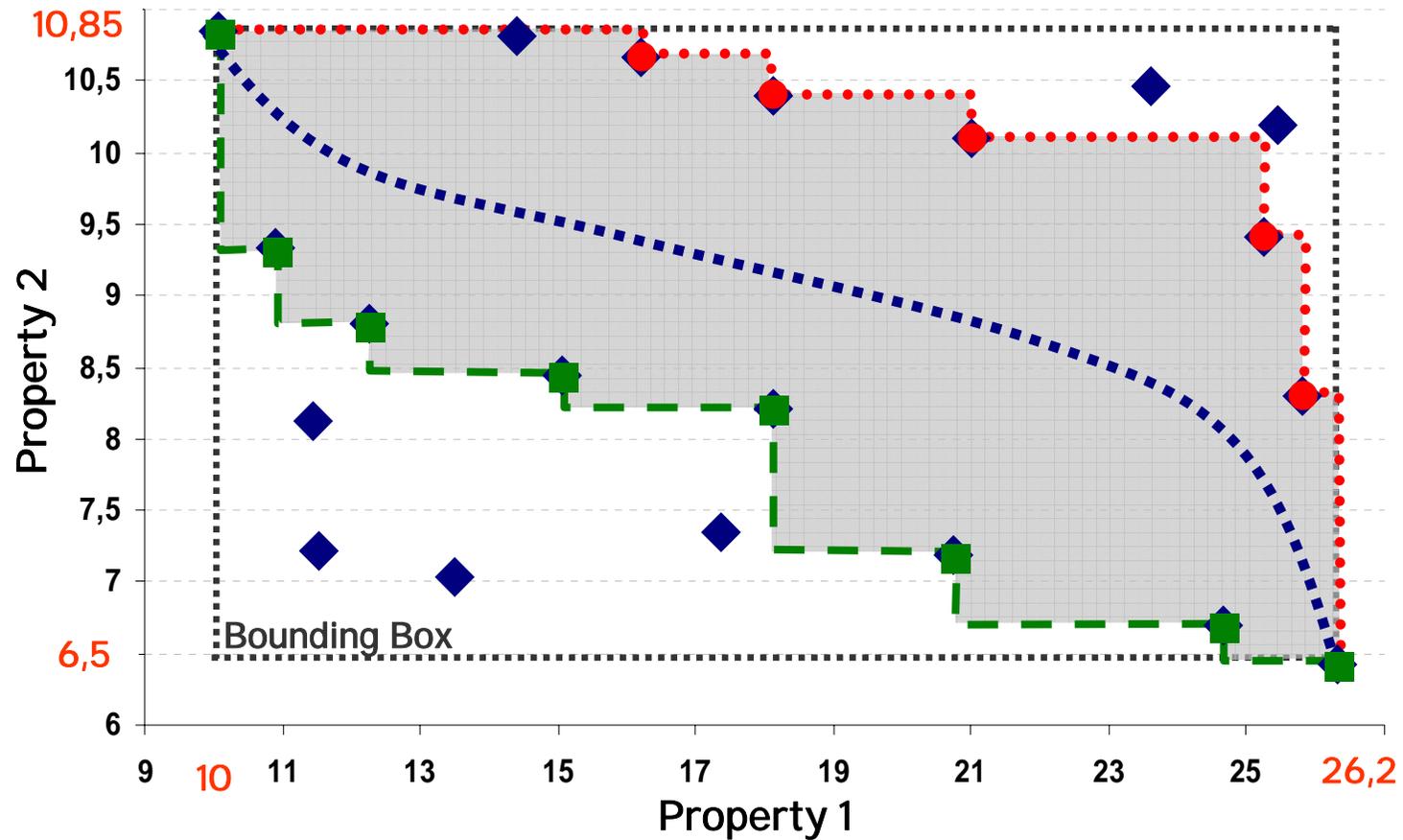
- Extension for stochastic sensitivity analysis for robustness optimization
- Idea: bound search space containing the sought-after sensitivity front
 - Bounding working Pareto-front \mathcal{F}^n
 - evaluated Pareto-optimal working points
 - Bounding non-working Pareto-front \mathcal{F}^{nw}
 - evaluated Pareto-optimal non-working points
- Bounding Pareto-fronts can be used to derive multi-dim. robustness metrics (later)



Bounding the Search Space (2)

- Space between bounding Pareto-fronts is called interesting region
 - Variation operators use algorithm ensuring that generated offsprings (points) are contained in interesting region
 - Below bounding non-working Pareto-front
 - Above bounding working Pareto-front
- Efficiently focuses exploration effort

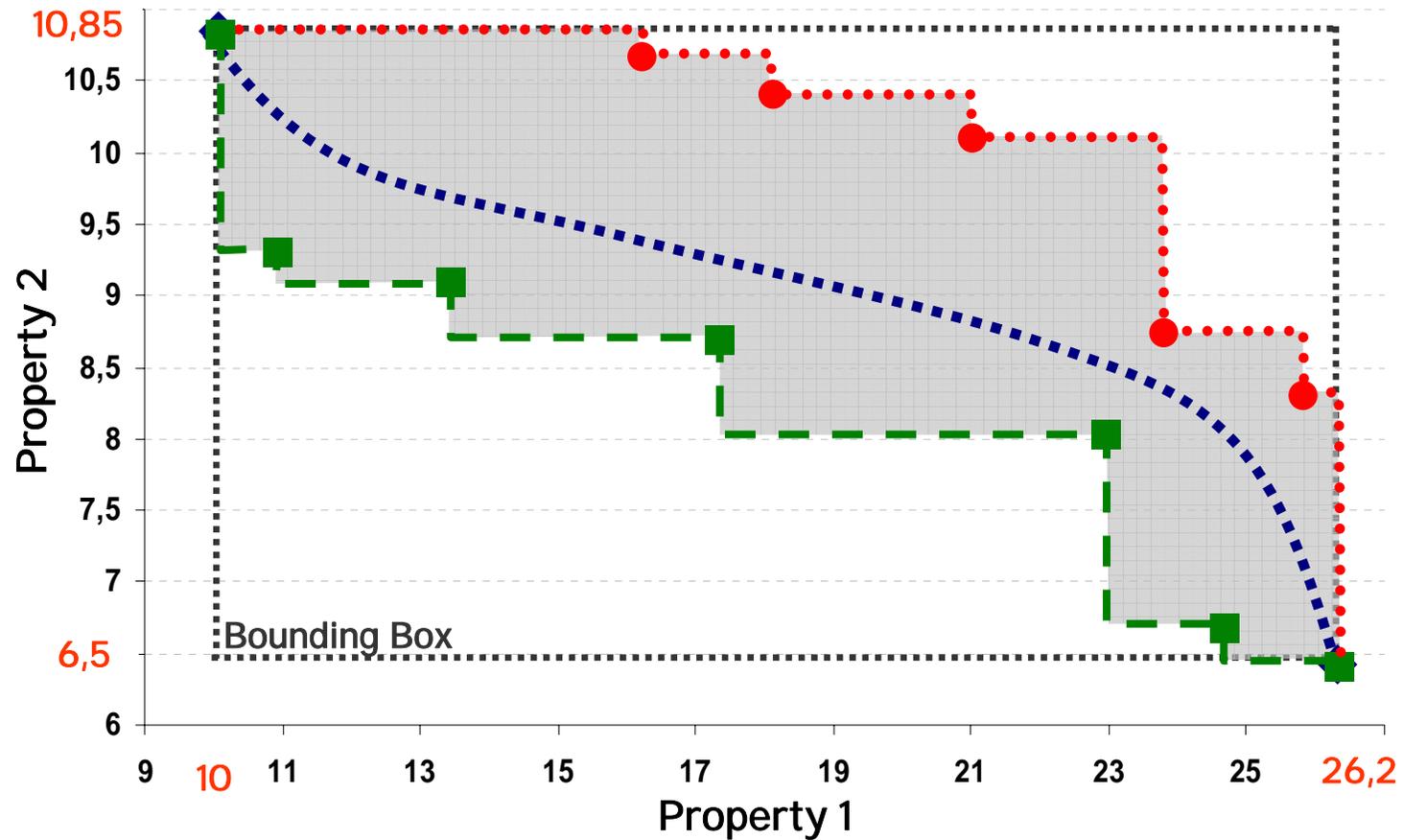
Bounding the Search Space (3)



Random Crossover (1)

- Takes as input two parent points to create two offspring points
- The two parent points define hypercube in which the created offspring points are randomly placed
- Simple standard operator that locally refines the approximation of the sought-after sensitivity front

Random Crossover (3)



Front Convergence Mutate (1)

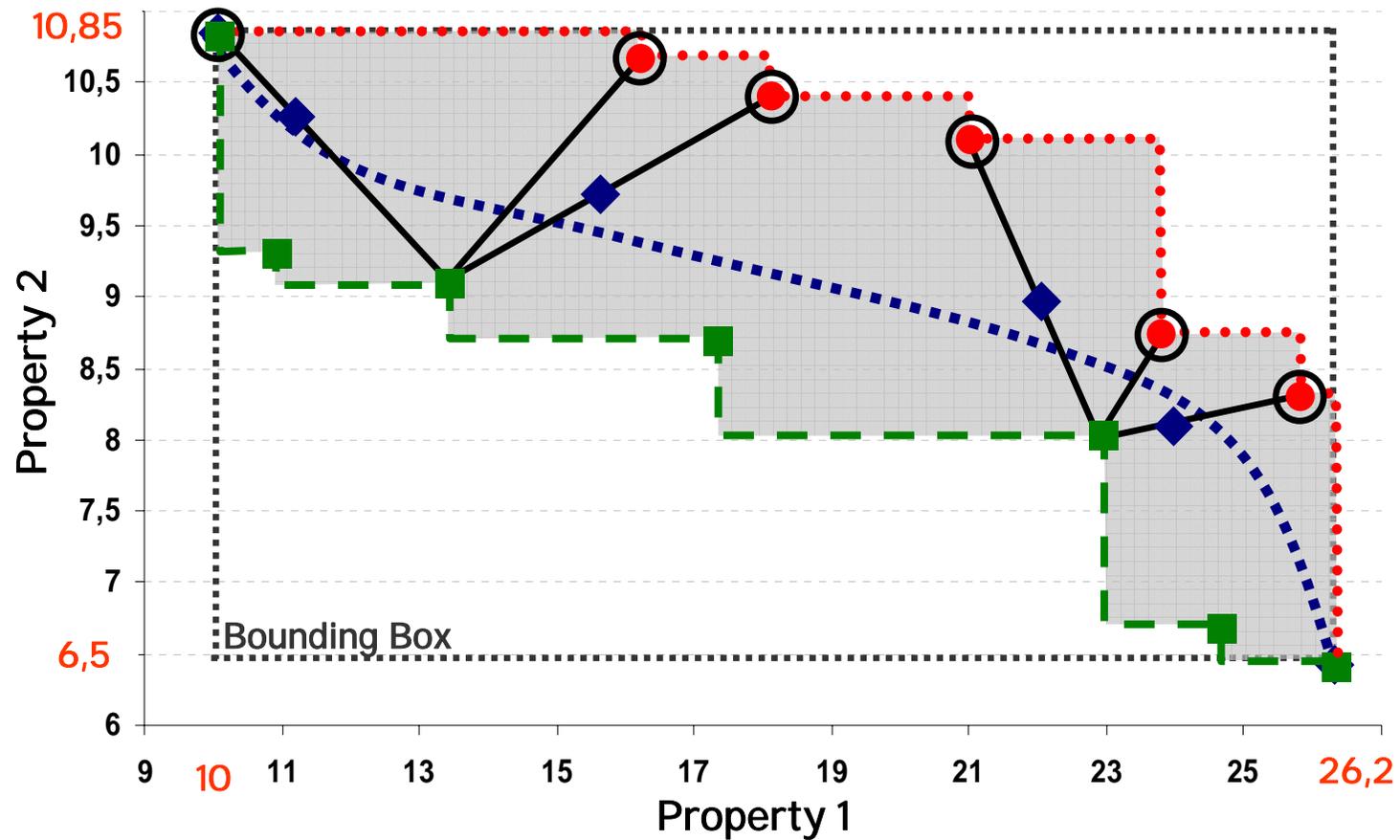
- Takes as input one parent point to produce one offspring point
- Heuristic operator adapted to optimization problem
- Increases convergence speed
- Directly supports the convergence of the bounding Pareto-fronts



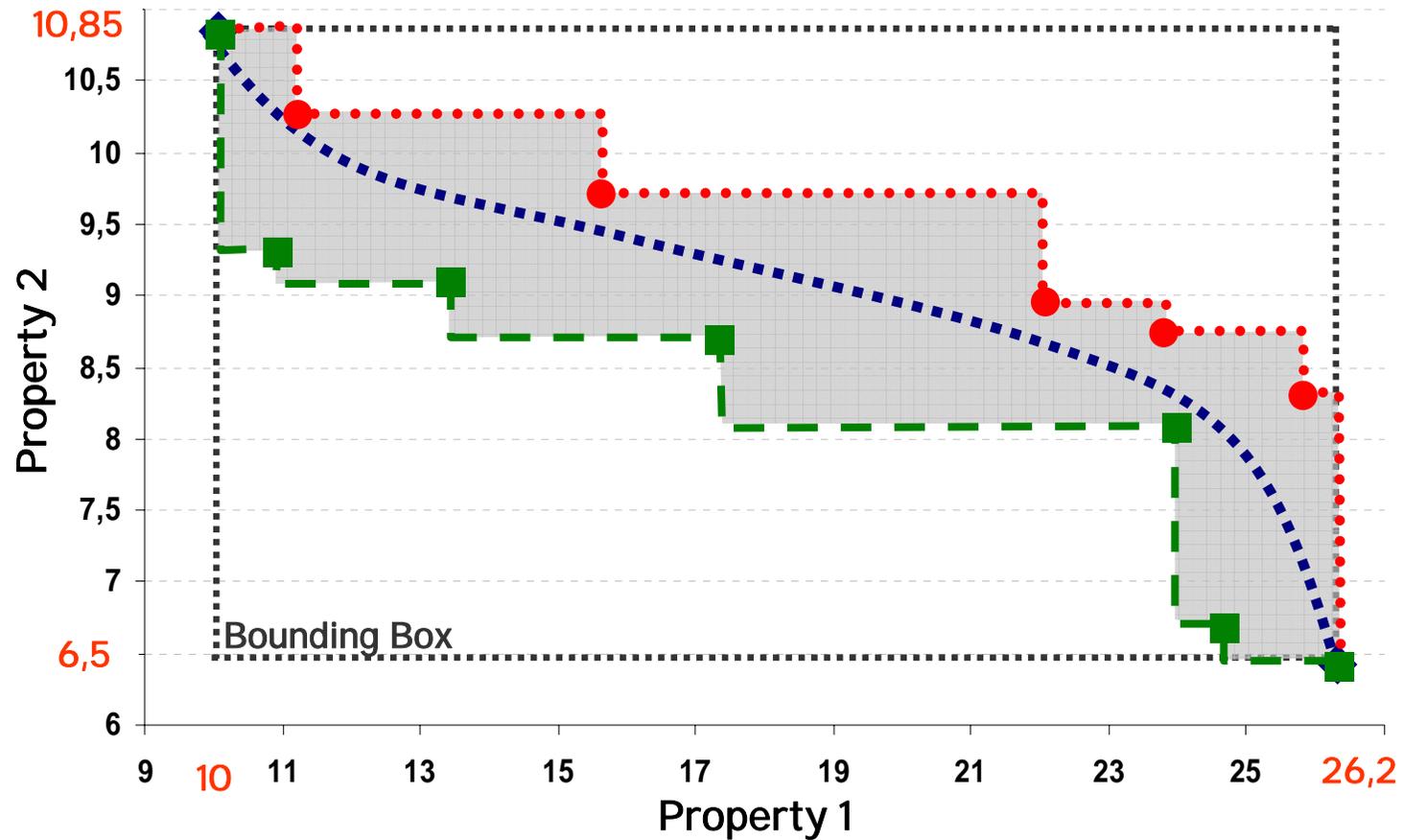
Front Convergence Mutate (2)

- **Strategy:**
 - **Determine X closest points on opposite Pareto-front**
 - **Choose randomly one of these points**
 - **Place offspring point randomly on straight line connecting the parent point and the chosen random point**

Front Convergence Mutate (3)



Front Convergence Mutate (4)

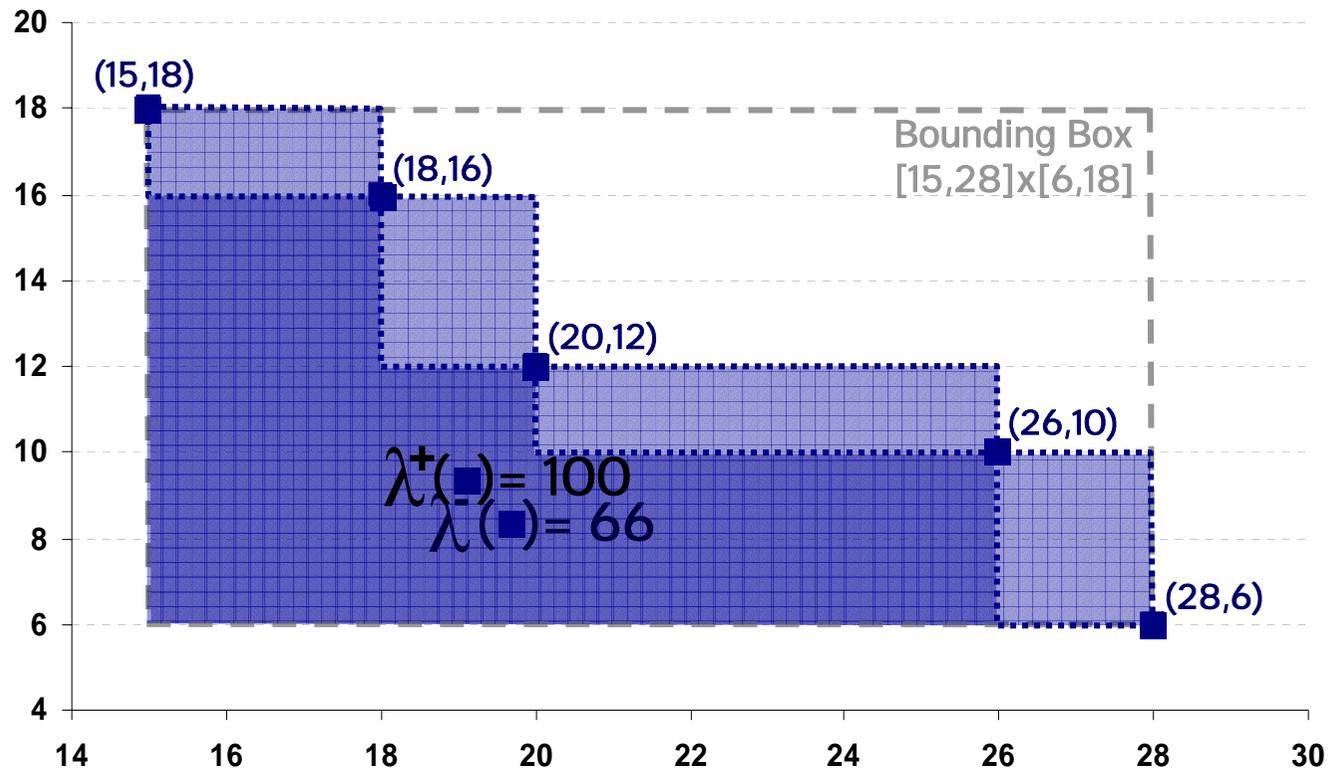


Hypervolume Calculation

- Hypervolume as basis of the proposed robustness metrics
- Hypervolume is defined in a given hypercube and associated to a point set
- Two different notions of hypervolume
 - inner hypervolume λ^- : Volume of space Pareto-dominated by the given points inside the given hypercube
 - outer hypervolume λ^+ : Volume of space Pareto-dominated by all points not Pareto-dominating any of the given points

Hypervolume Calculation (2)

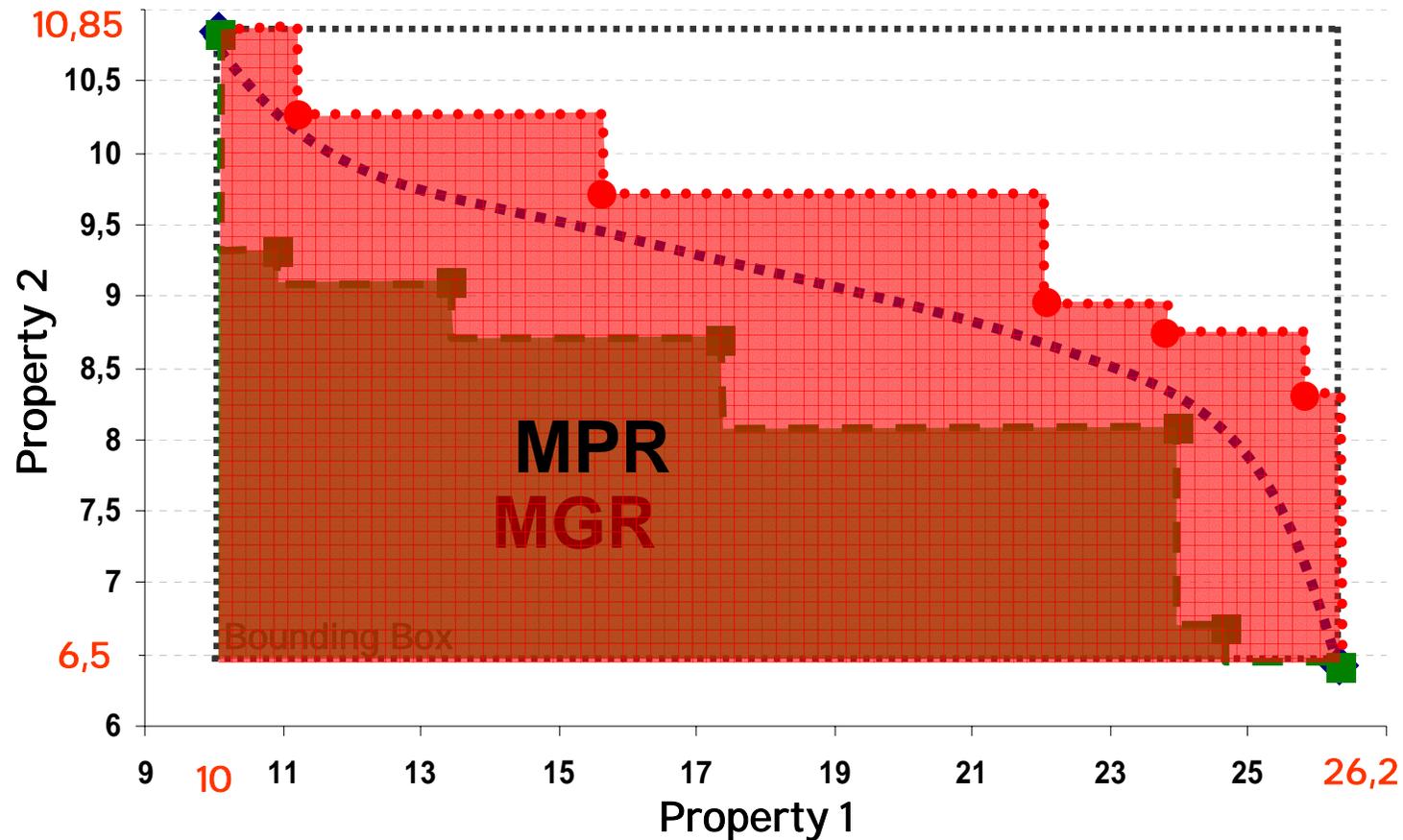
- 2D-case
 - inner hypervolume: lower step function
 - outer hypervolume: upper step function



Robustness Metrics

- Given a set of properties we want to achieve robustness for ...
- ... use stochastic sensitivity analysis to derive upper and lower robustness bounds
 - **Minimum Guaranteed Robustness (MGR)**
 - Defined as inner hypervolume of the bounding working Pareto-front \mathcal{F}^w
 - **Maximum Possible Robustness (MPR)**
 - Defined as outer hypervolume of the bounding non-working Pareto-front \mathcal{F}^{nw}

Robustness Metrics (2)



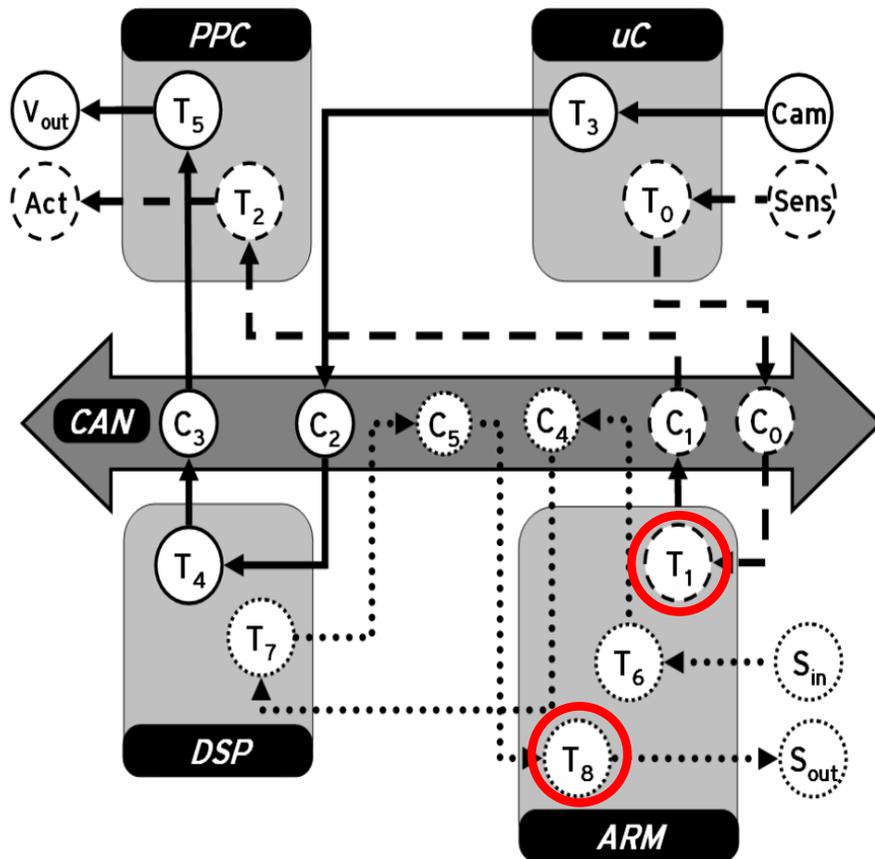
Obviously: $MGR \leq \text{Real Robustness} \leq MPR$

Robustness Exploration

- **Idea: Pareto-optimize MGR and MPR**
- **Advantages**
 - **Stochastic sensitivity analysis is scalable**
 - **Little computational effort necessary to reasonably bound robustness potential of given configuration**
 - **In-depth analysis can be performed once interesting configurations are identified (i.e. high MGR or high MPR)**
 - **Perfectly suited for robustness optimization**

Example System

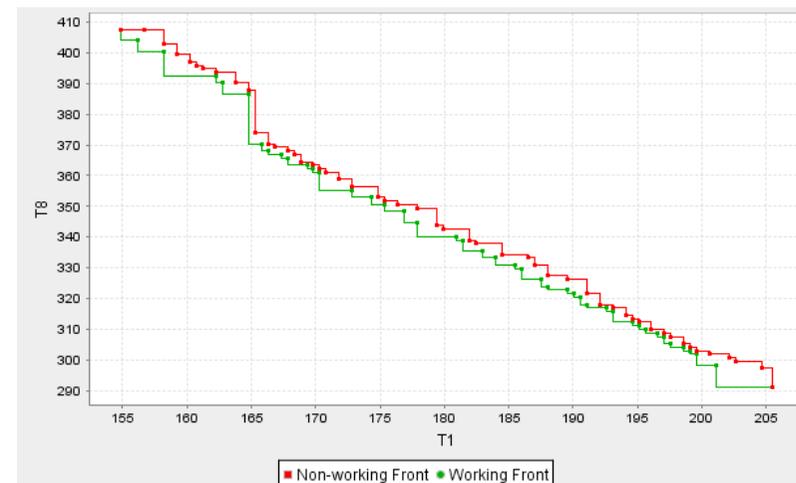
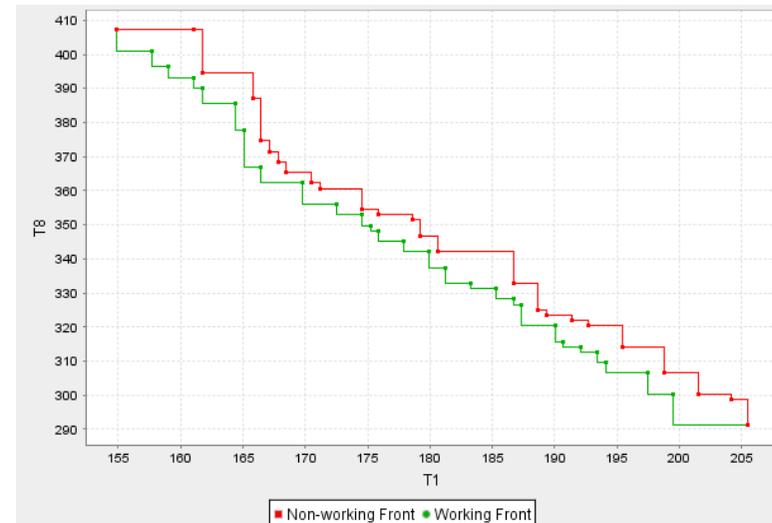
- Distributed embedded system
- 4 computational resource ...
- ... connected via CAN bus
- 3 applications
 - $Sens \rightarrow Act$
 - $S_{in} \rightarrow S_{out}$
 - $Cam \rightarrow V_{out}$



Approximation Quality (1)

- Approximation after 100 evaluations (20 sec)
- MGR = 2447
- MPR = 2937

- Approximation after 200 evaluations (40 sec)
- MGR = 2580
- MPR = 2813

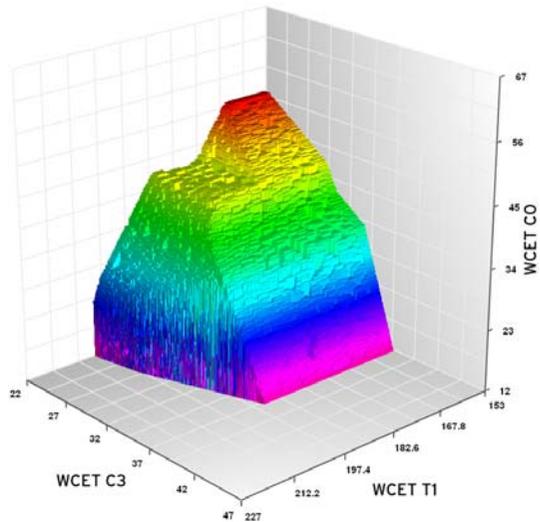


Approximation Quality (2)

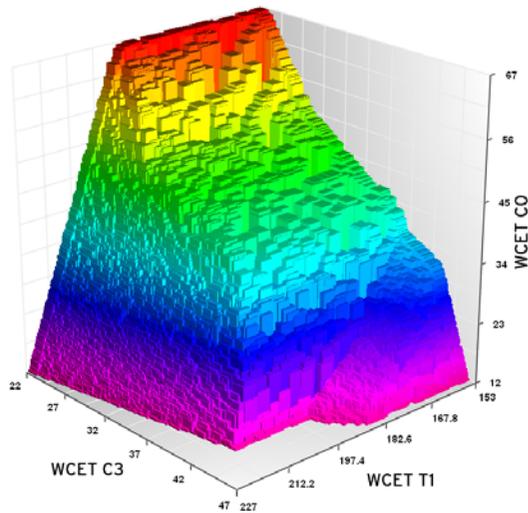
- Approximation after 300 evaluations (60 sec)
- MGR = 2632
- MPR = 2777



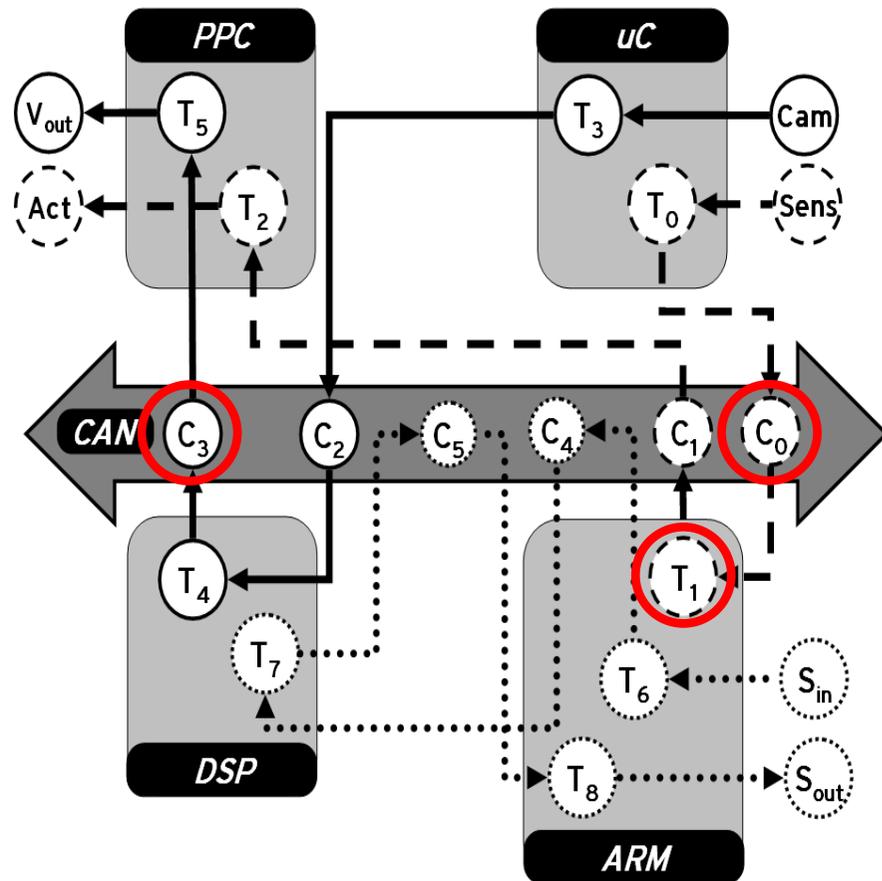
3D - Robustness Maximization (1)



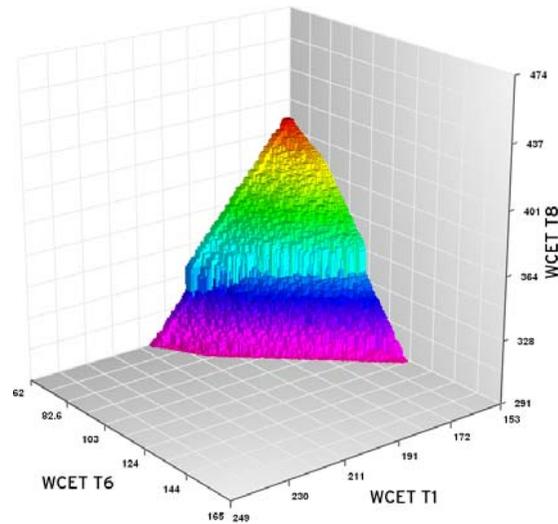
Original configuration



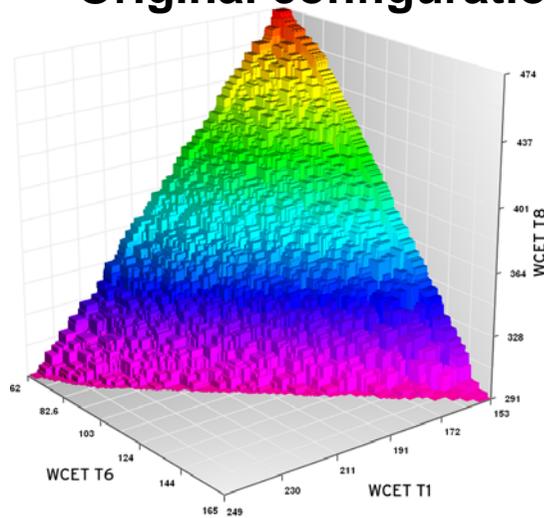
Optimized configuration



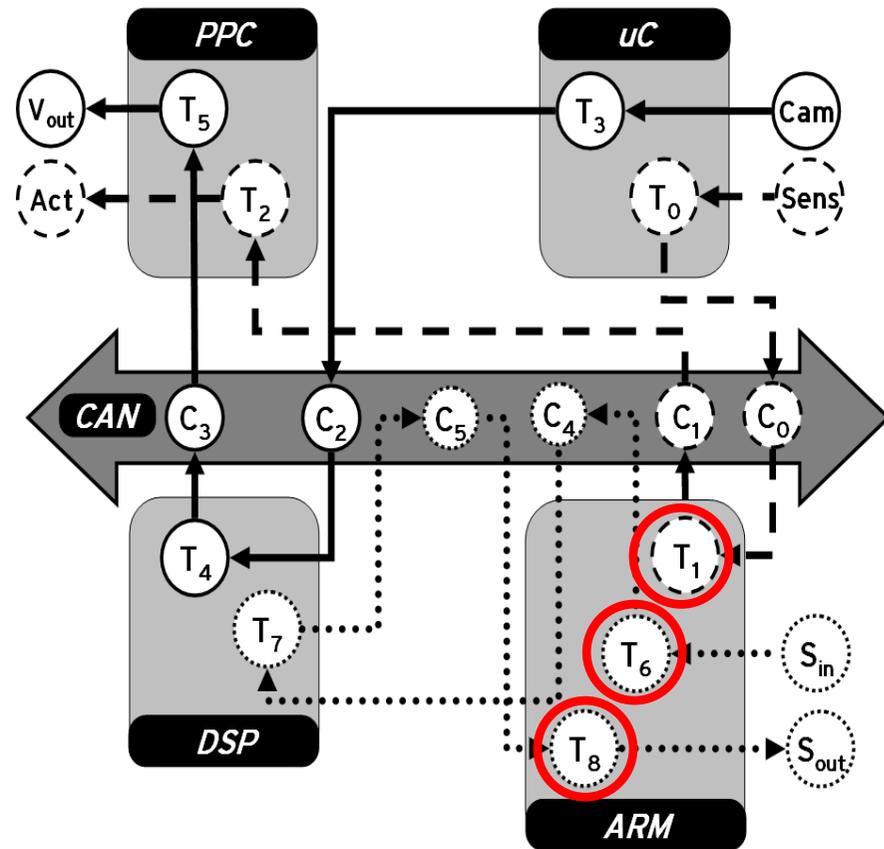
3D - Robustness Maximization (2)



Original configuration



Optimized configuration



Conclusion

- **Robustness to system property variations**
- **Scalable stochastic sensitivity analysis perfectly suited for robustness optimization**
- **Metrics expressing lower and upper system robustness bounds ...**
- **... enable efficient integration of robustness criteria into design space exploration**