Towards an Automatic Parametric WCET Analysis

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Method

- We present a practical implementation of a method to calculate a parametric WCET
- The method was first presented by Björn Lisper, WCET'03
- It obtains a formula representing the WCET in terms of input variables of the program
- Works for possibly unstructured, deterministic and terminating programs

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Contribution

- Showing that the analysis can be implemented for simple programs
- Reducing complexity of the analysis by reducing the number of variables
- Simplifying Pugh's method for counting solutions of presburger formulae to count points inside convex polyhedra
- Identifying the need for simplification of the resulting parametric WCET formula

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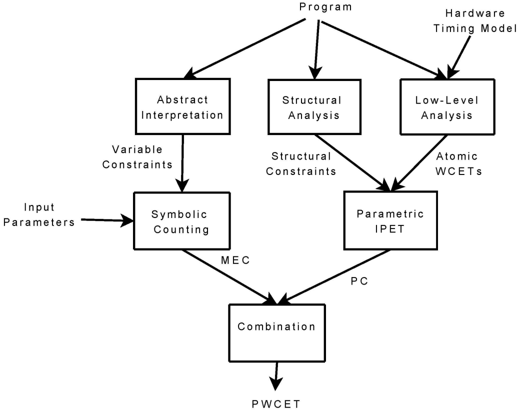
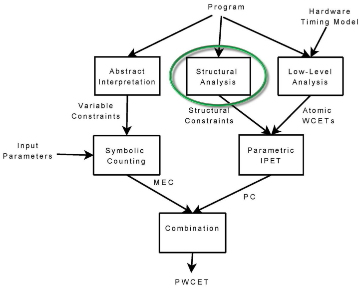
Workflow

Outline of the method presented at WCET'03



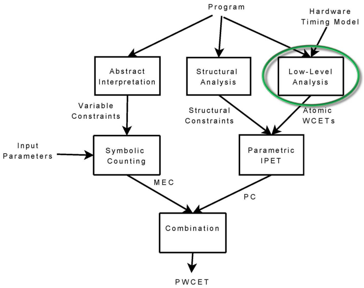
Structural Analysis



Structural Analysis

- Derives constraints on execution counts
- Imposed by the CFG structure only.

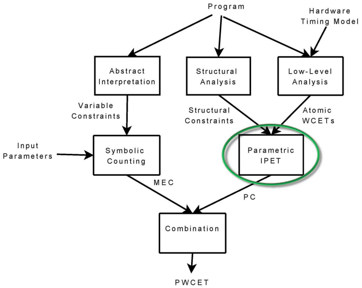
Low-Level Analysis



Low-Level Analysis

- Obtains WCETs for atomic parts of a program

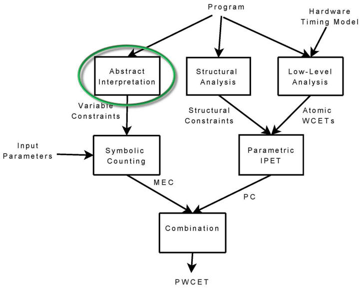
Parametric IPET



Parametric IPET

- Maximises $\vec{c}^{\mathrm{T}}\vec{x}$ subject to the structural constraints
- The execution counts are assumed to be bounded from above by *execution count parameters*
- The result is a formula in terms of these parameters

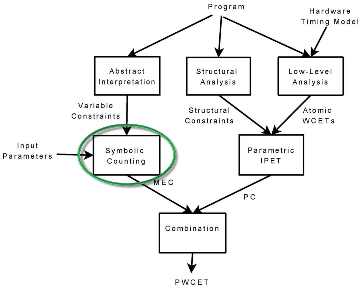
Abstract Interpretation



Abstract Interpretation

- Derives constraints on the values of the program variables

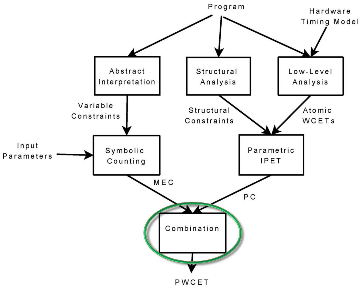
Symbolic Counting



Symbolic Counting

- Counts the maximum number of possible environments (run-time states) at each program point
- The result of the abstract interpretation constrains which environments are possible
- The result is symbolic in the input parameters

Combination



Combination

- The WCET formula obtained from the parametric IPET is symbolic in execution count parameters
- The execution counts are bounded by the possible number of environments
- The execution count parameters are substituted by the functions from the symbolic counting

Assumptions and Definitions

- Modular implementation of the different phases using C++ and Haskell
- Programs are modelled as CFGs
  - Variables are integer valued or Booleans
  - The CFG has entry, exit, assignment, conditional and merge nodes
  - No pointers, arrays or function calls
  - Assignments and conditionals are considered to be either linear or unknown

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Assumptions and Definitions cont'd

- Each program P has a set of variables $\mathcal{V}_P$
- Each program has a set of input variables $\mathcal{I}_P \subseteq \mathcal{V}_P$ and each input variable
  - Affects program flow
  - Remains unchanged in the program part under analysis
- The atomic WCETs for each program point is assumed to be a single constant
- Structural analysis is trivial since the program is represented as a CFG

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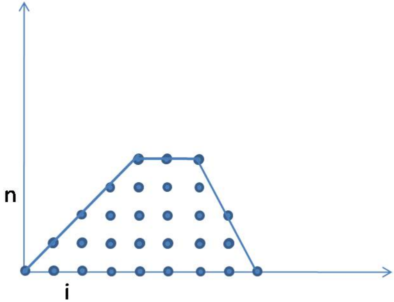

- Abstract Interpretation obtains a super set of the possible environments
- We have implemented the *polyhedral domain* with a existing library called "New Polka"
- Encloses the possible environments inside a convex polyhedron

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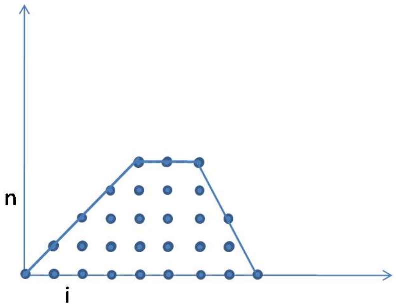

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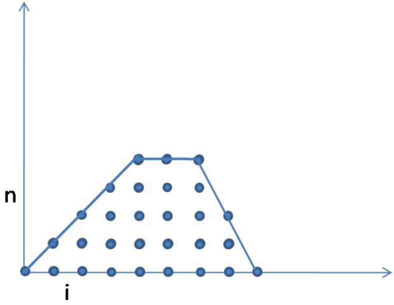

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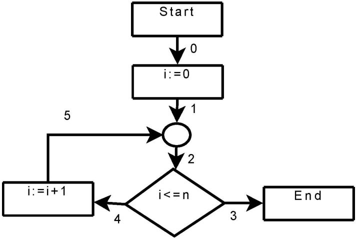
Example



$$
\begin{aligned}
a_0 &= \top \\
a_1 &= \{i = 0\} \\
a_2 &= \{0 \leq i \leq n+1\} \\
a_3 &= \{i \geq 0, i \geq n+1\} \\
a_4 &= \{0 \leq i \leq n\} \\
a_5 &= \{1 \leq i \leq n+1\}
\end{aligned}
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Pugh's Method

- W. Pugh proposed a method for counting solutions to presburger formulae (SIGPLAN'94)
- The method computes *generalised sums* which are sums over several variables with complex constraints

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Contribution I

Problem The method requires complex data structures and is not formulated as an algorithm

Solution By restricting the constraints to be convex polyhedra and to only summarise over polynomials, we are not exposed to the full complexity of the method and can make a straightforward implementation

Drawback In some cases some precision is lost

Example

Since n affects program flow and is not changed during execution, it is considered an input variable to the program

$$
\begin{aligned}
a_0 &= \top & &\Rightarrow \textstyle\sum_{i \in \mathbb{Z}^2} 1 = \infty \\
a_1 &= \{i = 0\} & &\Rightarrow \textstyle\sum_{i \in \{0\}} 1 = 1 \\
a_2 &= \{0 \leq i \leq n+1\} & &\Rightarrow n + 2 \text{ if } n \geq -1 \\
a_3 &= \{i \geq 0, i \geq n+1\} & &\Rightarrow \textstyle\sum_{i \geq 0, n+1} 1 = \infty \\
a_4 &= \{0 \leq i \leq n\} & &\Rightarrow n + 1 \text{ if } n \geq 0 \\
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Parametric Integer Programming

- Invented by P. Feautrier 1988
- Essentially a parameterised version of the (dual) simplex algorithm
- There is a tool called PipLib that solves parametric integer problems
- Can be used as parametric IPET calculation

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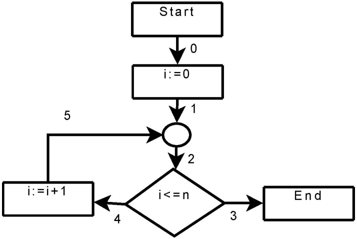
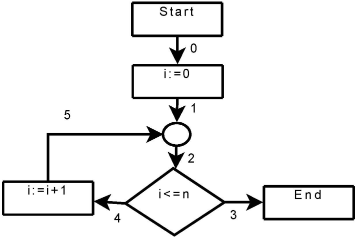


This program with assumed WCET of ten clock cycles for each program point gives the following IPET problem:

Maximise $10\sum_{i=0}^{5} x_i$ subject to

$$x_0 = 1$$
$$x_1 = x_0$$
$$x_2 = x_1 + x_5$$
$$x_2 = x_3 + x_4$$
$$x_3 = 1$$
$$x_5 = x_4$$
$$x_i \leq p_i$$

Example



PipLib Produces (when converted to human readable)

$\mathrm{Max} = \lambda(p_0, p_1, p_2, p_3, p_4, p_5).$
$\mathrm{if}(p_0, p_1, p_2, p_3 \geq 1 \wedge p_2 < p_4, p_5)$ then
$30p_2 + 10$ else if
$(p_0, p_1, p_2, p_3 \geq 1 \wedge p_2 < p_4 \wedge p_5 \geq p_2)$
then $(30p_5 + 40)$ else if ...

Contribution II

Problem Parametric ILP has exponential worst-case complexity and produces very large output files

Solution The structural constraint matrix imposes a basis of the unknown variables which can be used in the PILP-problem instead of the variables themselves

Reducing variables

- Most unknowns in the system of linear equations obtained from structural analysis can be expressed as linear combinations of a basis of variables
- By Jordan-Gauss elimination we can obtain this basis
- Solving the IPET problem in terms of this basis reduces the number of unknowns and the general complexity of the problem
- As an example, doing the above on the given program example, we have reduced six unknowns to one unknown

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From PipLib:

$$\mathrm{PWCET} = \lambda n.$$
$$\text{if}(c_0, c_1, c_2, c_3 \geq 1 \wedge c_2 < c_4, c_5) \text{ then}$$
$$30c_2 + 10 \text{ else if}$$
$$(c_0, c_1, c_2, c_3 \geq 1 \wedge c_2 < c_4 \wedge c_5 \geq c_2)$$
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Conclusions

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- We have presented a simplified Pugh's method to count solutions to presburger formulae to count integer points inside polyhedra
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Future Work

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- Do a full evaluation of the method with benchmarks
- Investigate how simplifications can be made on the final formula and the intermediate results
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The end

- Thank you!