

Quantitative Verification and Synthesis, of Embedded Systems

Kim G. Larsen

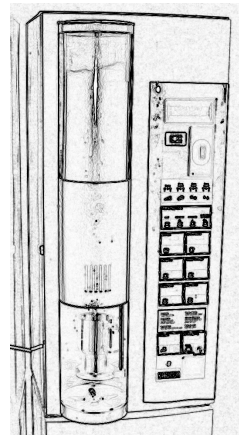
CISS – Aalborg University
DENMARK



Embedded Systems



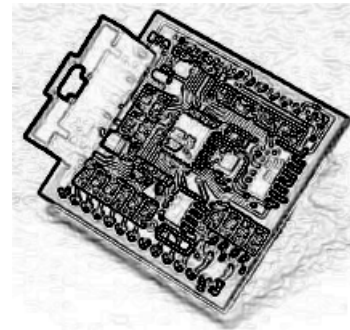
Plant
Continuous



sensors



actuators



**Controller
Program**
Discrete

Eg.: Realtime Protocols
Pump Control
Air Bags
Robots
Cruise Control
ABS
CD Players
Production Lines

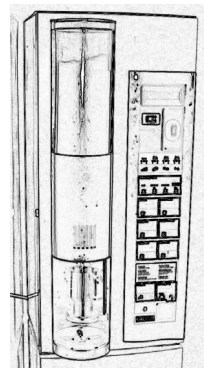
Quantities:

- timing
- energy
- memory
- bandwidth
- uncertainties

Verification



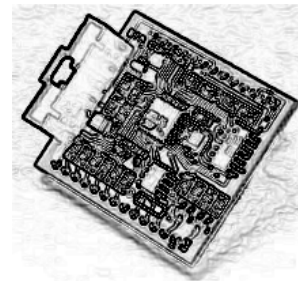
Plant
Continuous



sensors

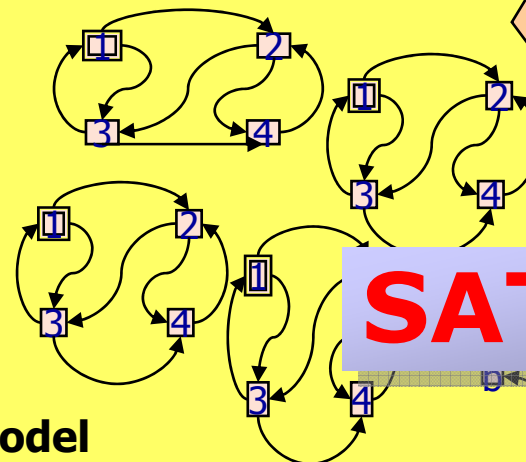
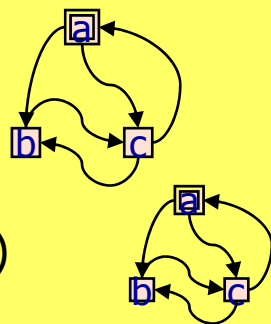
actuators

Controller Program
Discrete



Model
of
tasks
(automatic?)

Model
of
environment
(user-supplied /
non-determinism)



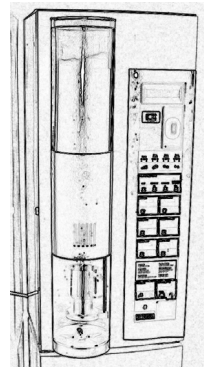
UPPAAL Model

SAT ϕ ??

Synthesis



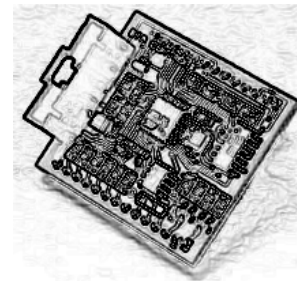
Plant
Continuous



sensors

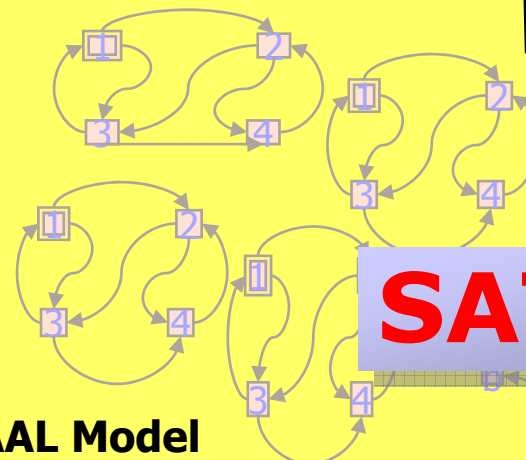
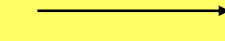
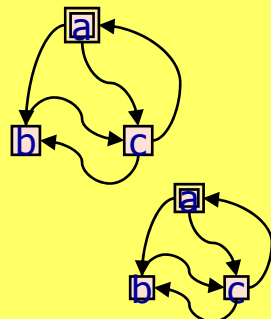
actuators

Controller Program
Discrete



Synthesis
of
tasks
(automatic)

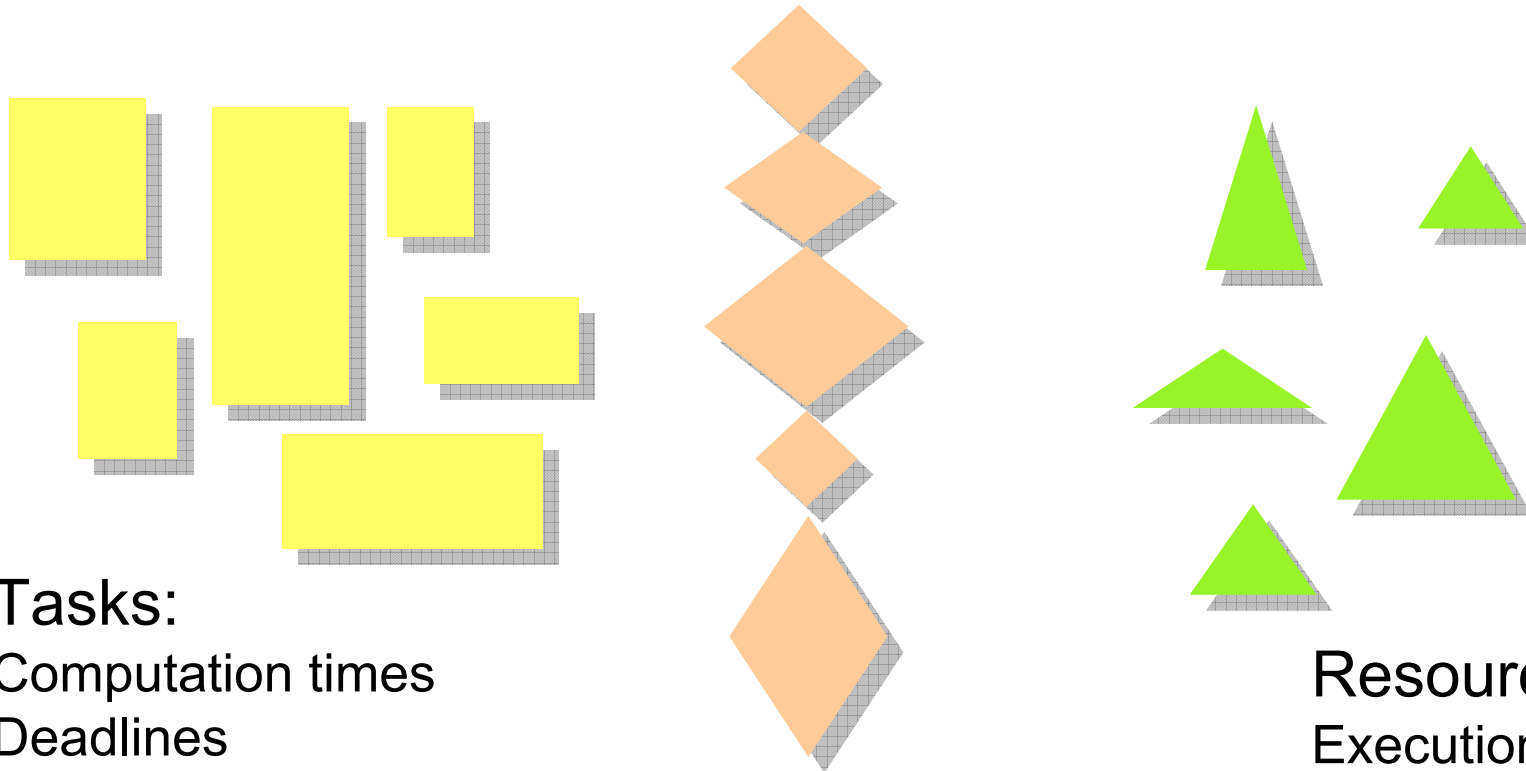
Model
of
environment
(user-supplied)



SAT ϕ !!

Partial UPPAAL Model

Embedded Systems → Scheduling



Tasks:

Computation times
Deadlines
Dependencies
Arrival patterns
uncertainties

Scheduling Principles (OS)
EDF, FPS, RMS, DVS, ..

Resources

Execution platform
PE, Memory
Networks
Drivers
uncertainties

Approach – Timed Automata



- Schedulability
 - Verify that given SP ensures deadlines.

- Performance Evaluation
 - Estimate resources (e.g. energy) required by given

CLASSIC

- Scheduling Synthesis
 - (optimal) SP given objective.
 - Scheduling: SP controls

TIGA

TALK:

What can we do?
What can we do **efficiently**?
What would we **like to** do?
What **can not** be done?

Timed Automata

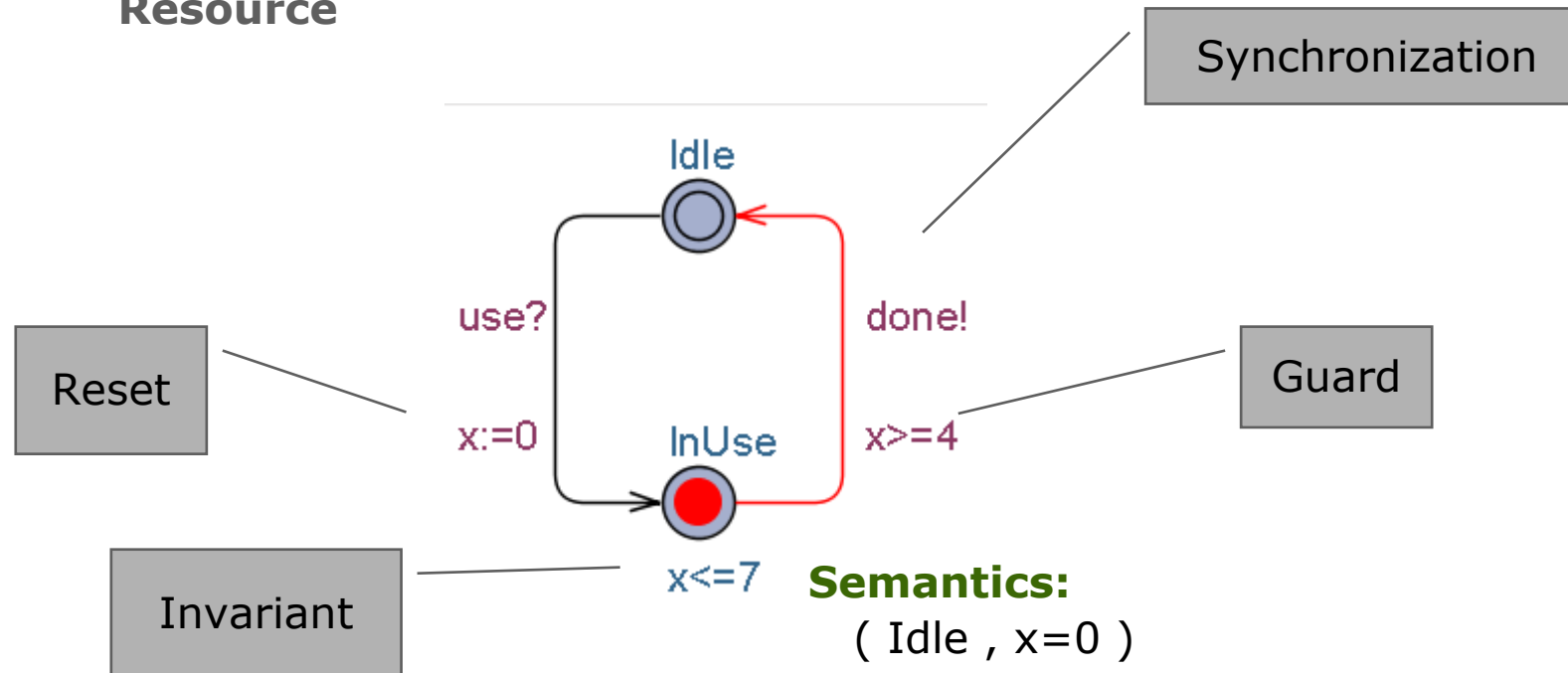


Timed Automata

[Alur & Dill'89]



Resource



Semantics:

(Idle , x=0)

d(2.5) → (Idle , x=2.5)

use? → (InUse , x=0)

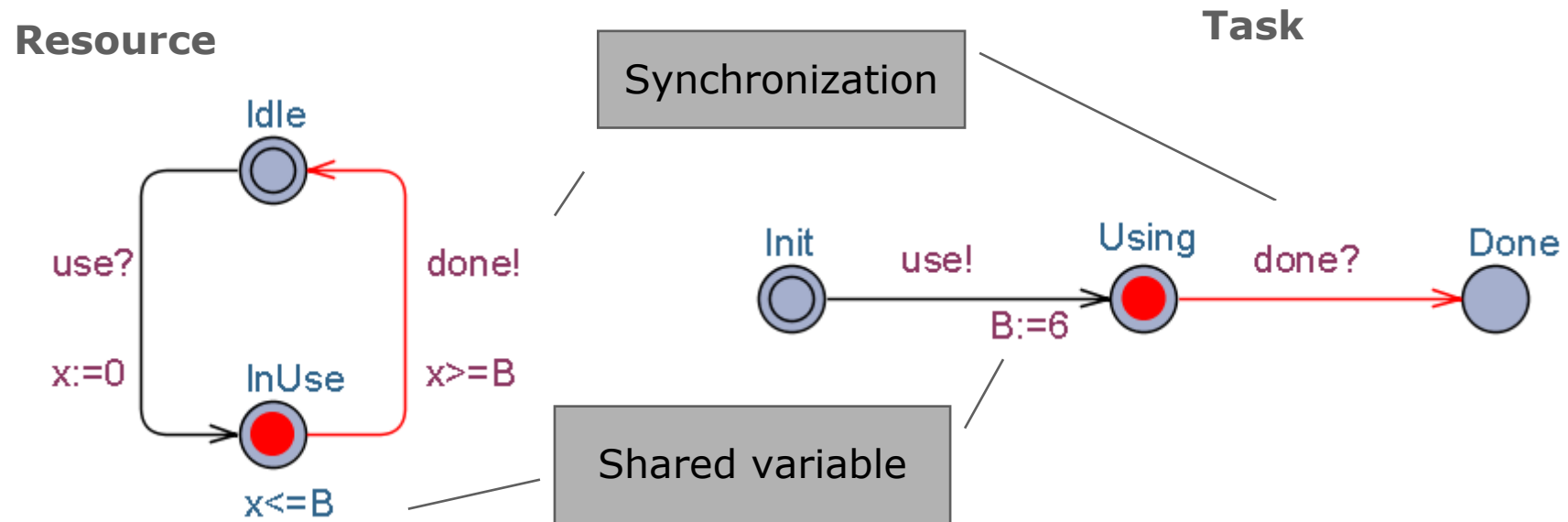
d(5) → (InUse , x=5)

done! → (Idle , x=5)

d(3) → (Idle , x=8)

use? → (InUse , x=0)

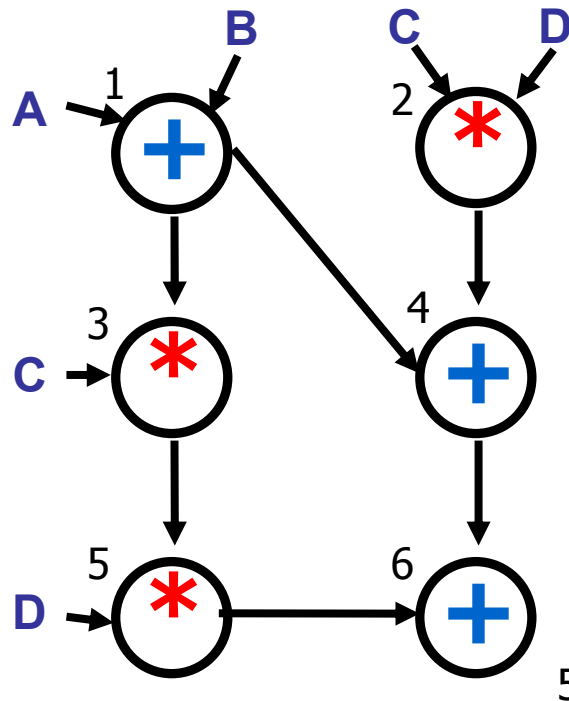
Composition



Semantics:

$(\text{Idle}, \text{Init}, B=0, x=0)$
 $d(3.1415) \rightarrow (\text{Idle}, \text{Init}, B=0, x=3.1415)$
 $\text{use} \rightarrow (\text{InUse}, \text{Using}, B=6, x=0)$
 $d(6) \rightarrow (\text{InUse}, \text{Using}, B=6, x=6)$
 $\text{done} \rightarrow (\text{Idle}, \text{Done}, B=6, x=6)$

Task Graph Scheduling – Example



Compute :
 $(D * (C * (A + B))) + ((A + B) + (C * D))$

using 2 processors

P1 (fast)

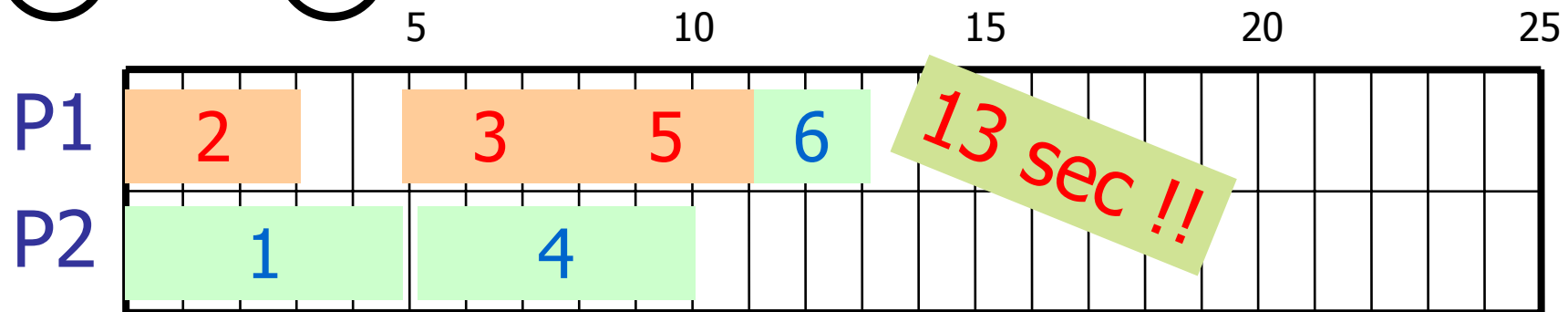


+	2s
*	3s

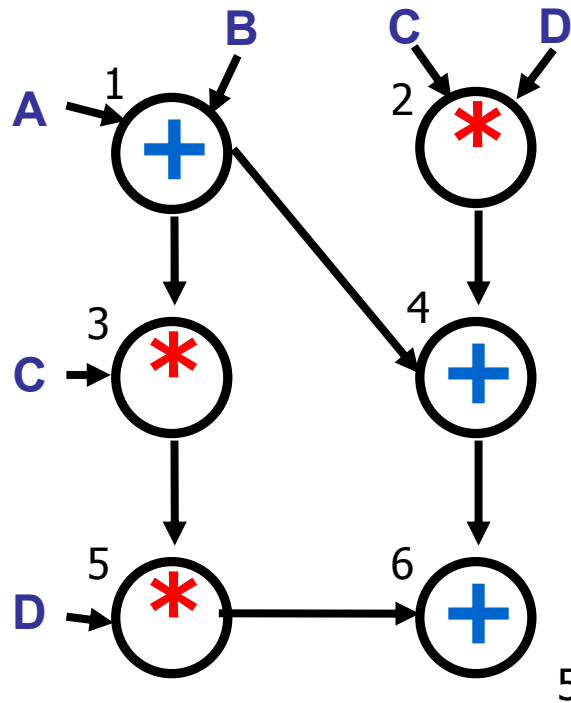
P2 (slow)



+	5s
*	7s



Task Graph Scheduling – Example



Compute :
 $(D * (C * (A + B)) + ((A + B) + (C * D)))$

using 2 processors

P1 (fast)

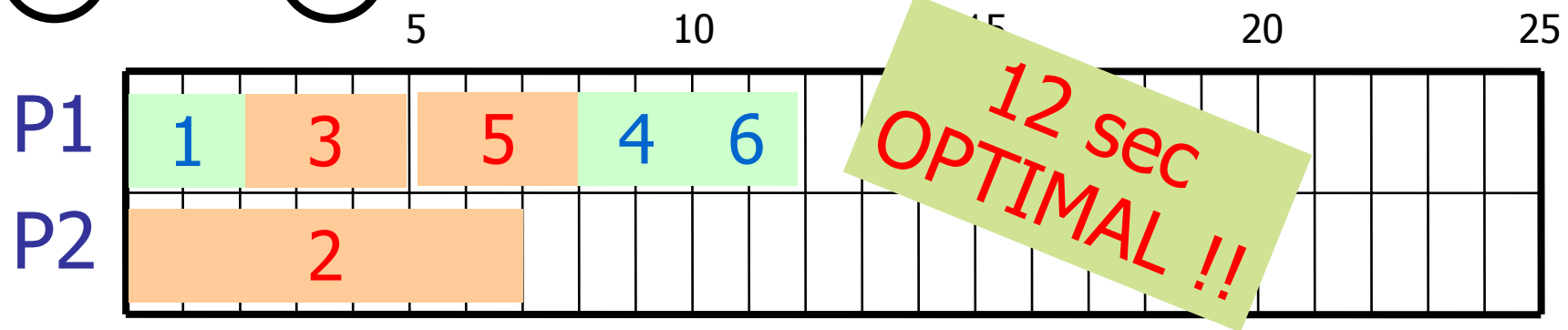


+	2s
*	3s

P2 (slow)



+	5s
*	7s

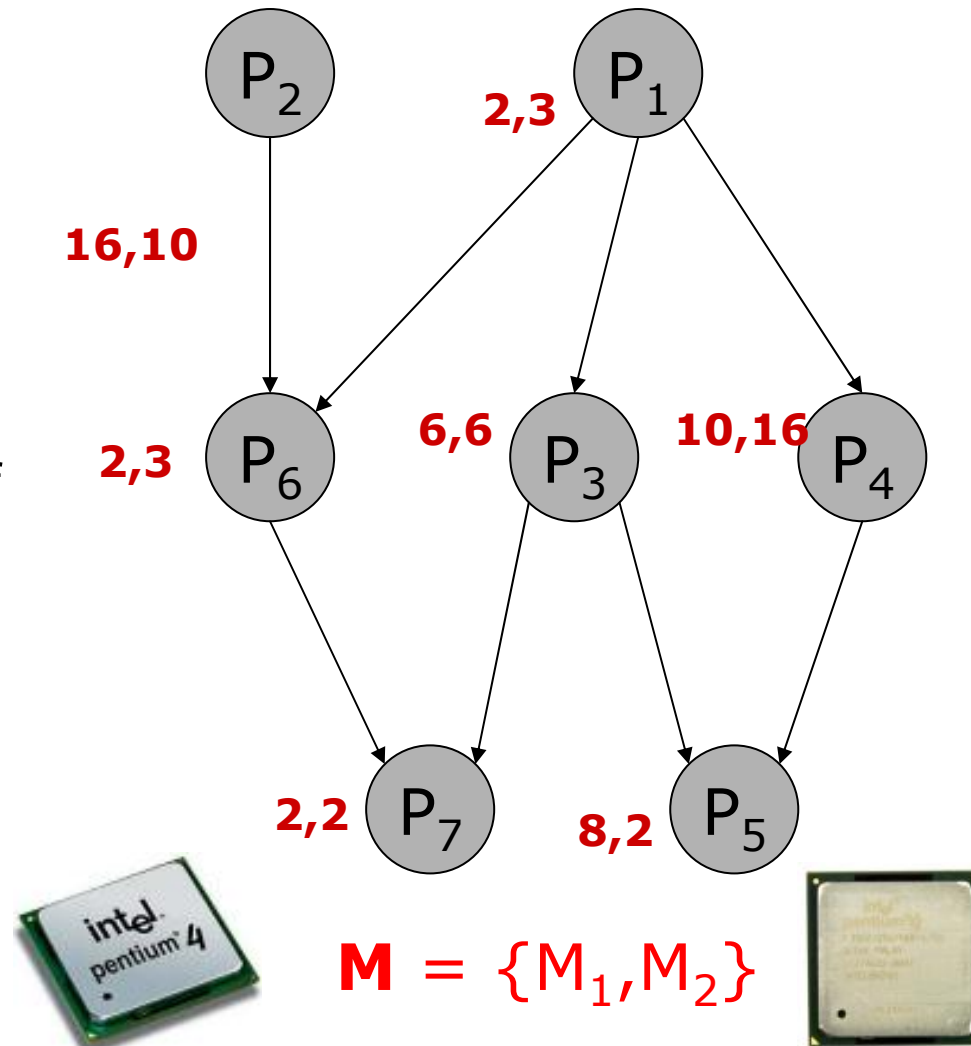


Task Graph Scheduling

Optimal Static Task Scheduling



- Task $\mathbf{P} = \{P_1, \dots, P_m\}$
- Machines $\mathbf{M} = \{M_1, \dots, M_n\}$
- Duration $\Delta : (\mathbf{P} \times \mathbf{M}) \rightarrow \mathbb{N}_1$
- $< : \text{p.o. on } \mathbf{P} \text{ (pred.)}$
- A task can be executed only if all predecessors have completed
- Each machine can process at most one task at a time
- Task cannot be preempted.
- Compute schedule with minimum completion-time!



Task Graph Scheduling

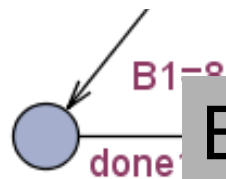
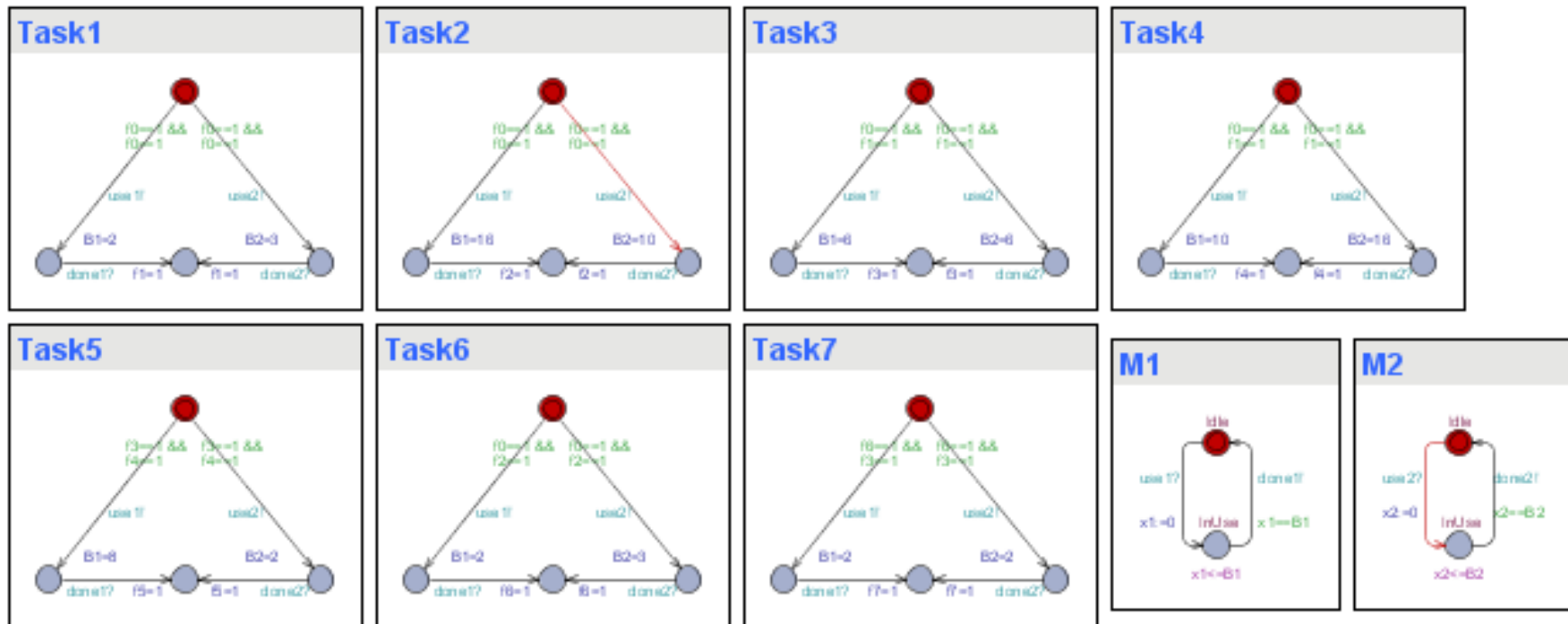
Optimal Static Task Scheduling



- Task $P = \{P_1, \dots, P_m\}$

P_2

P_1



$E \leftrightarrow (\text{Task1.End and } \dots \text{ and Task7.End})$

Experimental Results



name	#tasks	#chains	# machines	optimal	TA
001	437	125	4	1178	1182
000	452	43	20	537	537
018	730	175	10	700	704
074	1007	66	12	891	894
021	1145	88	20	605	612
228	1187	293	8	1570	1574
071	1193	124	20	629	634
271	1348	127	12	1163	1164
237	1566	152	12	1340	1342
231	1664	101	16	t.o.	1137
235	1782	218	16	t.o.	1150
233	1980	207	19	1118	1121
294	2014	141	17	1257	1261
295	2168	965	18	1318	1322
292	2333	318	3	8009	8009
298	2399	303	10	2471	2473



Symbolic A*
Branch-&-Bound
60 sec

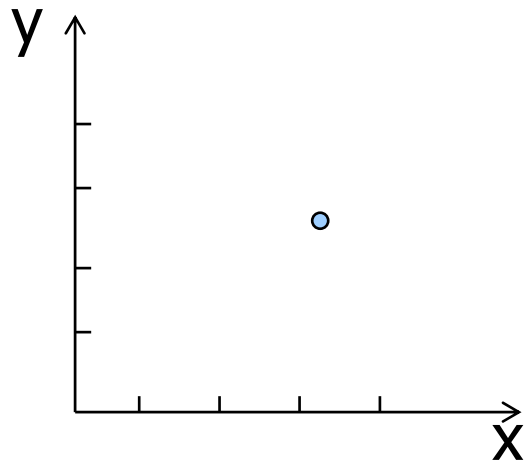
Abdeddaïm, Kerbaa, Maler

Zones -- From infinite to finite



State

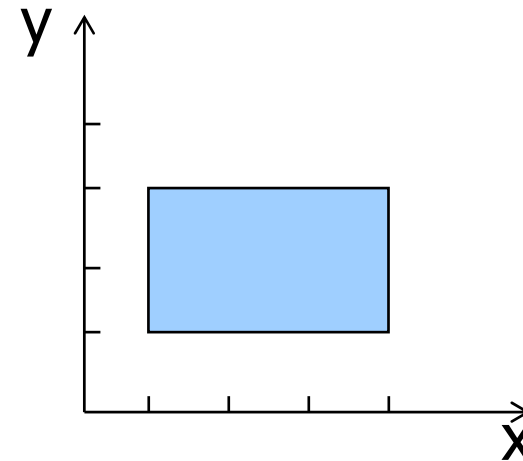
$(n, x=3.2, y=2.5)$



Symbolic state (set)

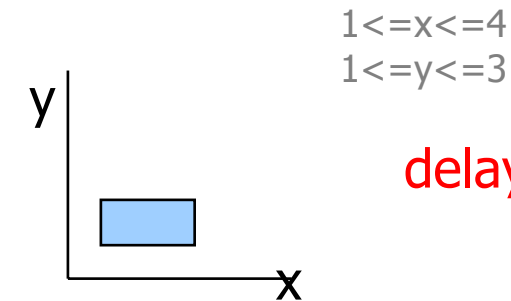
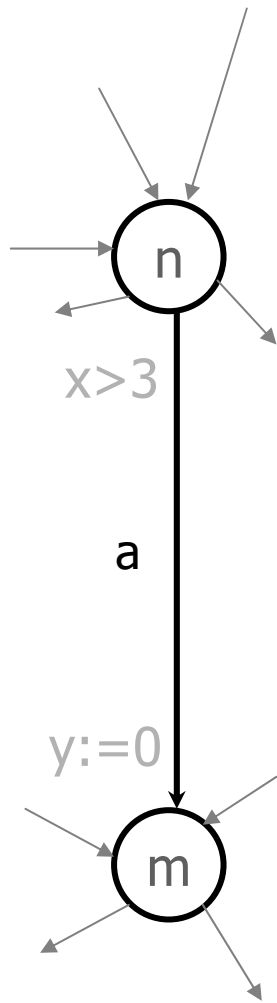
$(n, 1 \cdot x \cdot 4, 1 \cdot y \cdot 3)$

Zone:
conjunction of
 $x-y \leq n, x \leq y \leq x+n$

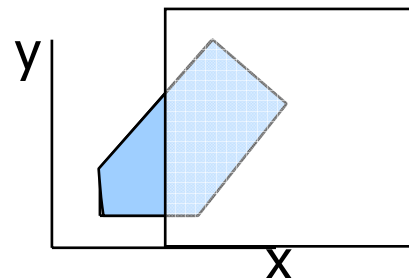
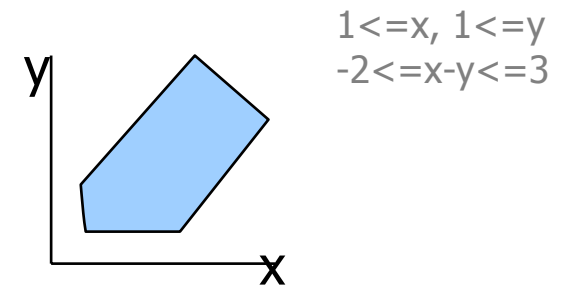


Symbolic Transitions

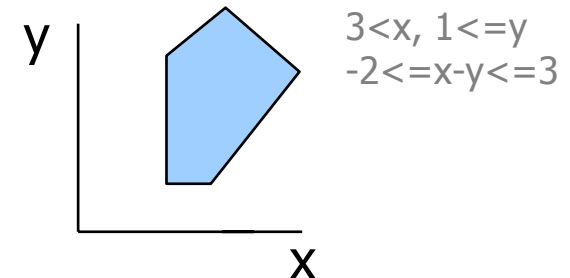
using Zones



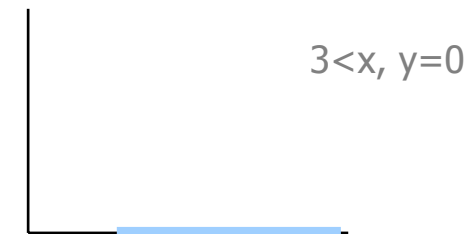
delays to



conjuncts to



projects to

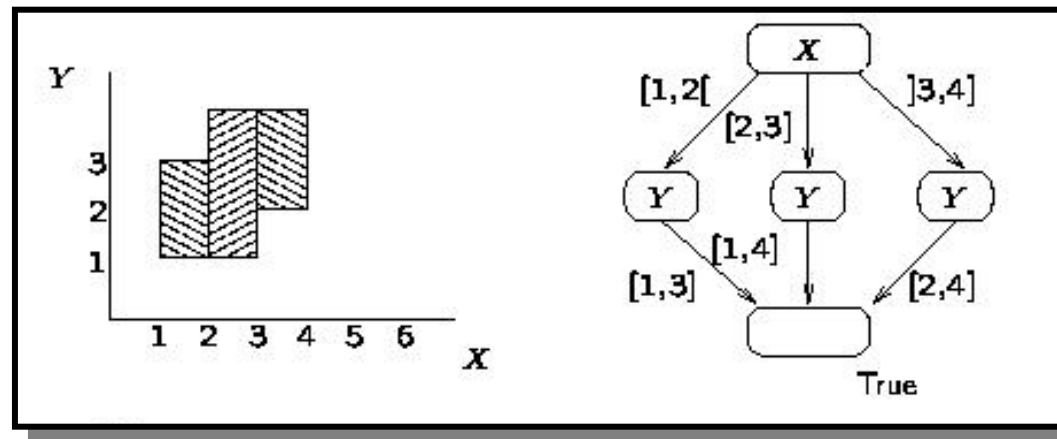
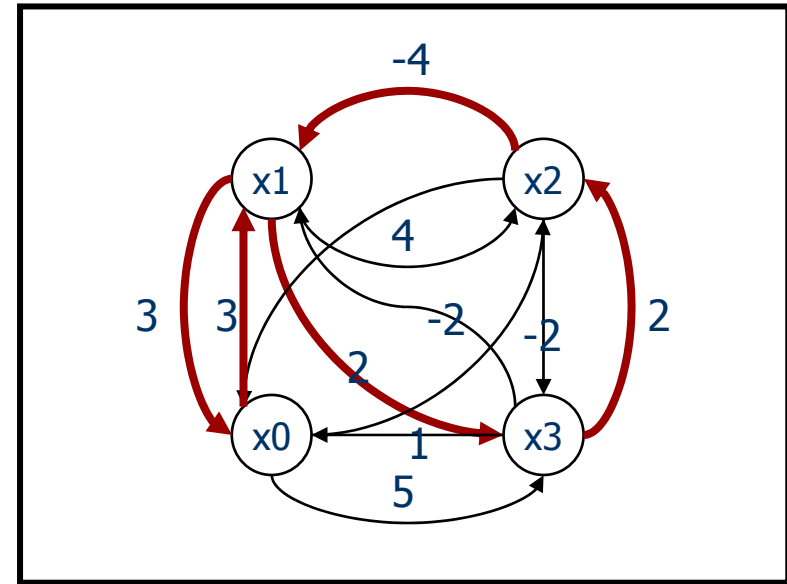


Thus $(n, 1 \leq x \leq 4, 1 \leq y \leq 3) = a \Rightarrow (m, 3 < x, y = 0)$

Datastructures for Zones



- Difference Bounded Matrices (DBMs)
- Minimal Constraint Form [RTSS97]
- Clock Difference Diagrams [CAV99]
- PW List [SPIN03]



Datastructures for Zones



■ Difference Bounded

UPPAAL DBM Library

The library used to manipulate DBMs in UPPAAL

help | Contact us



Alexandre David

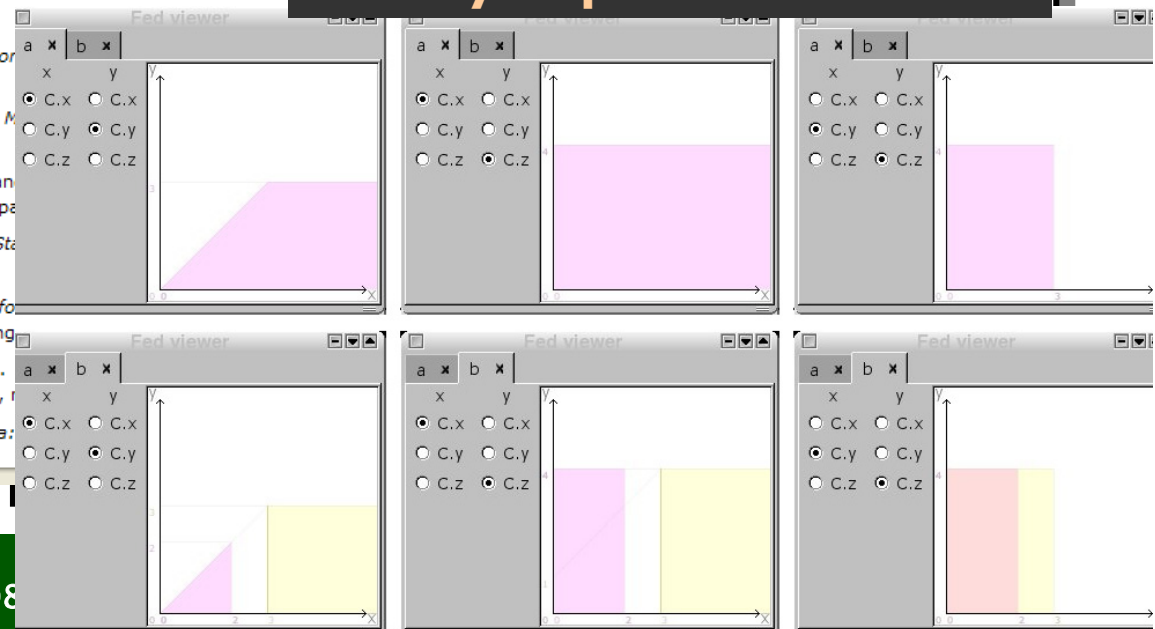
Latest News

Draft manual available.

23 Oct 2006

Elegant RUBY bindings for easy implementations

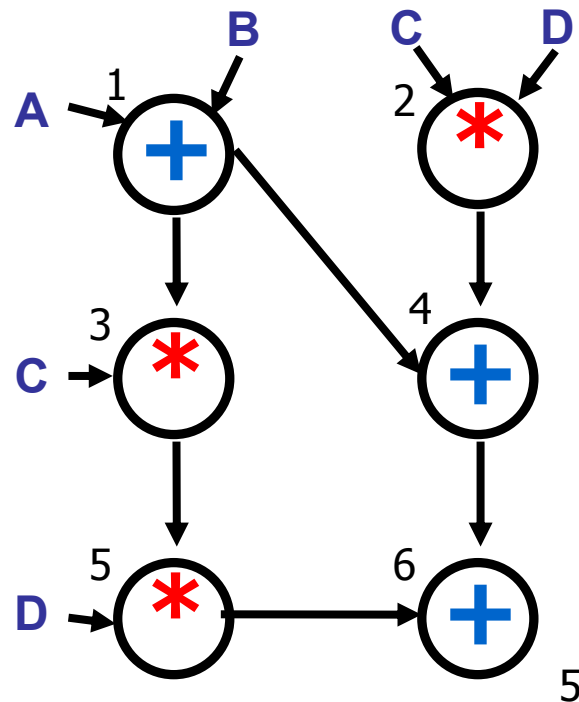
- [bengtsson02] Johan Bengtsson. *Clocks, DBM, and State Space Reduction*. PhD thesis, University of Gothenburg, 2002.
- [ad90] Rajeev Alur and David L. Dill. *Automata for Modeling and Verification of Synchronous Sequential Circuits and Data Paths*. In *Proceedings of the 11th Annual Symposium on Foundations of Computer Science*, pages 16-30. IEEE Press, 1990.
- [lpy97] Kim G. Larsen, Paul Pettersson, and Wang Yi. *UPPAAL: A User-Programmable Platform for Real-Time Systems*. In *Proceedings of the 14th Annual Symposium on Real-Time Systems*, pages 1-10. IEEE Press, 1997.
- [by04] Johan Bengtsson and Wang Yi. *Timed Automata: Theory and Practice*. LNCS 3008, Springer, 2004.



Priced Timed Automata

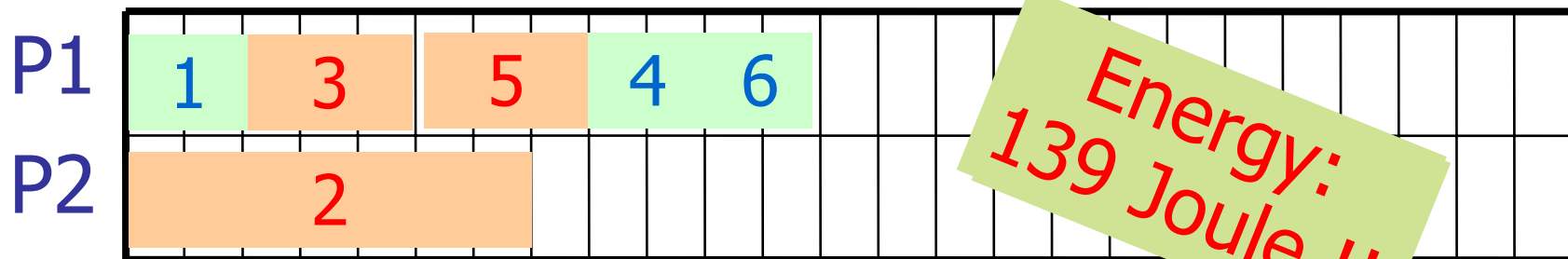
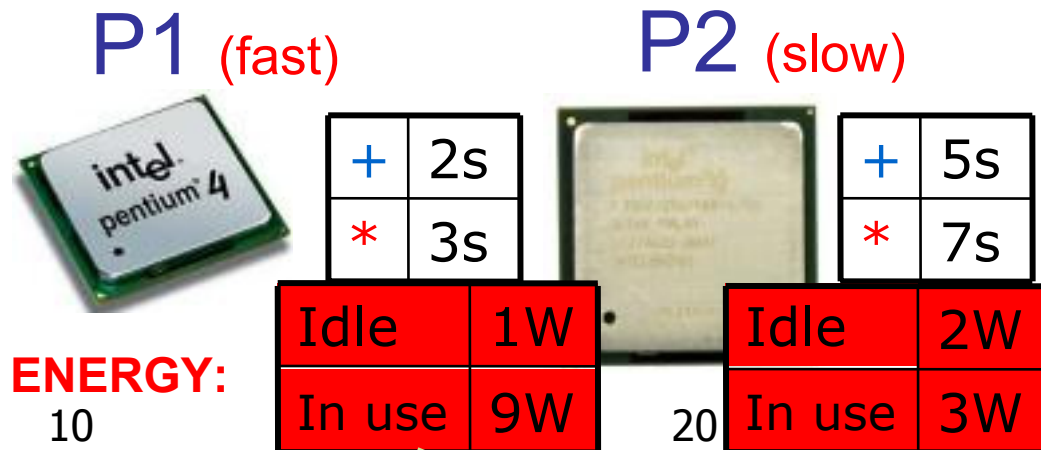


Task Graph Scheduling – Revisited



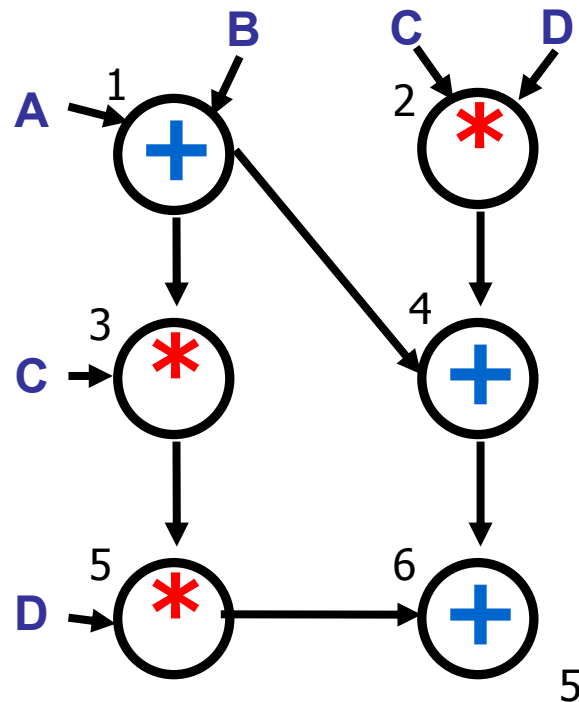
Compute :
 $(D * (C * (A + B)) + ((A + B) + (C * D)))$

using 2 processors



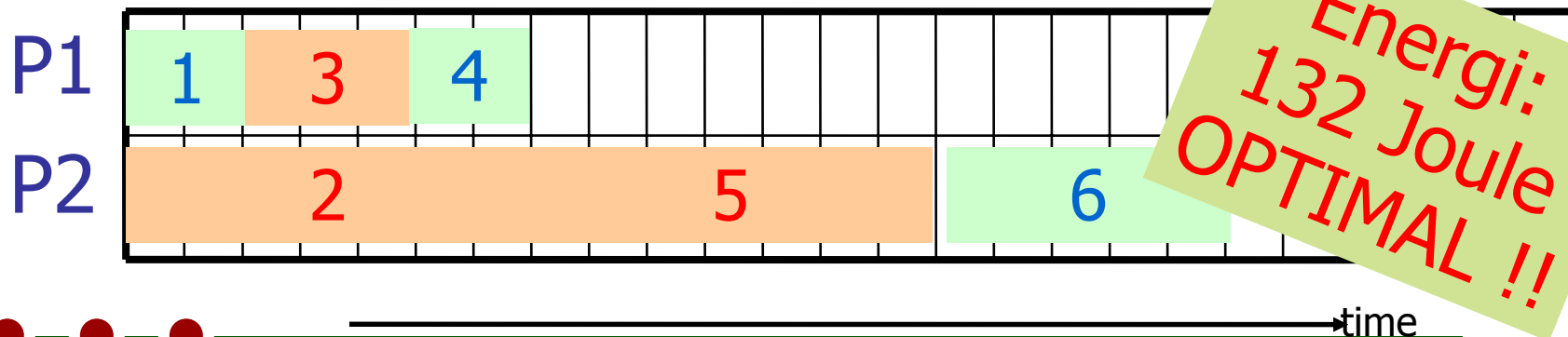
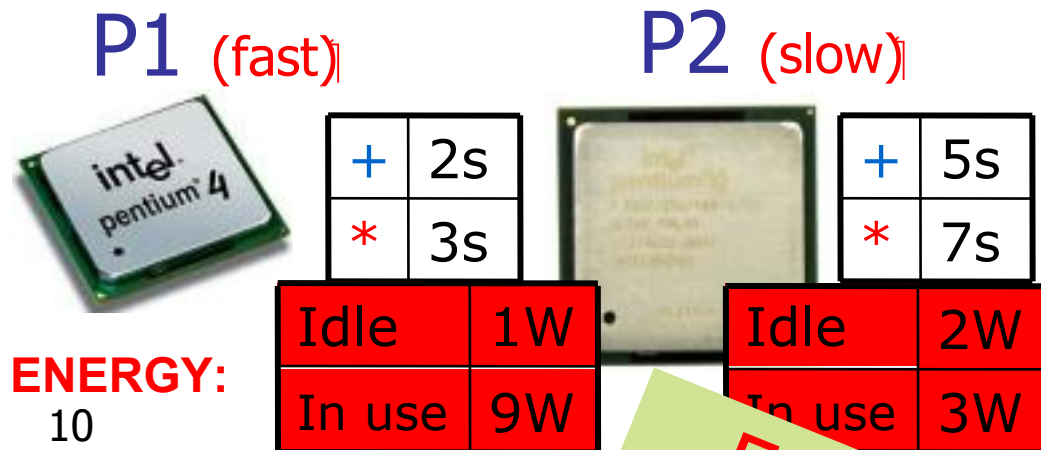
**Energy:
139 Joule !!**

Task Graph Scheduling – Revisited



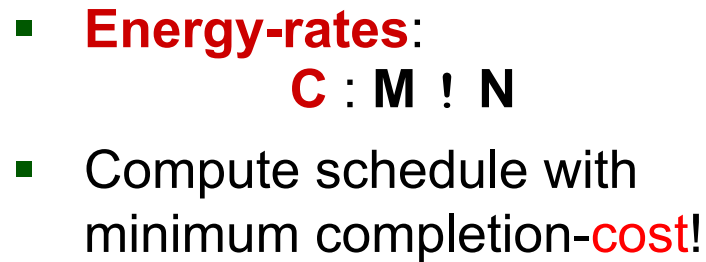
Compute :
 $(D * (C * (A + B)) + ((A + B) + (C * D)))$

using 2 processors



**Energi:
132 Joule
OPTIMAL !!**

Power-Optimality



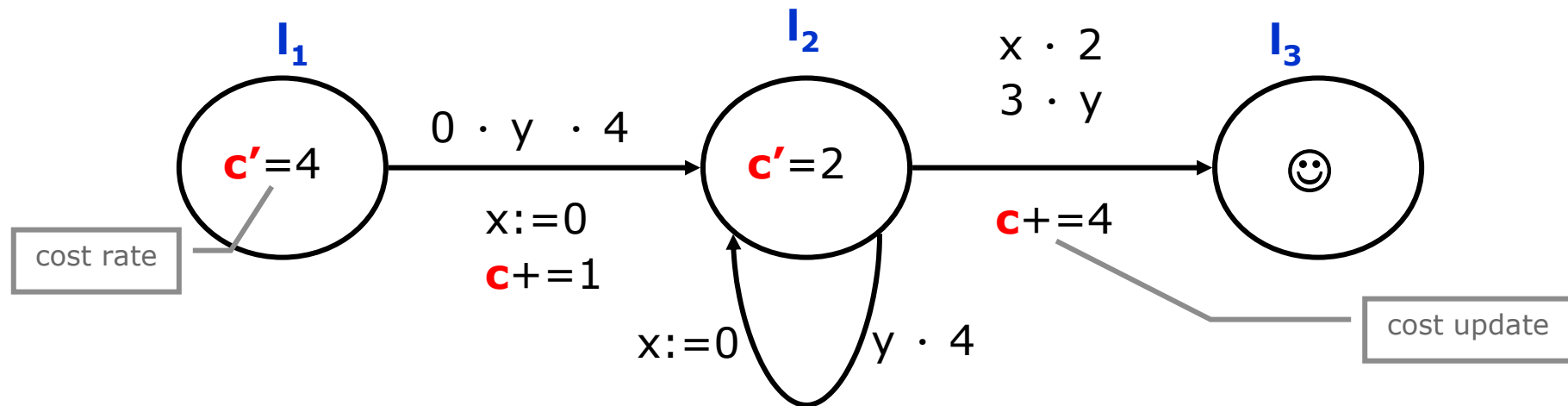
Priced Timed Automata



Behrmann, Fehnker, et al (HSCC'01)

Alur, Torre, Pappas (HSCC'01)

Timed Automata + **COST** variable



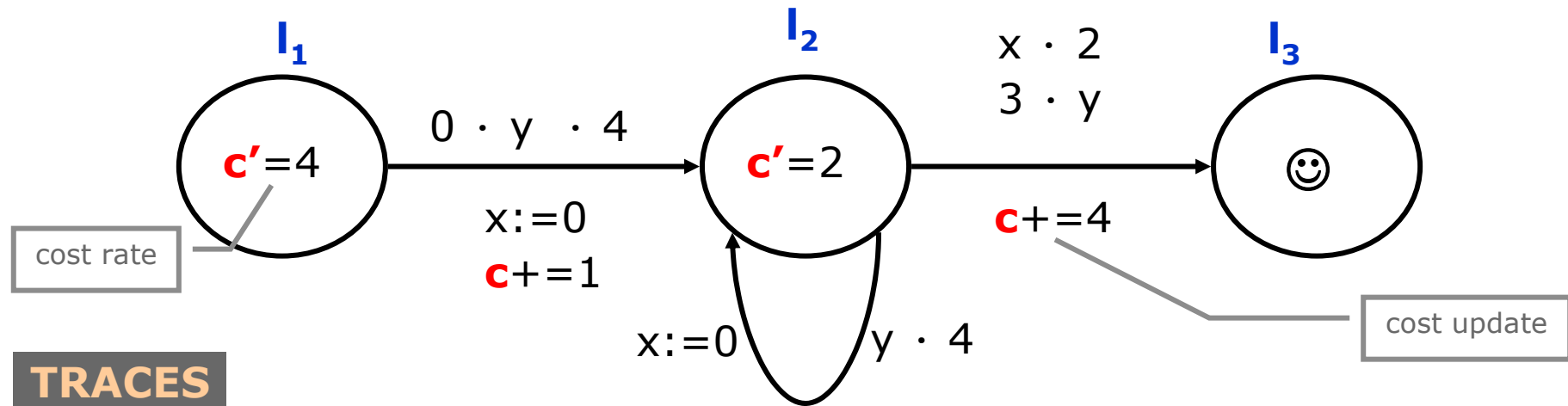
Priced Timed Automata



Behrmann, Fehnker, et al (HSCC'01)

Alur, Torre, Pappas (HSCC'01)

Timed Automata + **COST** variable



TRACES

$(l_1, x=y=0) \xrightarrow[\text{12}]{\varepsilon(3)} (l_1, x=y=3) \xrightarrow[\text{1}]{} (l_2, x=0, y=3) \xrightarrow[\text{4}]{} (l_3, -, -)$

$\Sigma c = 17$

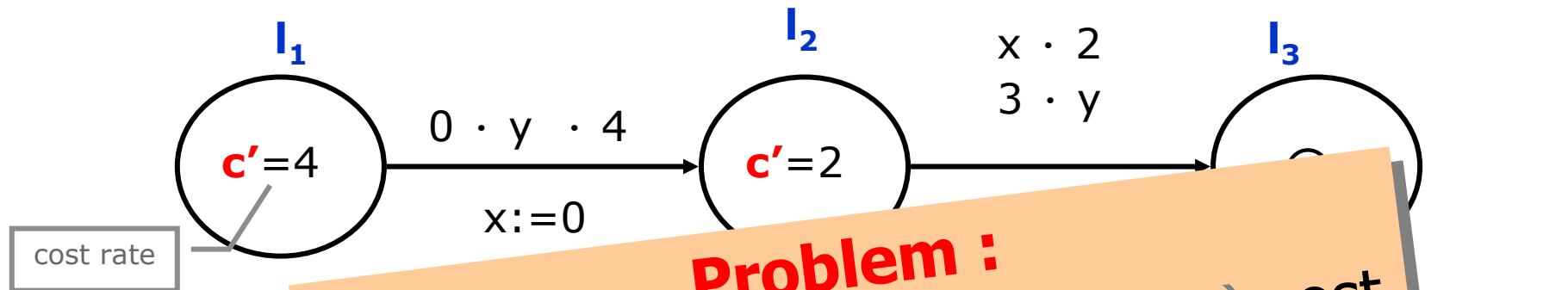
Priced Timed Automata



Behrmann, Fehnker, et al (HSCC'01)

Alur, Torre, Pappas (HSCC'01)

Timed Automata + **COST** variable



TRACES

$(l_1, x=y=0)$

$(l_1, x=y=0)^{\epsilon}$

$(l_1, x=y=0) \xrightarrow{1} (l_2, x=0, y=0)$

Problem :
Find the **minimum** (maximum) cost
of reaching location l_3

Efficient Implementation:
CAV'01 and TACAS'04

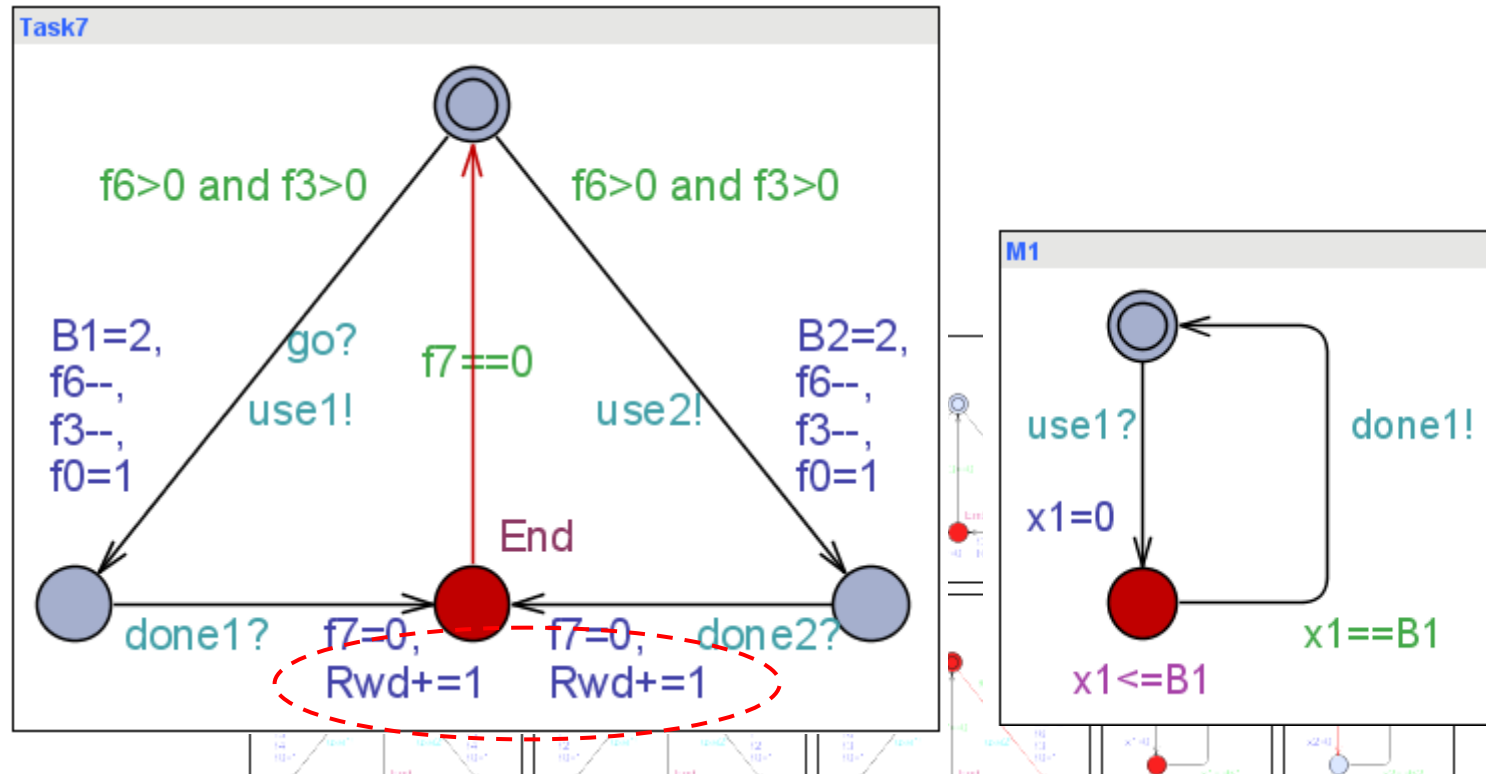
**Competitive with MILP
and commercial tool (Axxon)**

$\Sigma c=17$

$(l_3, _, _)$
 $\Sigma c=16$

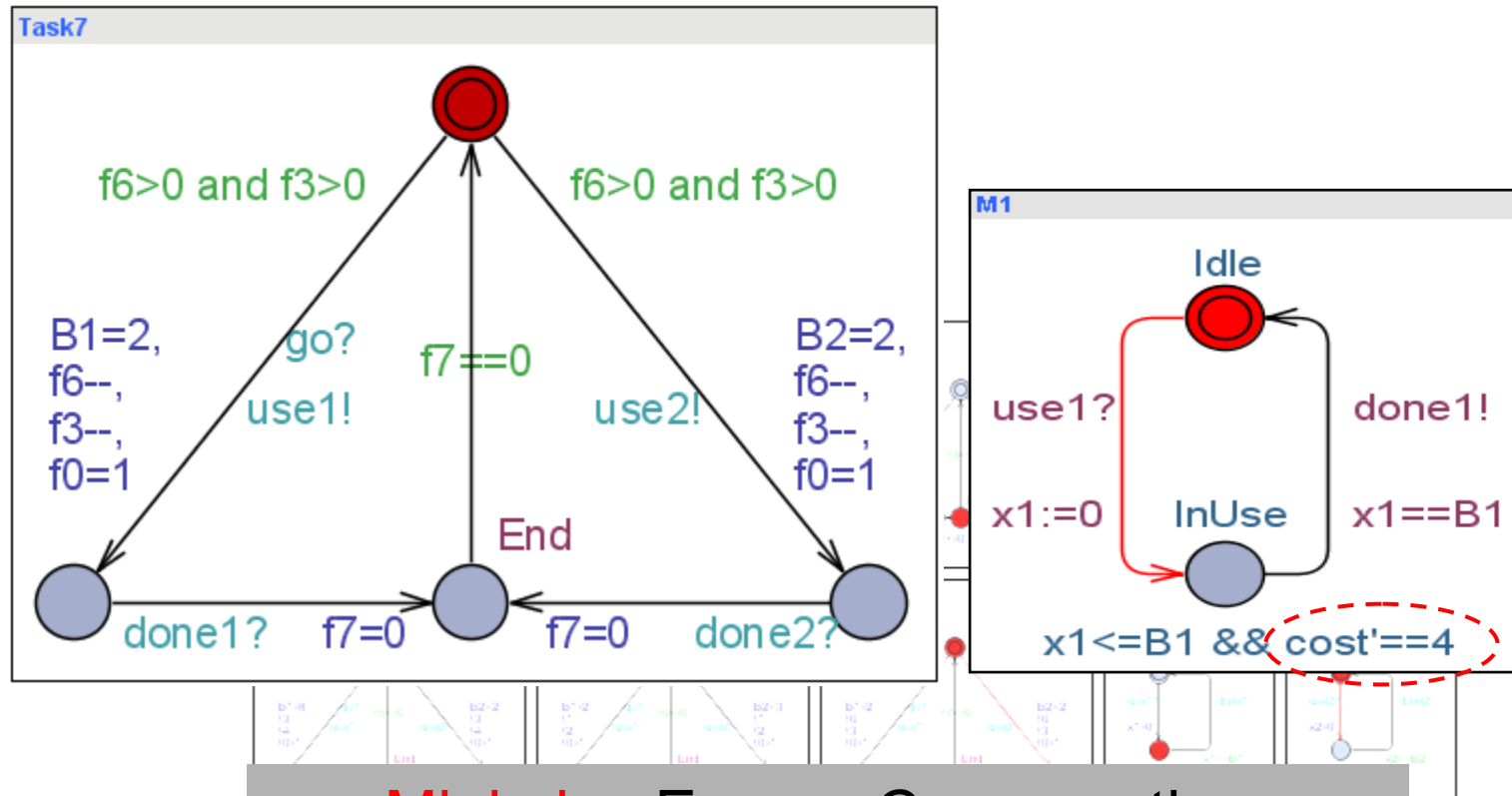
$(l_3, _, _)$
 $\Sigma c=11$

Optimal Infinite Scheduling



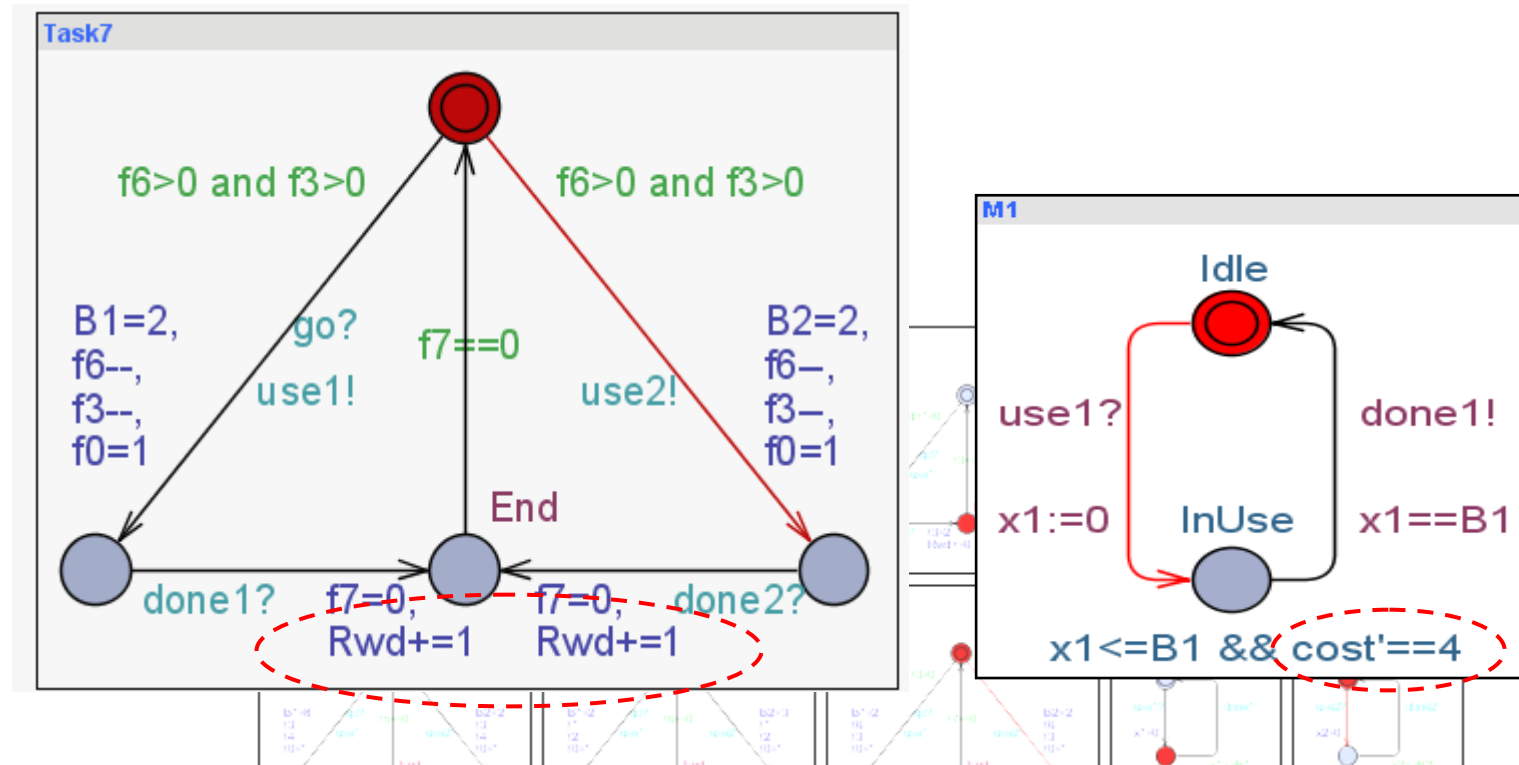
Maximize throughput:
i.e. maximize **Reward** / Time in the long run!

Optimal Infinite Scheduling



Minimize Energy Consumption:
i.e. minimize **Cost** / Time in the long run

Optimal Infinite Scheduling



Maximize throughput:
i.e. maximize **Reward** / **Cost** in the long run

HSCC04,FMSD07



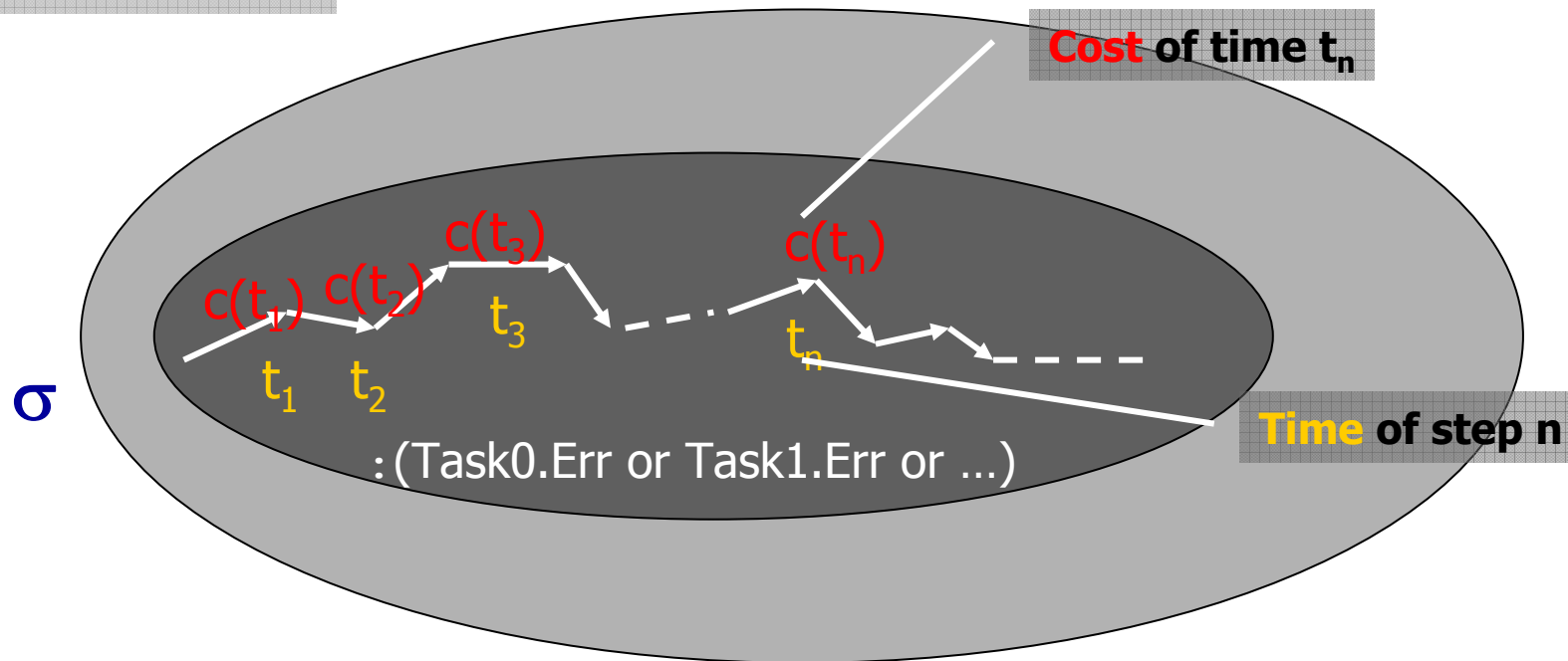
Optimal Schedule σ^* : $\text{val}(\sigma^*) = \inf_{\sigma} \text{val}(\sigma)$

Discount Optimality

$\lambda < 1$: discounting factor



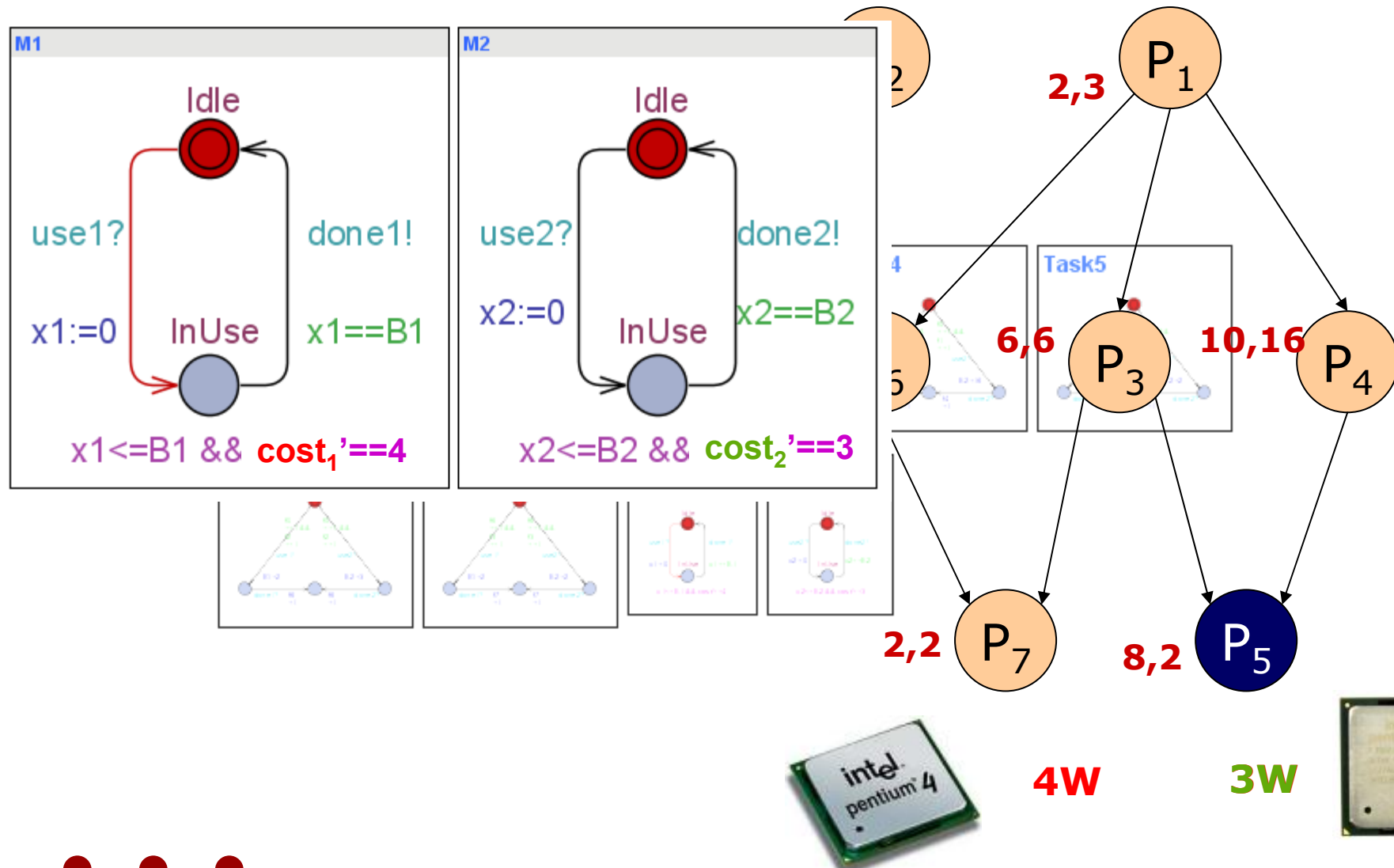
INFINITY'08



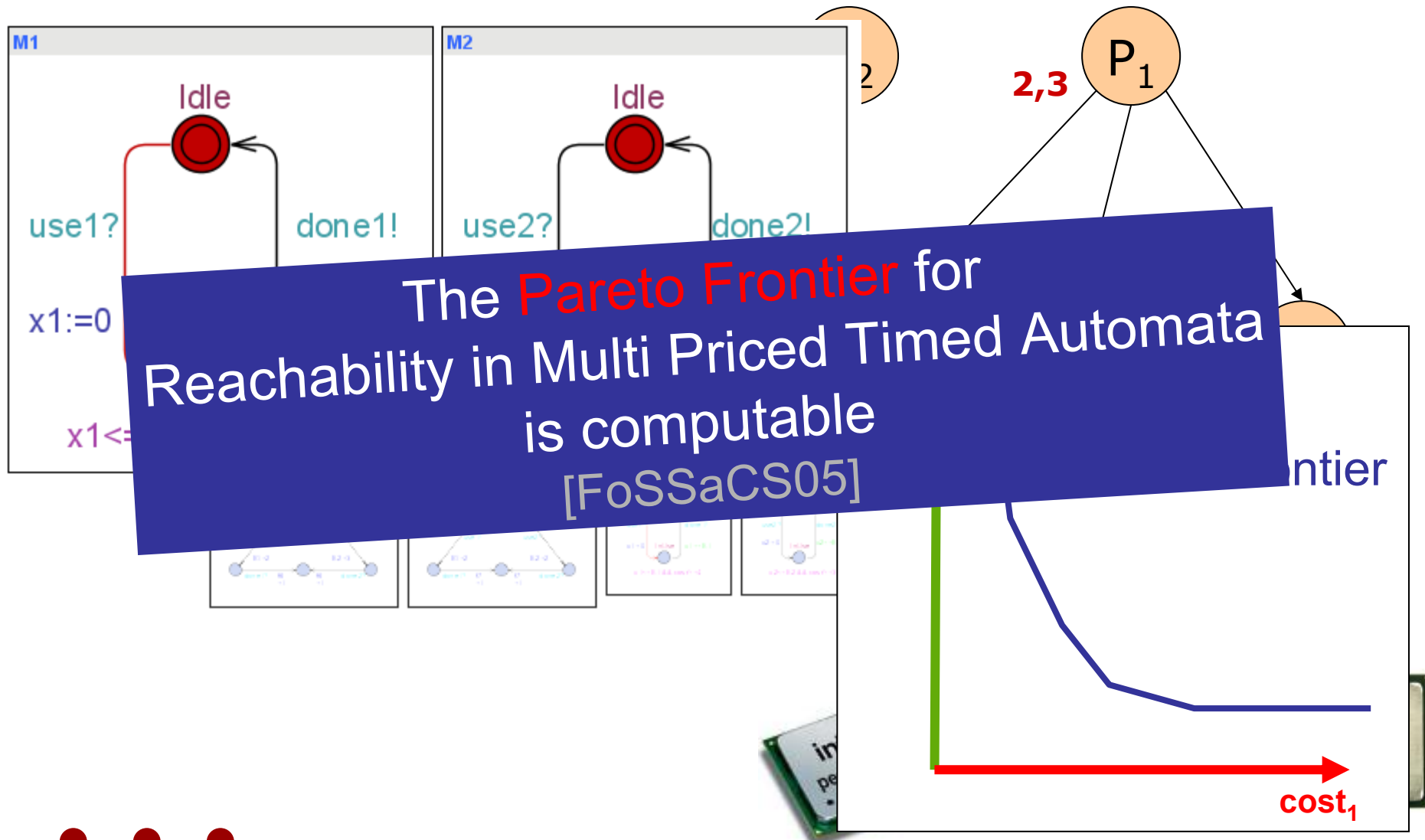
Value of path σ : $\text{val}(\sigma) = \int_{t=0}^{t=\infty} c(t) \lambda^t dt$

Optimal Schedule σ^* : $\text{val}(\sigma^*) = \inf_{\sigma} \text{val}(\sigma)$

Multiple Objective Scheduling



Multiple Objective Scheduling



Schedulability Analysis



Task Scheduling

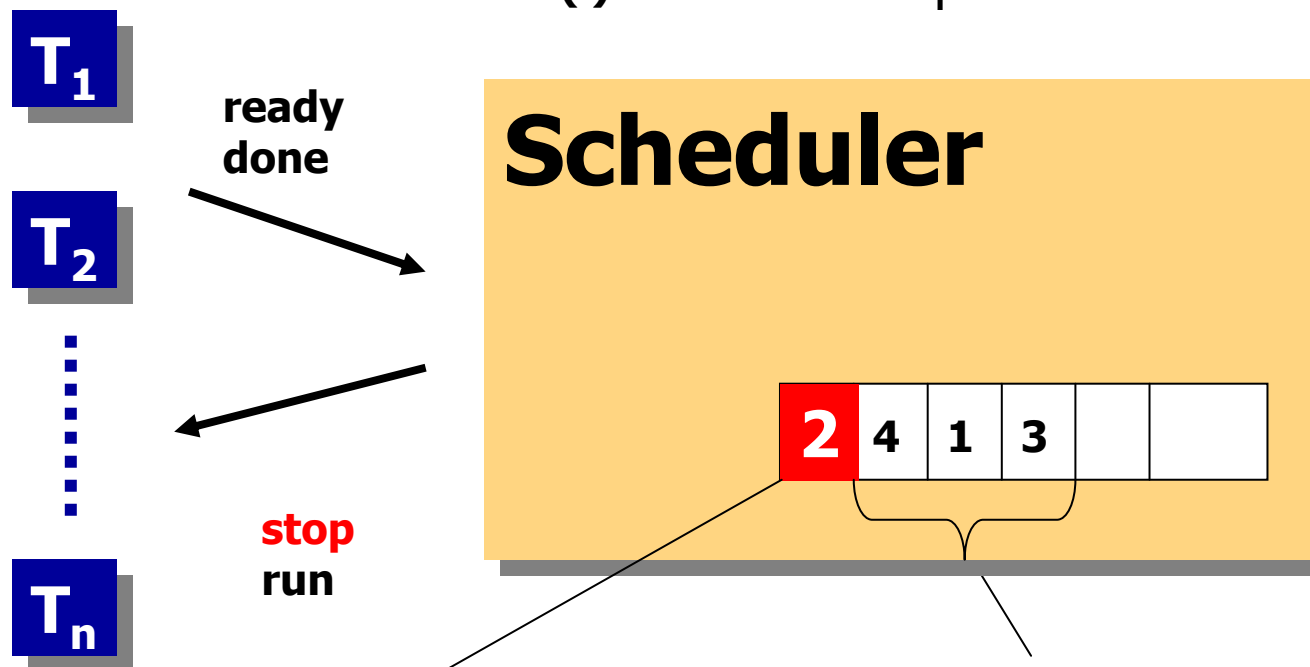
utilization of CPU



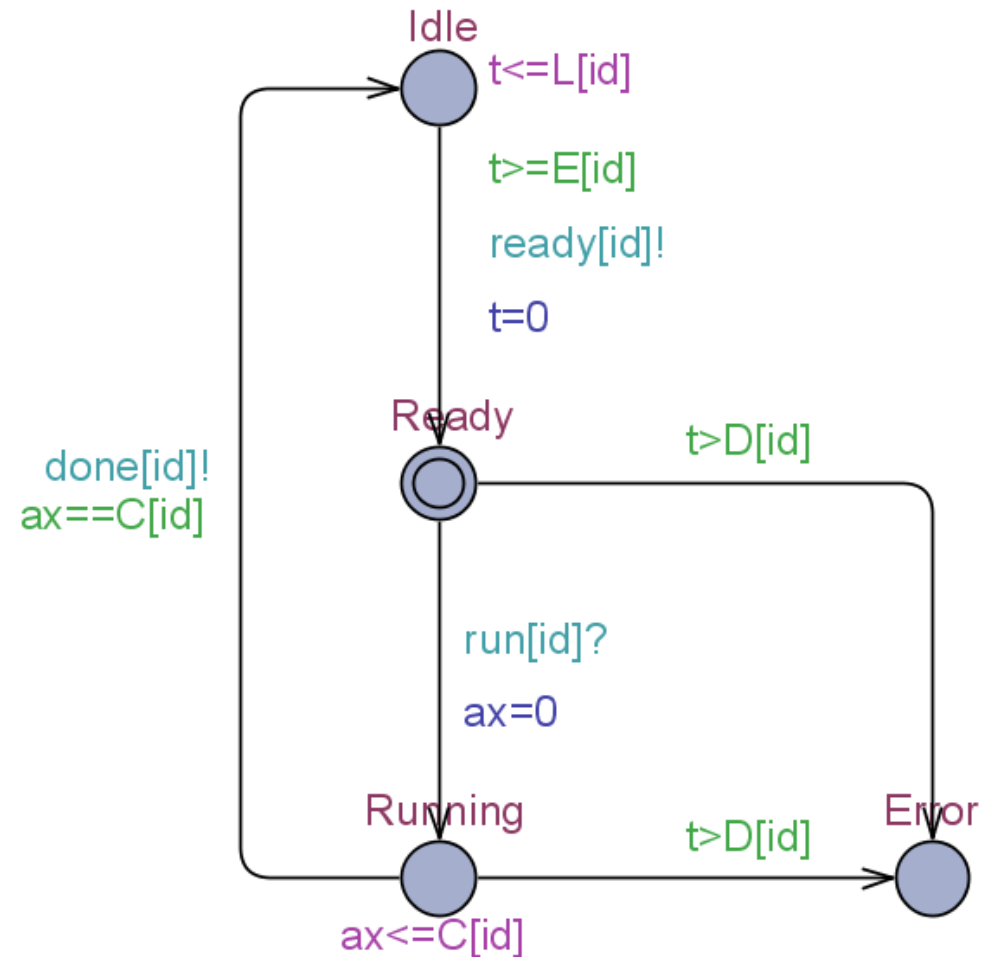
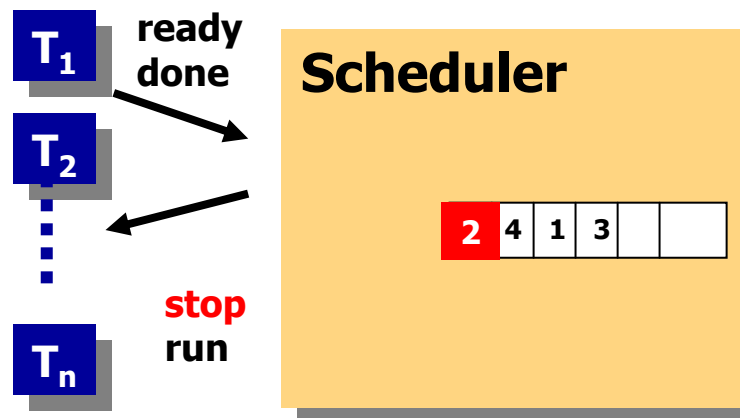
$P(i)$, $[E(i), L(i)]$, .. : period or
earliest/latest arrival or .. for T_i

$C(i)$: execution time for T_i

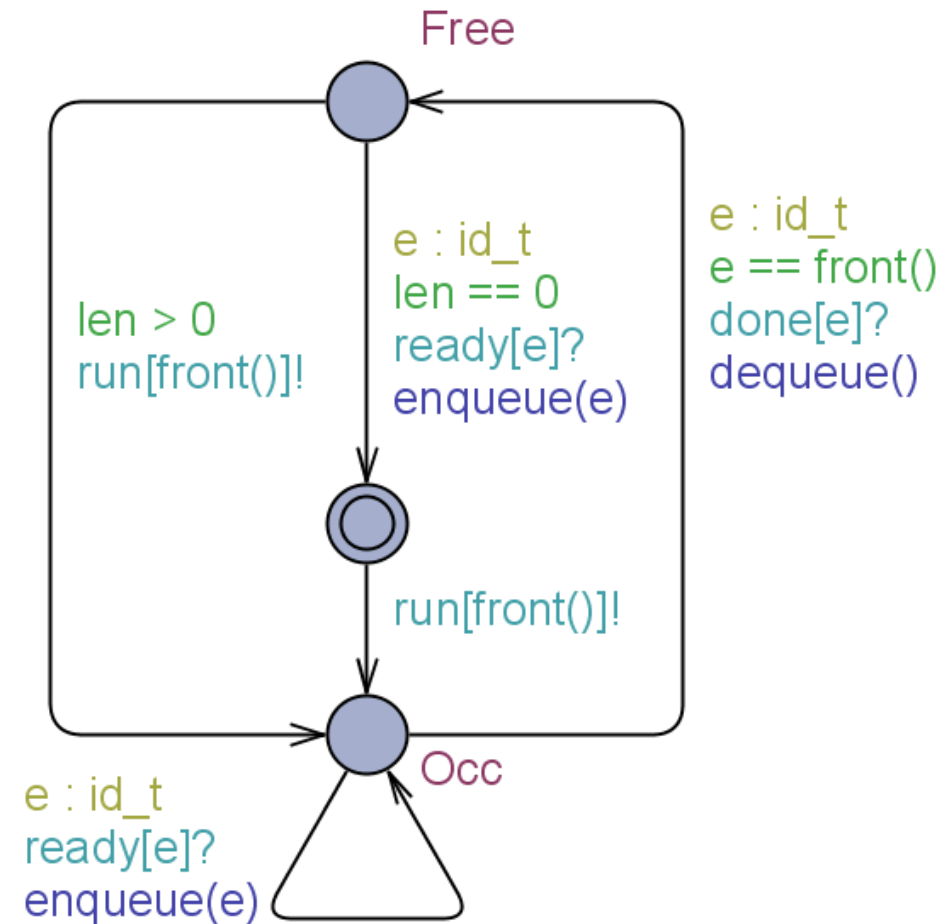
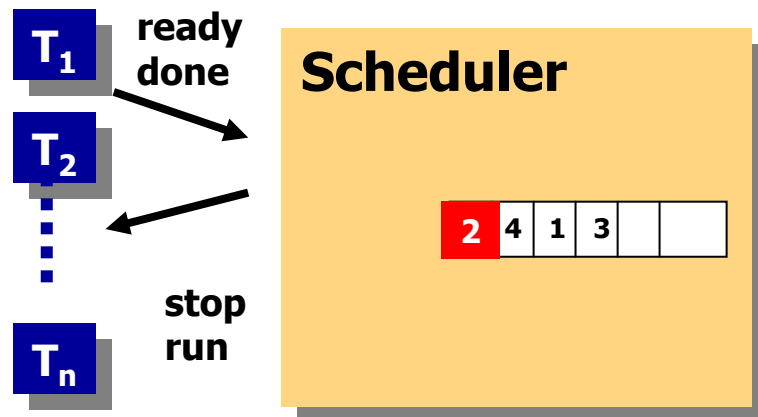
$D(i)$: deadline for T_i



Modeling Task

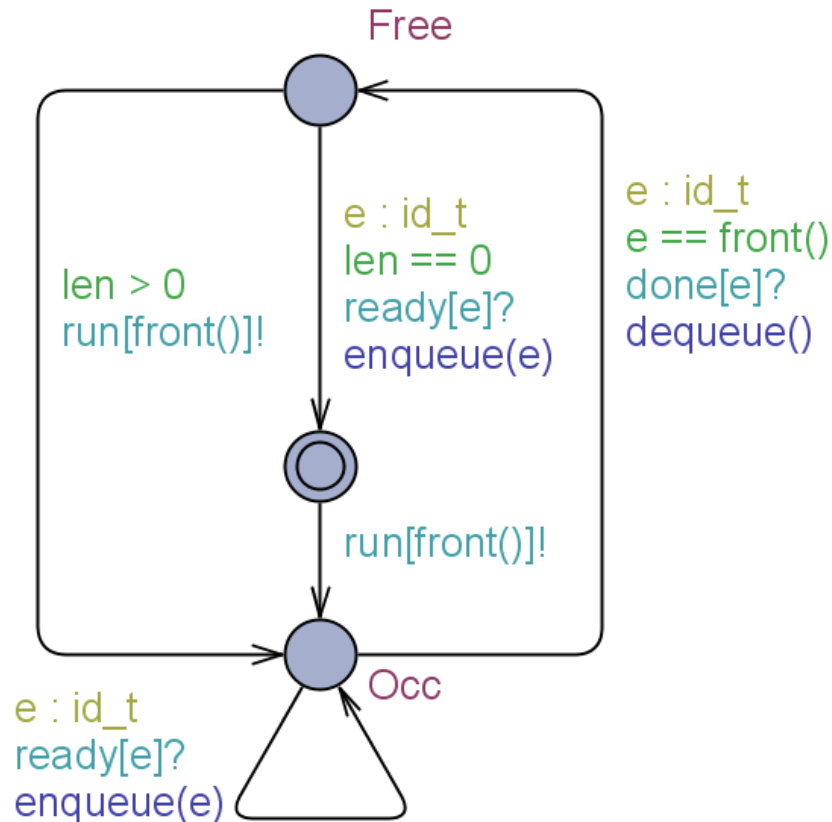


Modeling Scheduler



Modeling Queue

In UPPAAL 4.0
User Defined Function



```

// Put an element at the end of the queue
void enqueue(id_t element)
{
    int tmp=0;
    list[len++] = element;
    if (len>0)
    {
        int i=len-1;
        while (i>1 && P[list[i]]>P[list[i-1]])
        {
            tmp = list[i-1];
            list[i-1] = list[i];
            list[i] = tmp;
            i--;
        }
    }
}

```

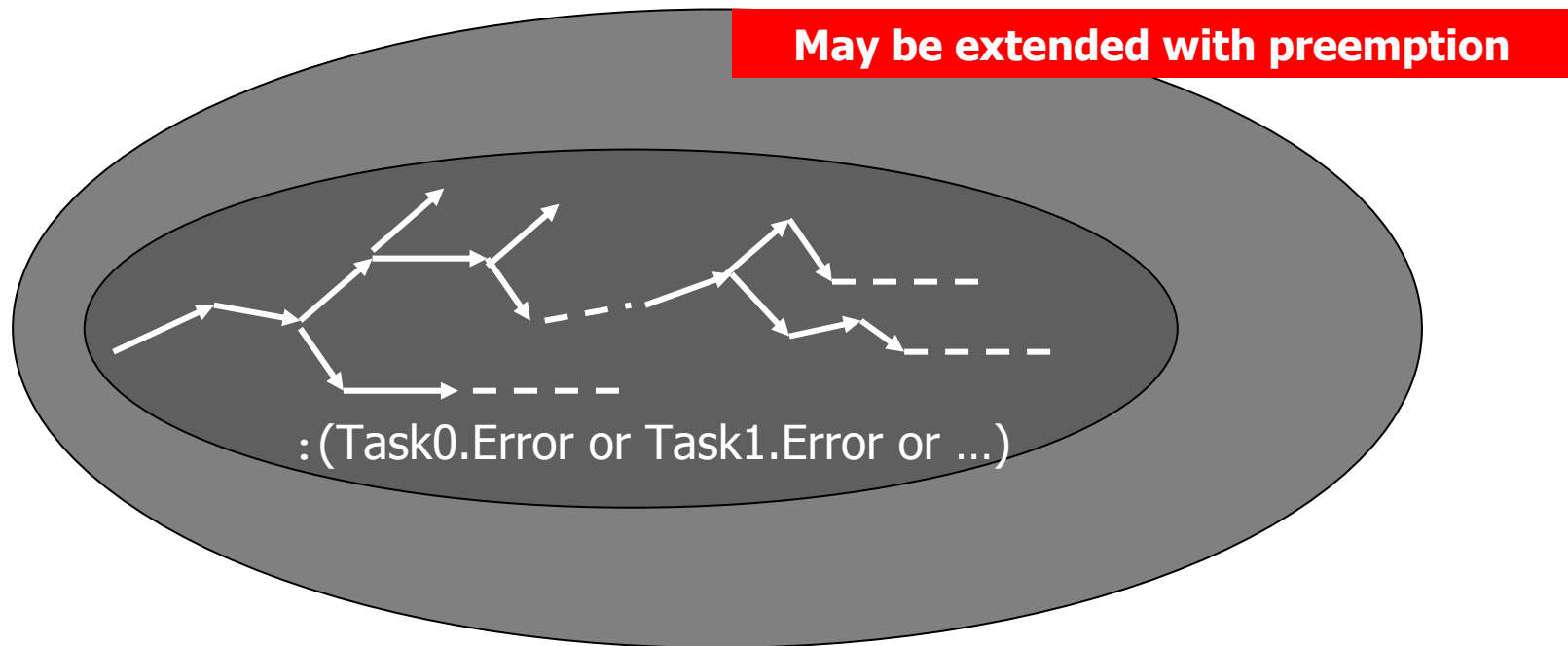
```

// Remove the front element of the queue
void dequeue()
{
    .....
}

```



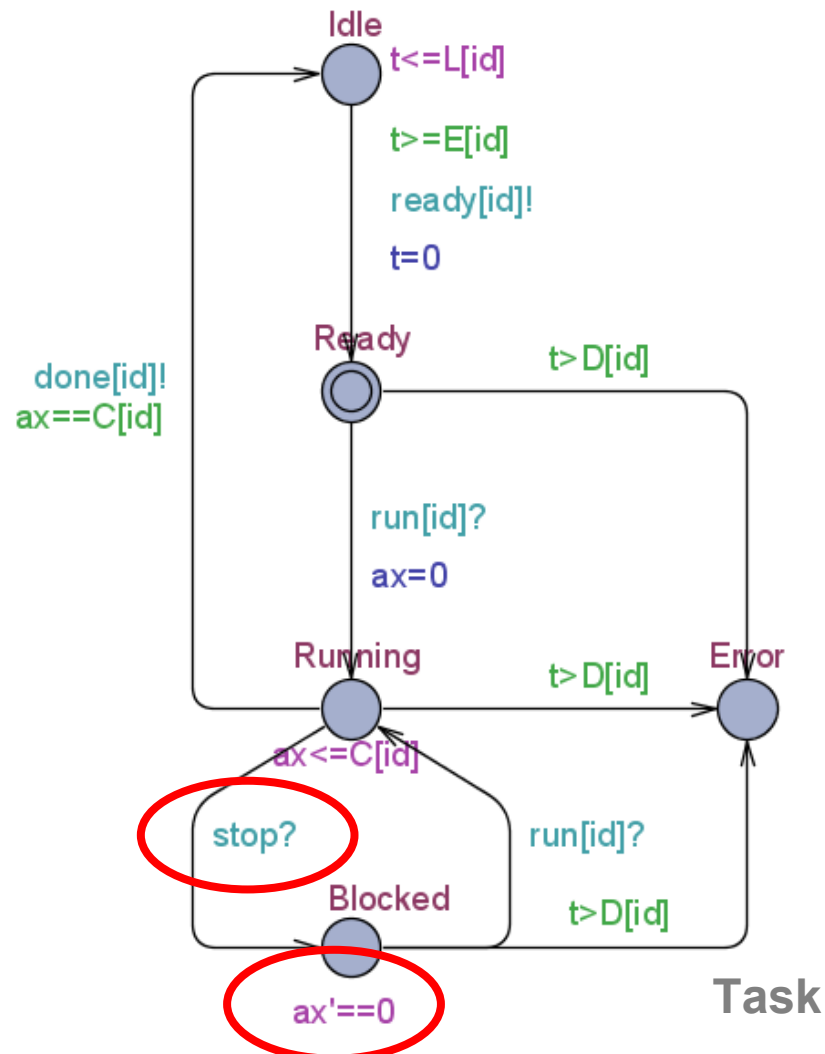
Schedulability = Safety Property



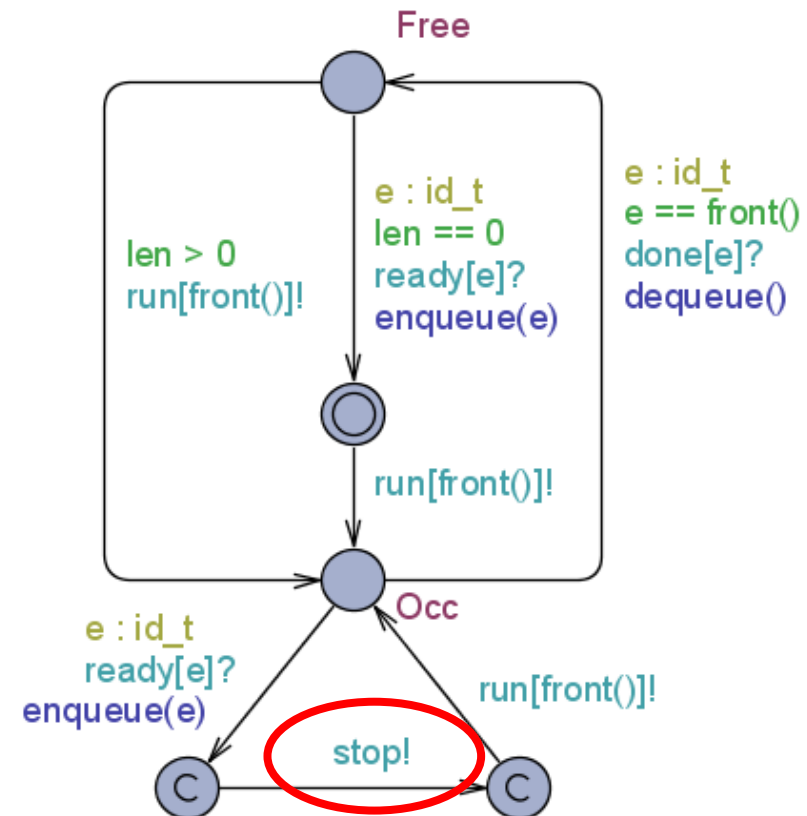
$A \square : (\text{Task0.Error or Task1.Error or ...})$



Preemption – Stopwatches!



Scheduler



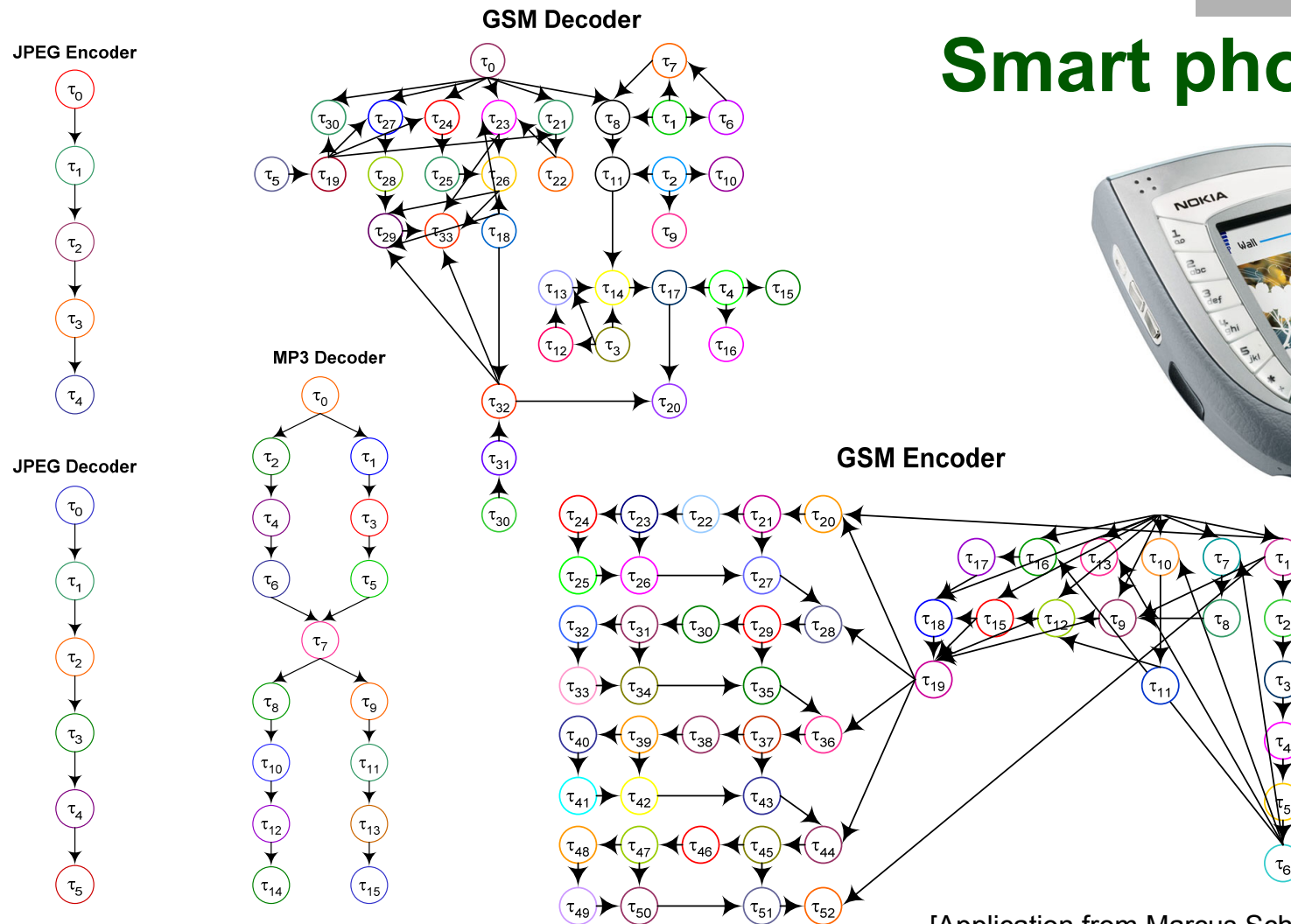
Defeating undecidability ☺

Handling realistic applications?



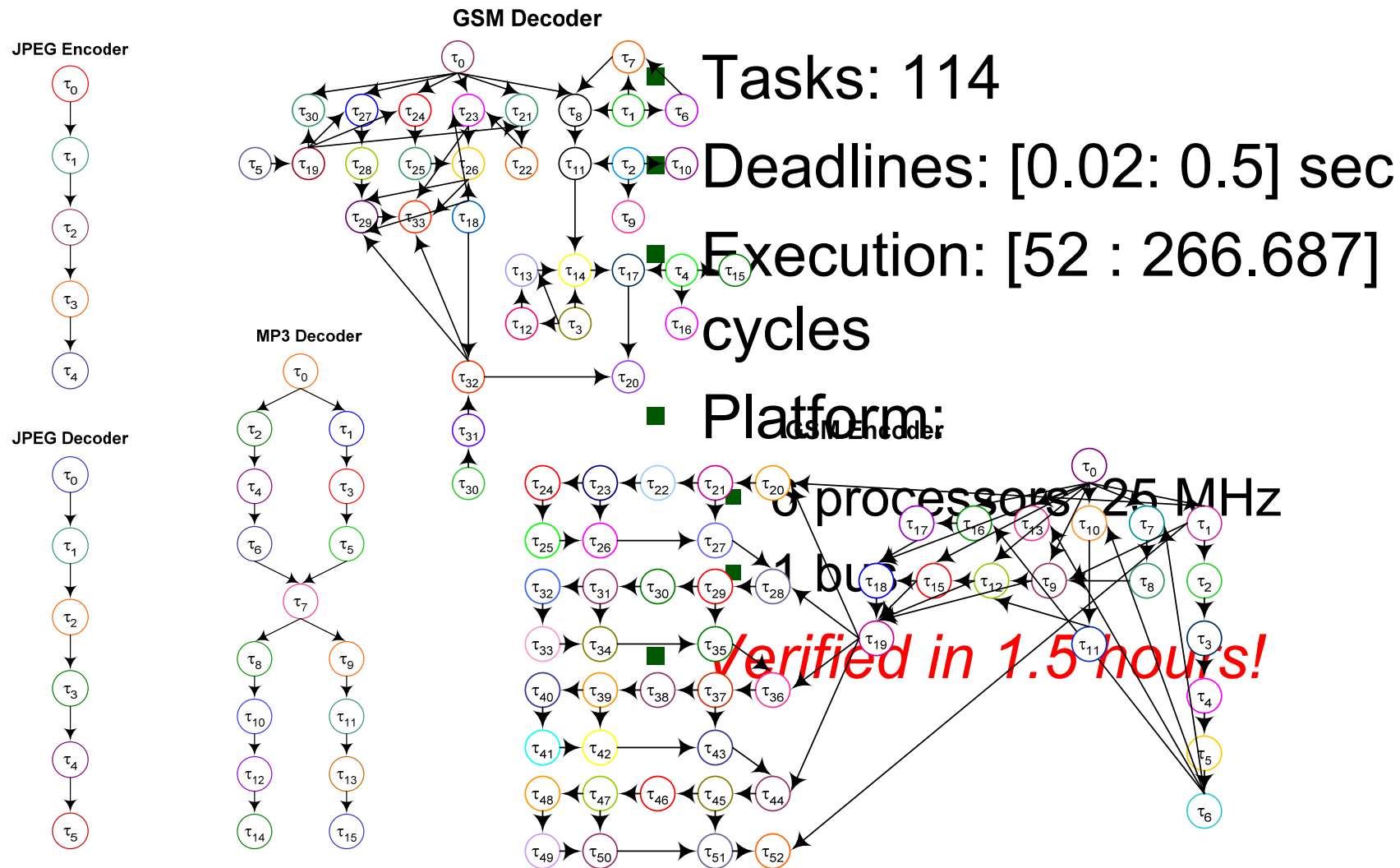
Jan Madsen / DTU

Smart phone:



[Application from Marcus Schmitz, TU Linköping]

Smart phone

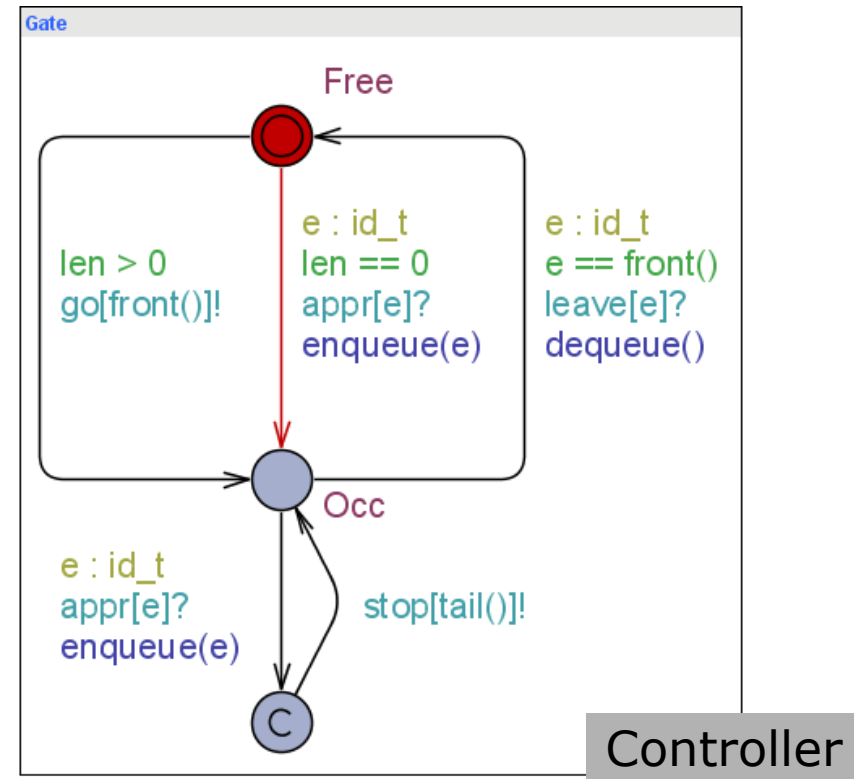
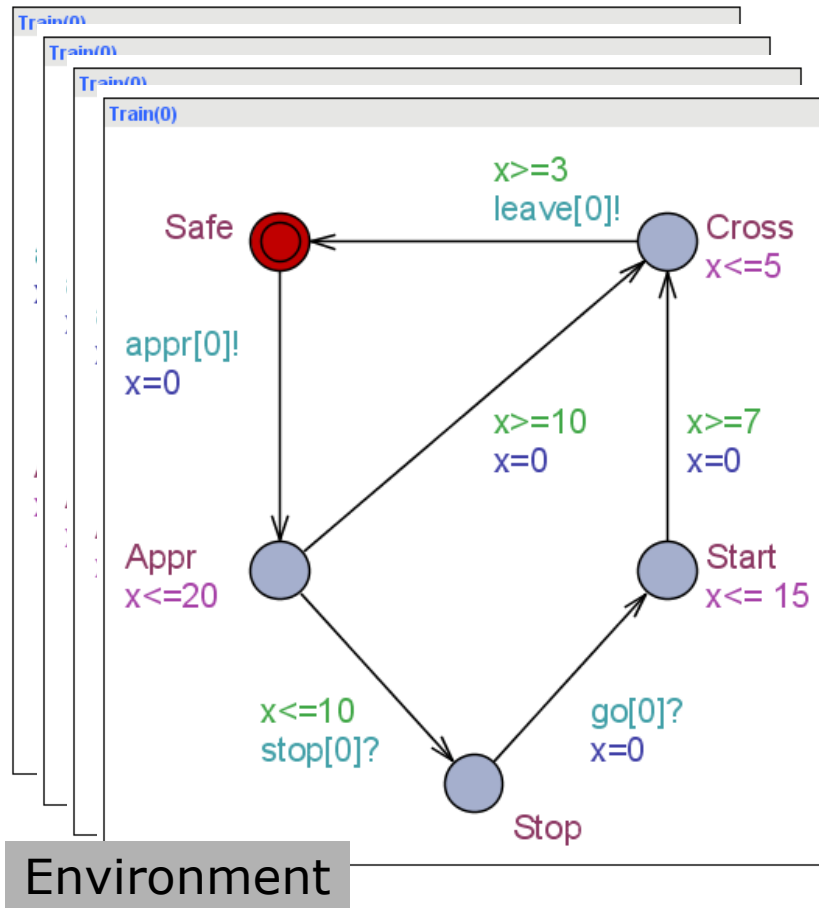


Synthesis

Timed Game Automata



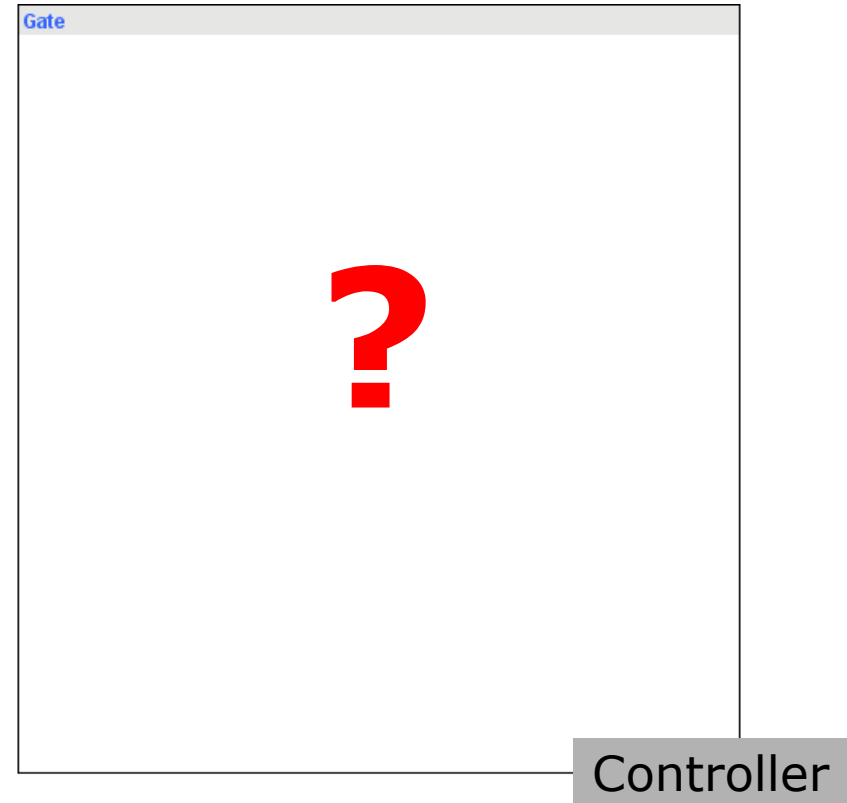
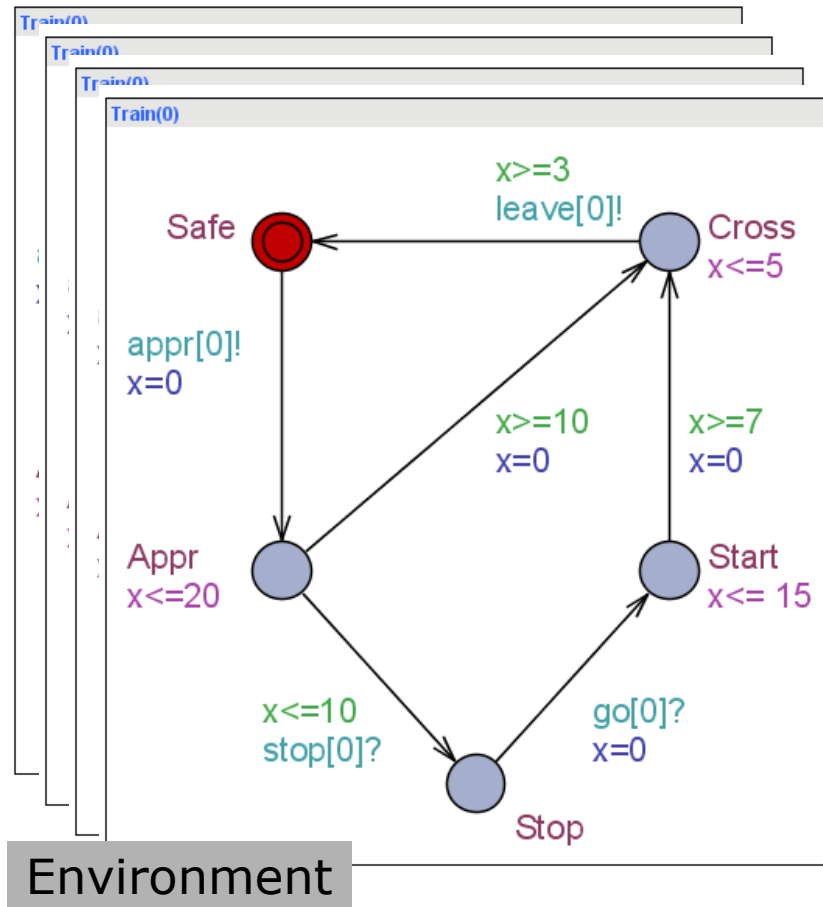
Model Checking (ex Train Gate)



ϕ : Never two trains at the crossing at the same time



Synthesis (ex Train Gate)



ϕ : Never two trains at the crossing at the same time



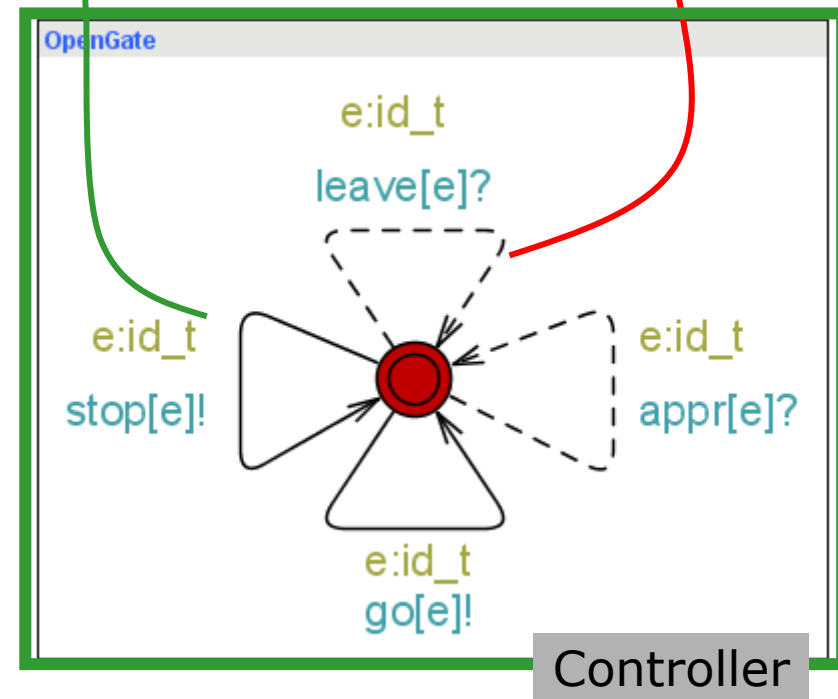
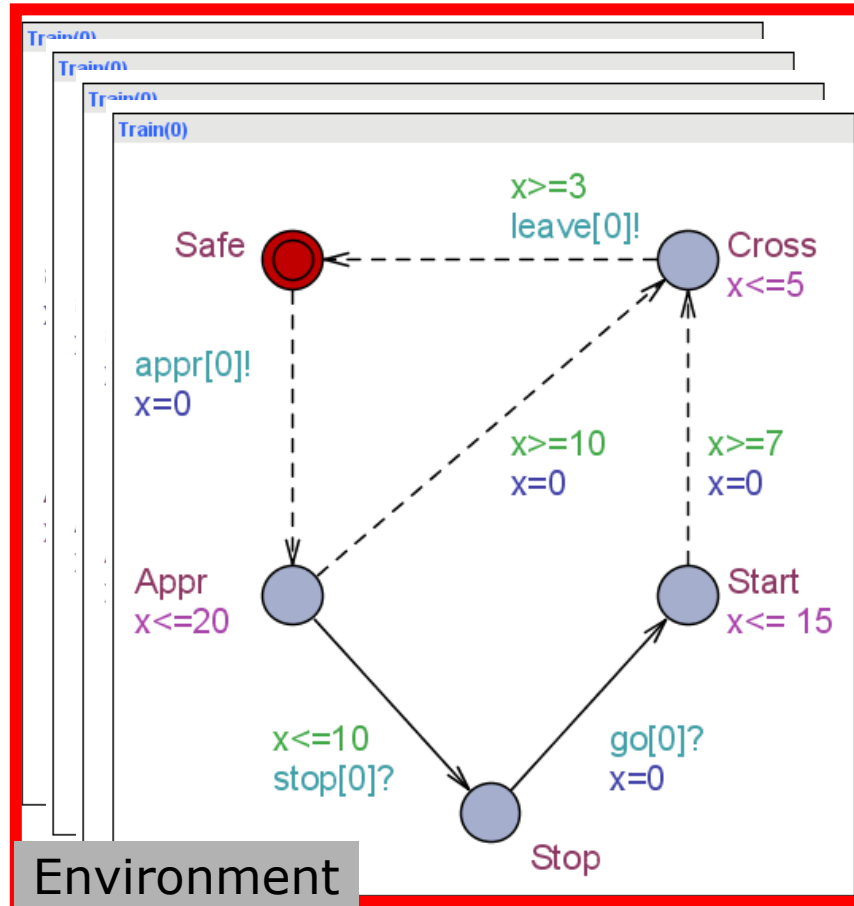
Synthesis

Two Player Game



Controllable

Uncontrollable



Find strategy for controllable actions st behaviour satisfies ϕ

ϕ : Never two trains at the crossing at the same time



Memoryless strategy:

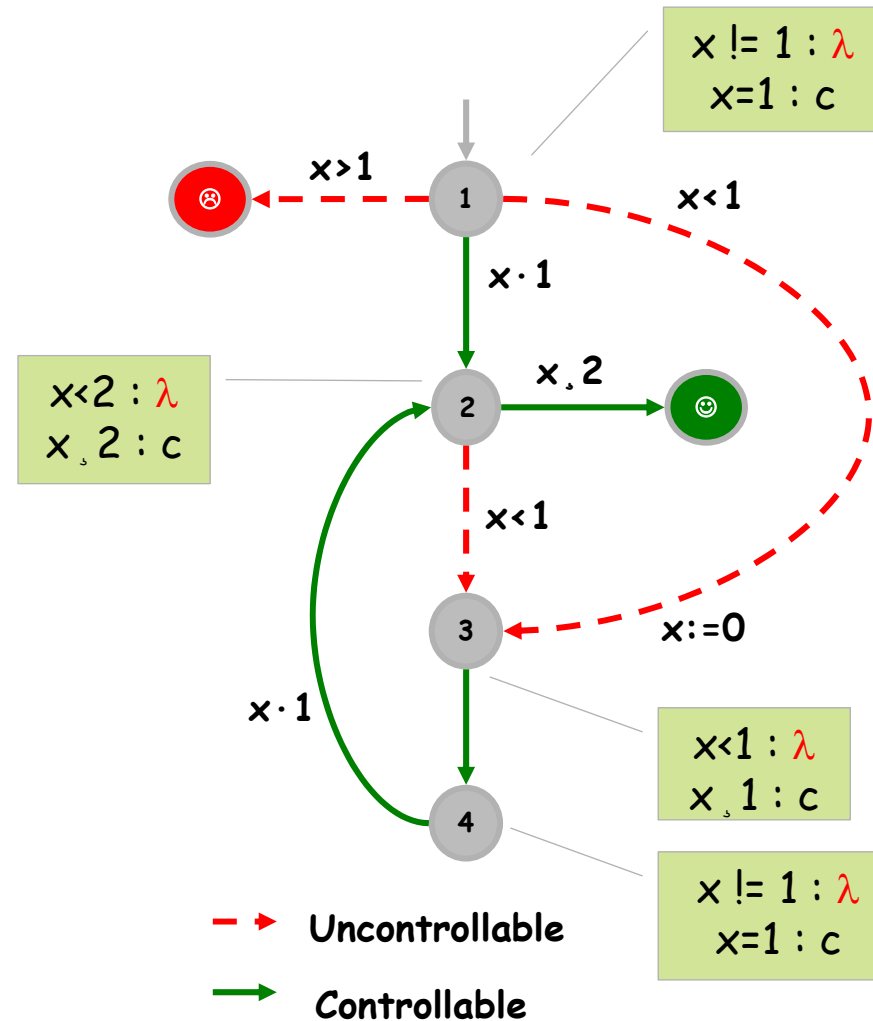
$$F : Q \rightarrow E_c \mid \lambda$$

Winning Run:

$$\text{States}(\rho) \stackrel{\circ}{\Delta} G \neq \emptyset$$

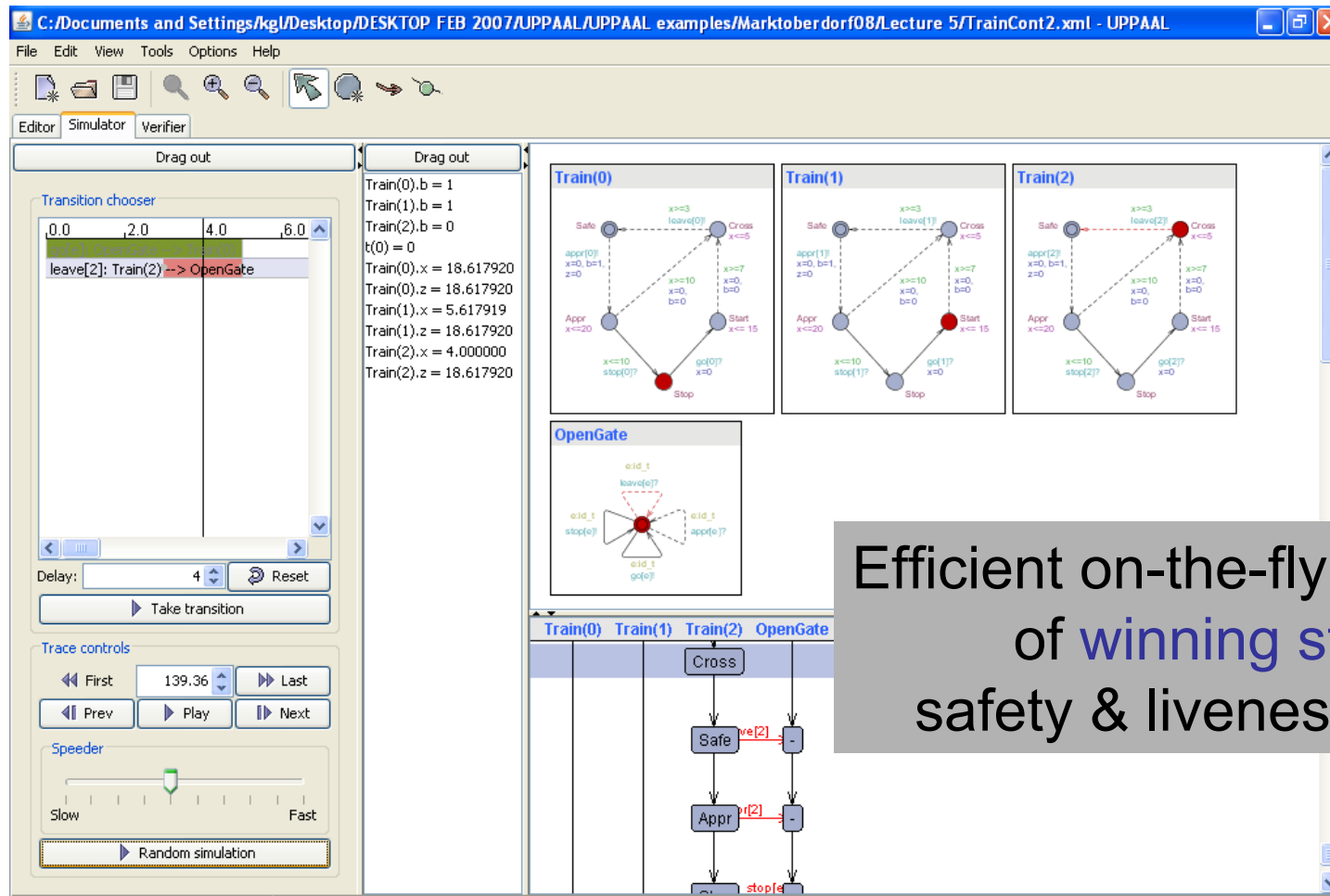
Winning Strategy:

$$\text{Runs}(F) \mid \mu \text{ WinRuns}$$



UPPAAL Tiga

Synthesis of winning strategies for *TIMED GAMES*



CONCUR05,
CAV07,
FORMATS07

Efficient on-the-fly generation
of winning strategies for
safety & liveness objectives

Open Problems



Decidability

- Priced Timed Games
 - Reachability & Model Checking
 - Safety
 - Negative cost rates
- Probabilistic Priced Timed Automata

Efficiency

- Timed Automata
 - Fully Symbolic (CDD)
 - Static Analysis & Slicing of C-code
- Timed Games
 - Alternating Logics
 - Partial Orderability
 - CEGAR



- Priced Timed Automata
 - Optimal Infinite Schedules for PTA (zone based)
 - Multi-Objective Optimal

Thanks for your attention!
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- APPLICATIONS
 - Live Sequence Charts
 - Gantt Chart
 - Probabilistic Timed Automata

Usability