Statistical-Based WCET Estimation and Validation

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Outline

- Measurement-Based WCET Estimation
- About the Data Set
- Extreme Value Theory
- WCET Prediction Algorithm
- Experimental Methodology and Results

Measurement-Based WCET Estimation

Modern processors are optimized for throughput with features such as

- Predictive branching
- Caching

Due to these CPU features

- Execution time can be difficult to model
- True worst case may be unreasonably long

Our Measurement-Based Approach

- Estimate WCET based on execution time measurements
- Use Extreme Value Theory (EVT) to produce estimate
- WCET estimate based on user requirement for probability of being exceeded
- Additional measurement data used to validate predictions

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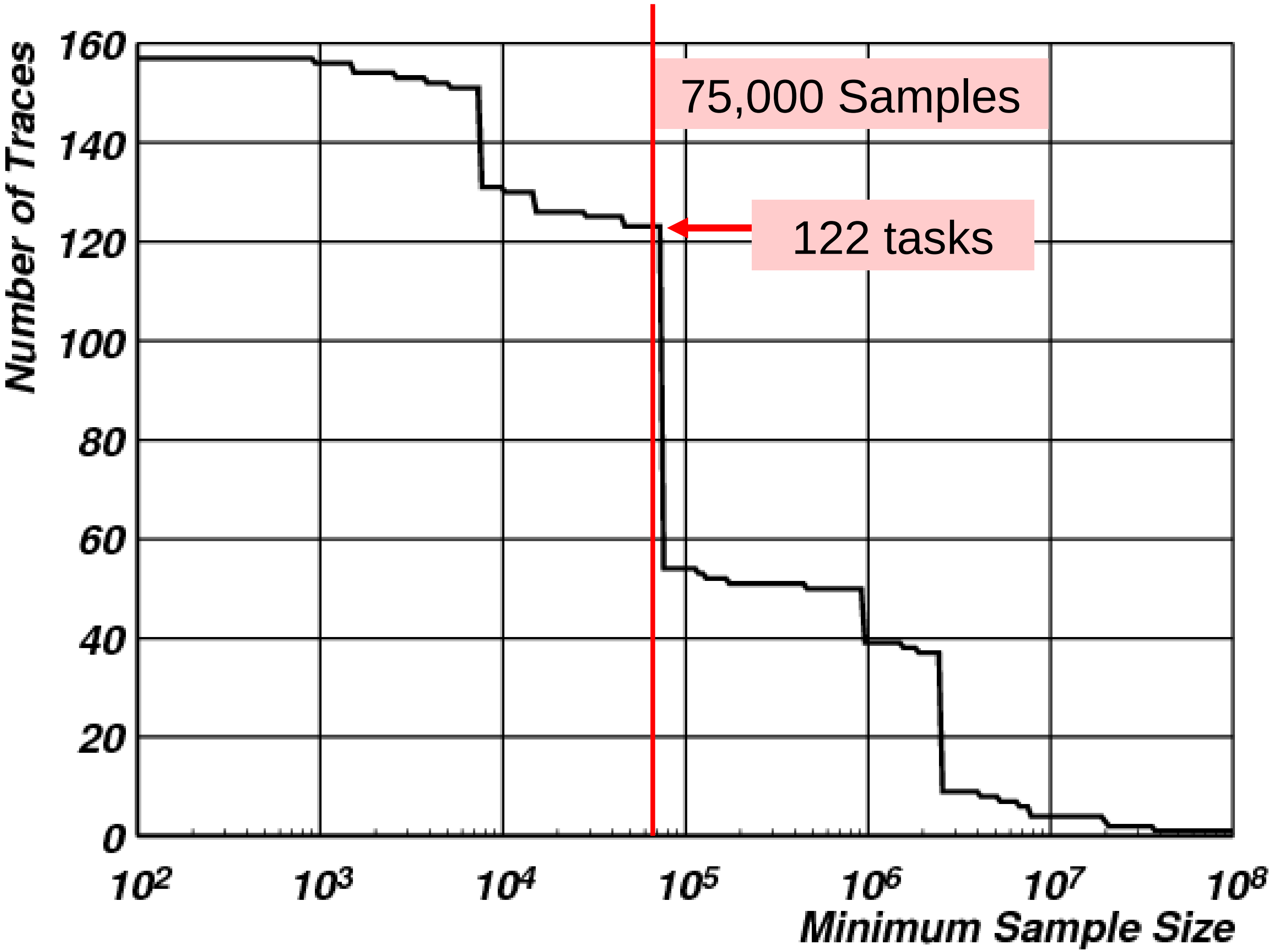
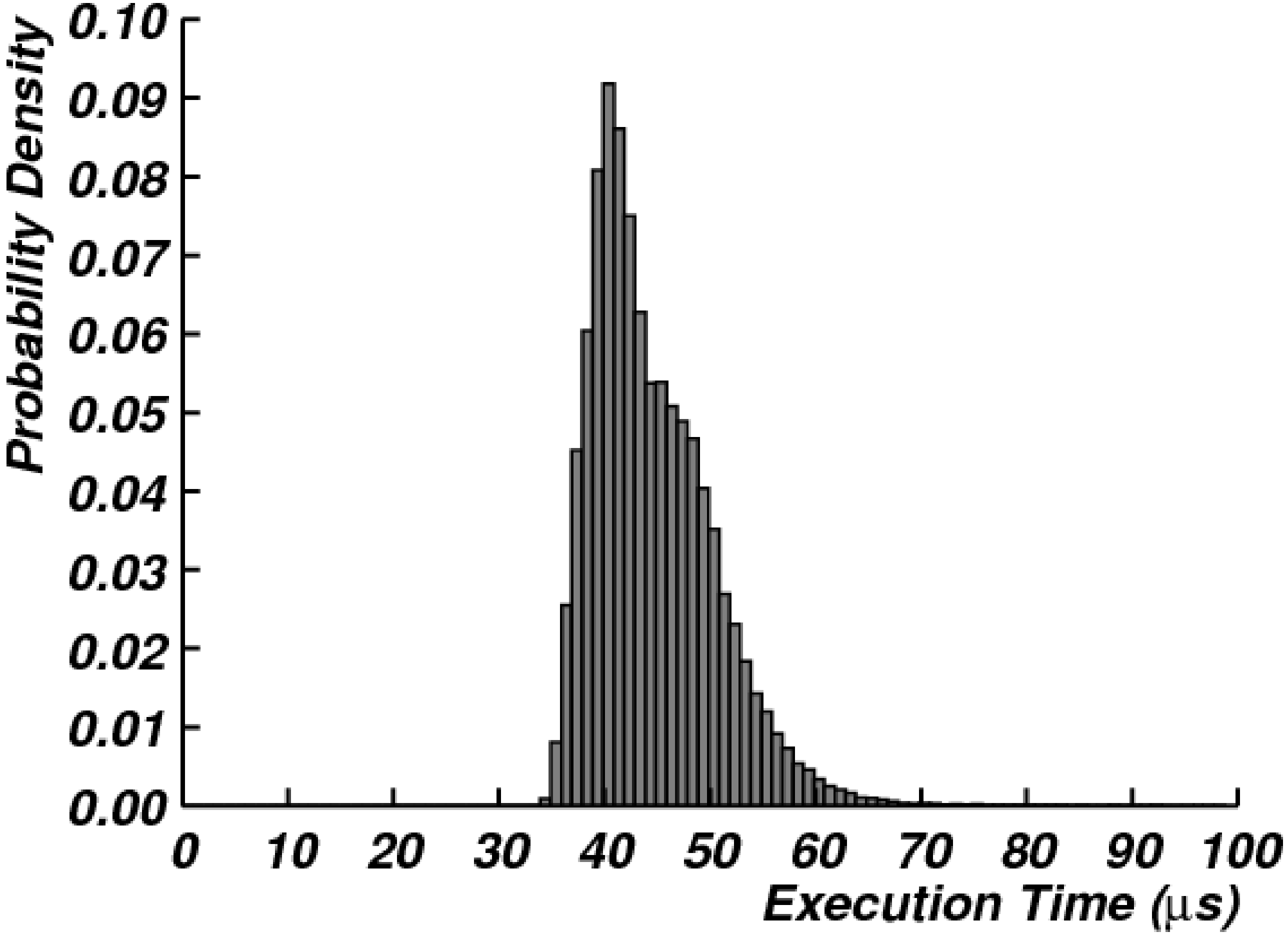
Data Set Overview

- Trace data taken from an embedded system
- Total of 154 tasks
- For each task
  - 25 runs of 5 min. each
  - 15 min. of data from each trace used for estimation
  - 110 min. of data from each trace used for validation
  - No data-dependant loops

Sample Sizes of Trace Data



Execution Time Distribution for Typical Task



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Extreme Value Theory

Branch of statistics used to reason about rare events

Models tail of distribution

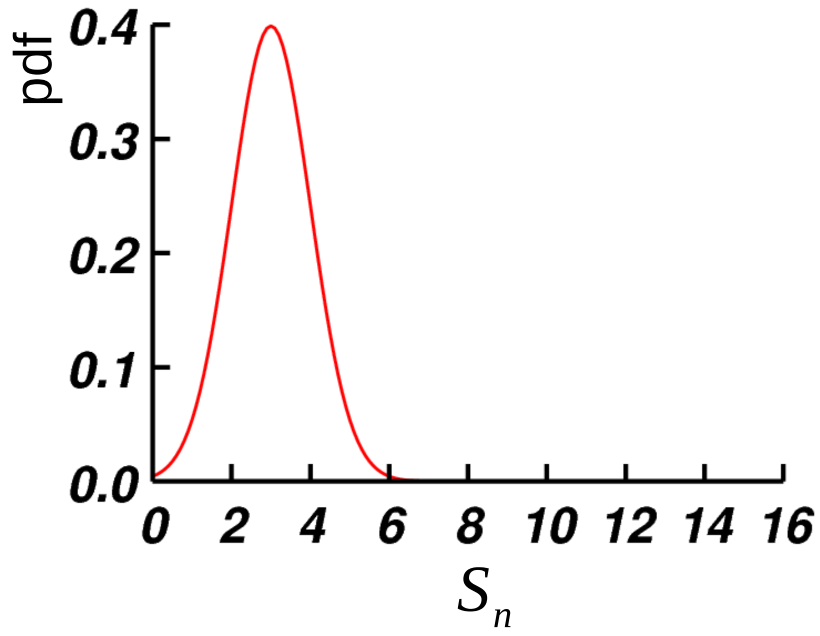
Used to model

- Insurance claims (e.g., floods)
- Finance (e.g., market risks)

Central Limit Theorem

The sum of a set of independent identically distributed random variables converges to the normal distribution.

$$S_n = X_1 + X_2 + \cdots + X_n$$

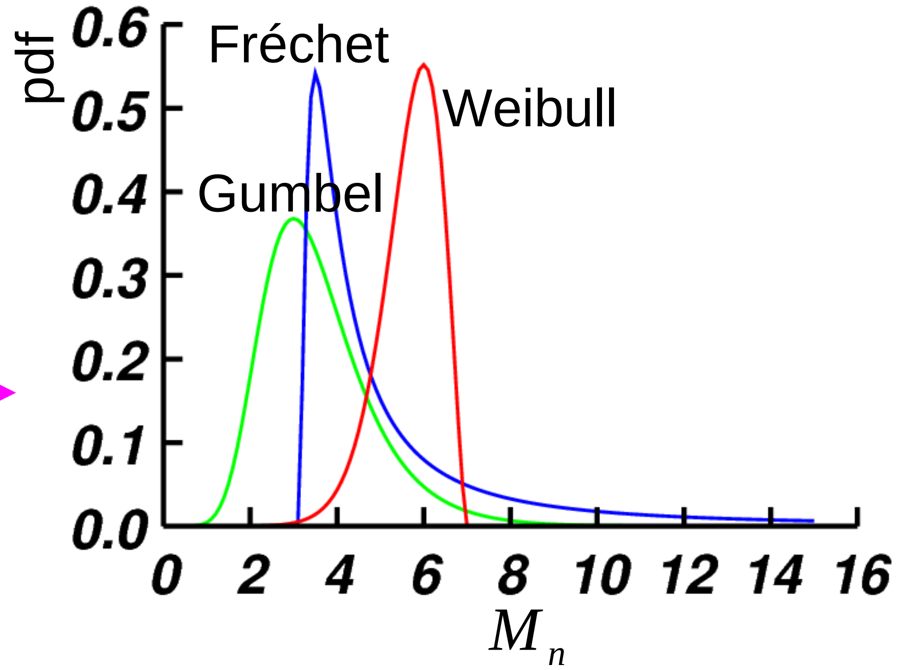


Fisher-Tippet Theorem

The maximum of a set of independent identically distributed random variables X_i converges to one of:

- Gumbel – if X_i has exponential tail
- Fréchet – if X_i has heavy tail
- Weibull – if X_i has bounded tail

$$M_n = \max(X_1, X_2, \cdots, X_n)$$



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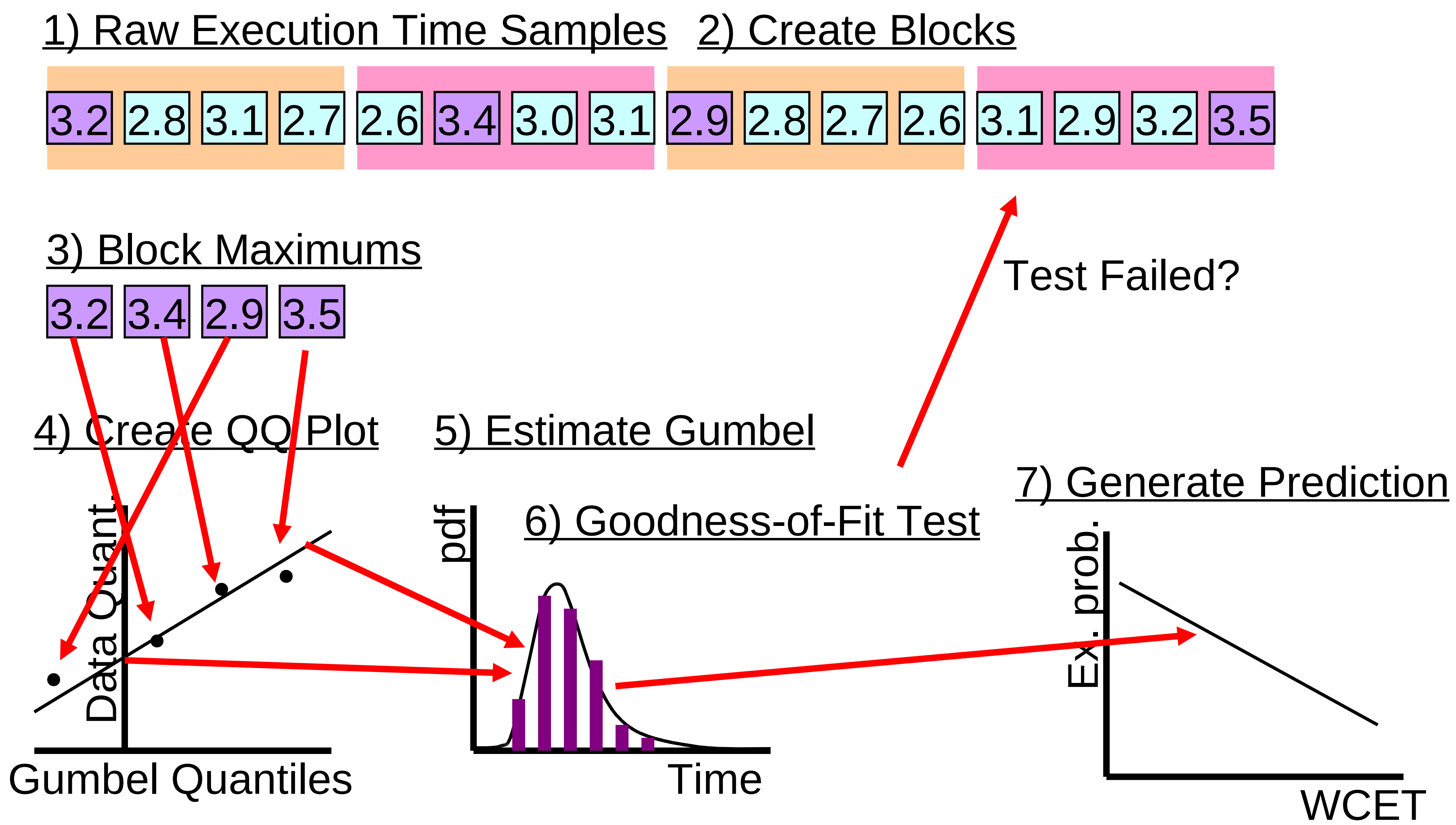
WCET Estimation Algorithm



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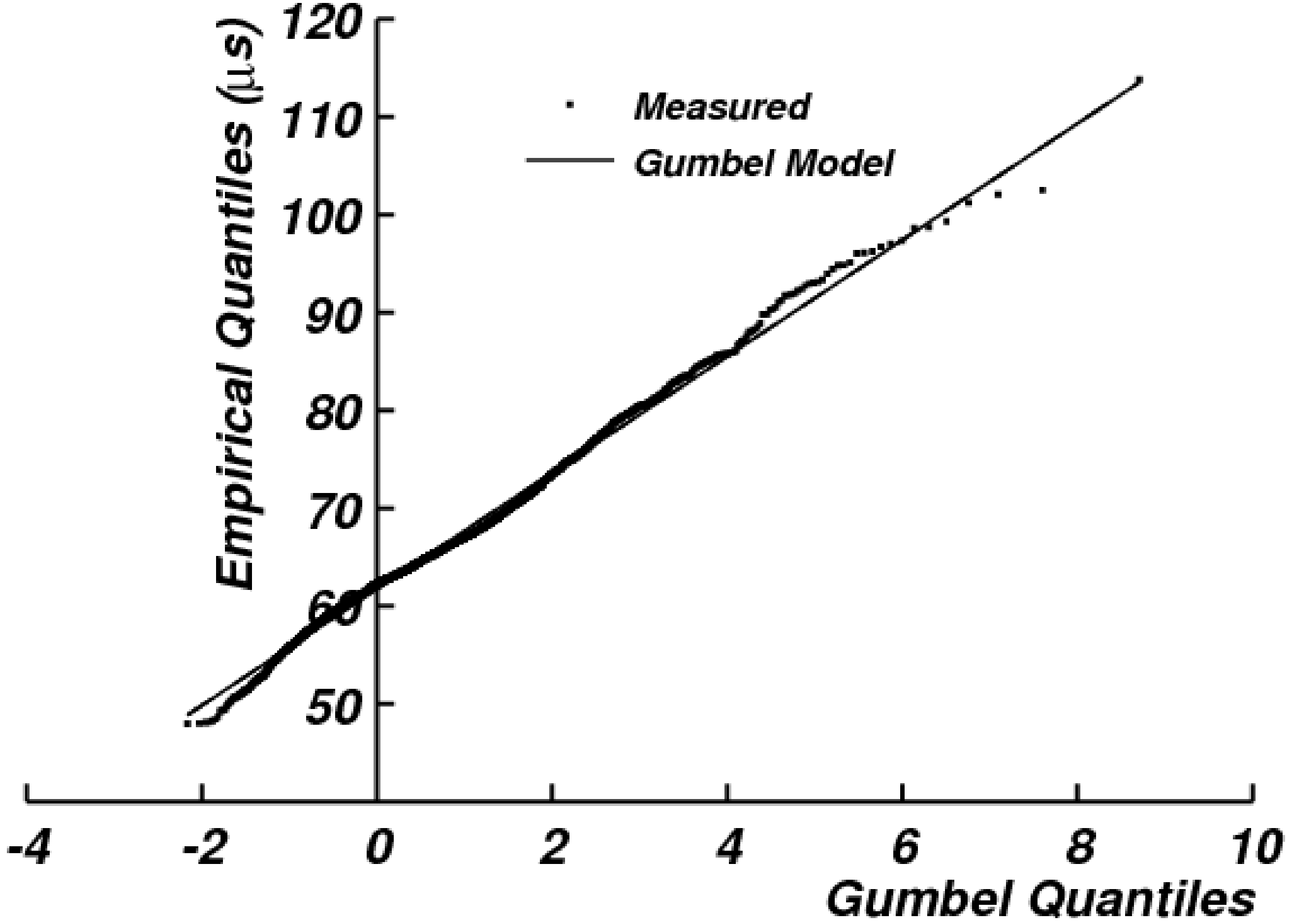
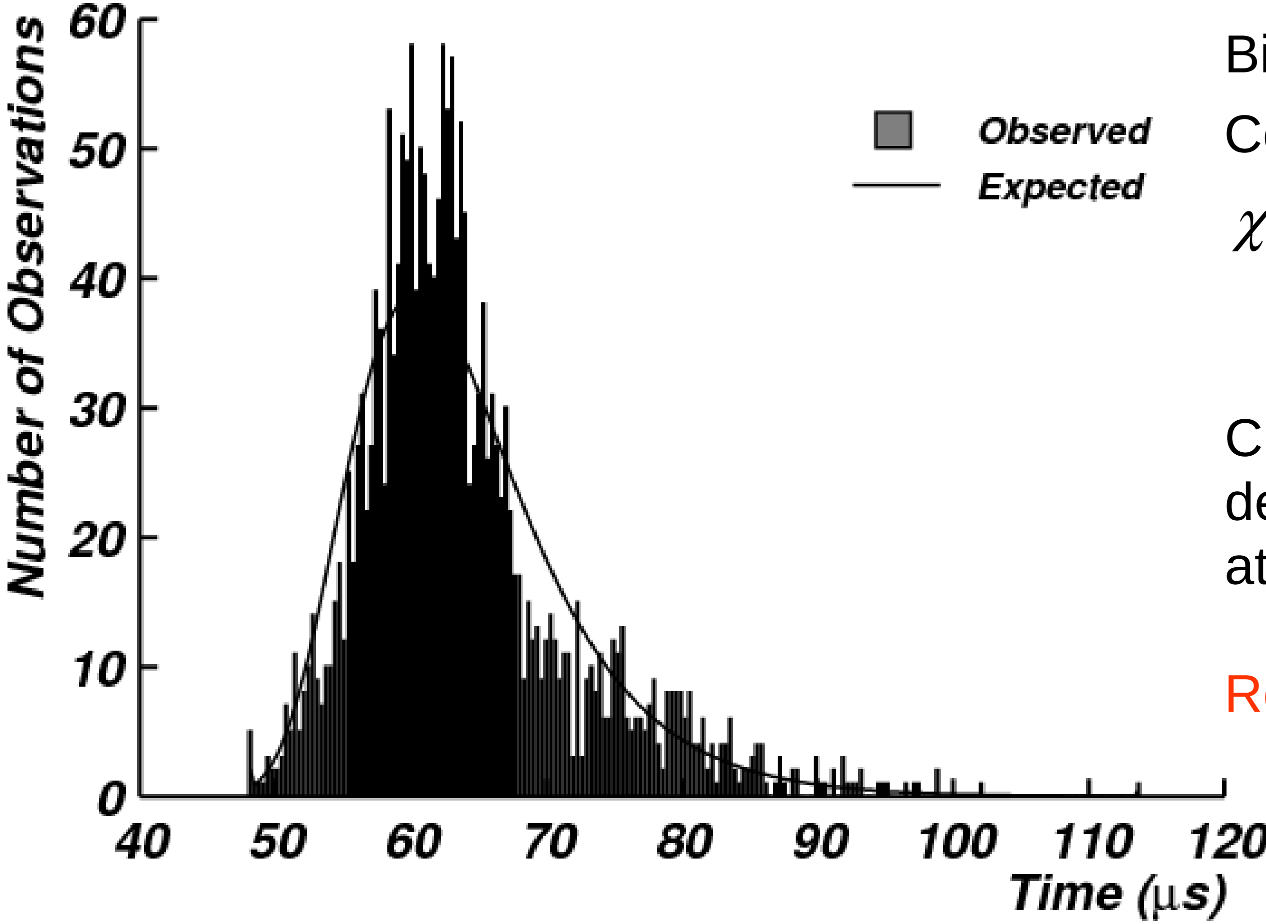
Quantile-Quantile Plot Example



Chi-Squared Test for Block Size 100



Bins: 100

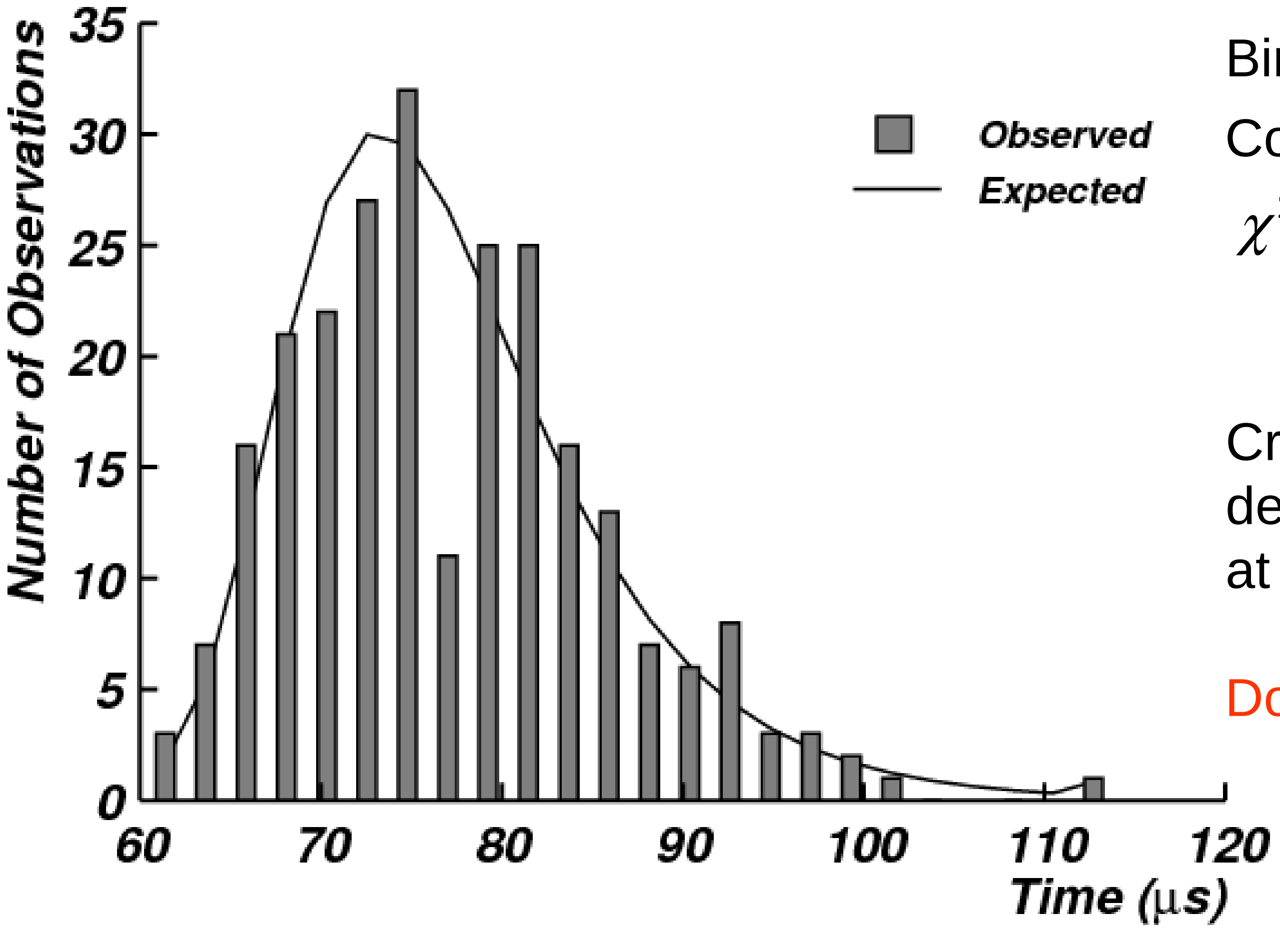
Collapsed Bins: 62

$\chi^2 = 236$

Critical value for 59 degrees of freedom at p=0.05 is 77.93

Reject

Chi-Squared Test for Block Size 400



Bins: 25

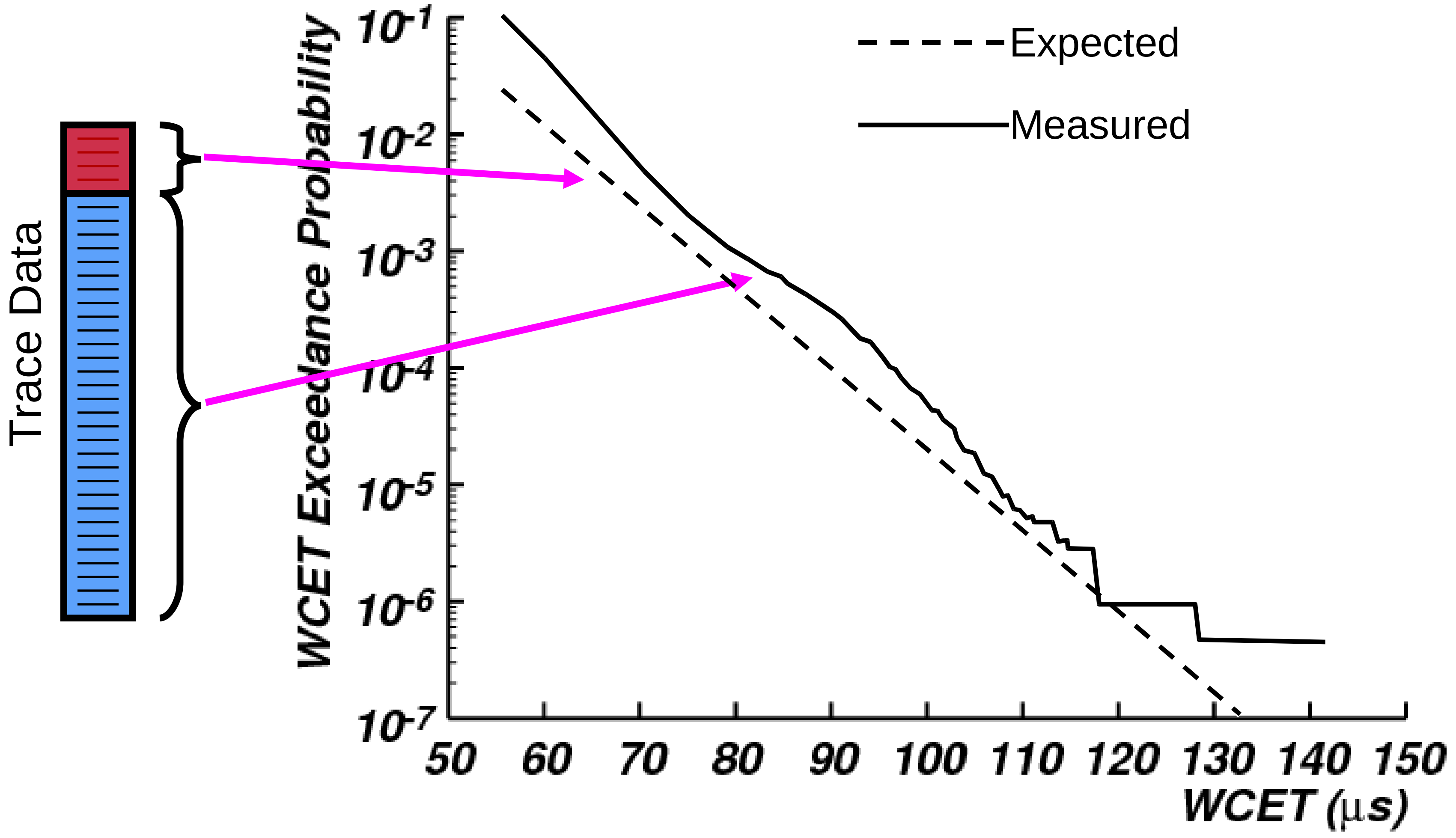
Collapsed Bins: 19

$\chi^2 = 19.67$

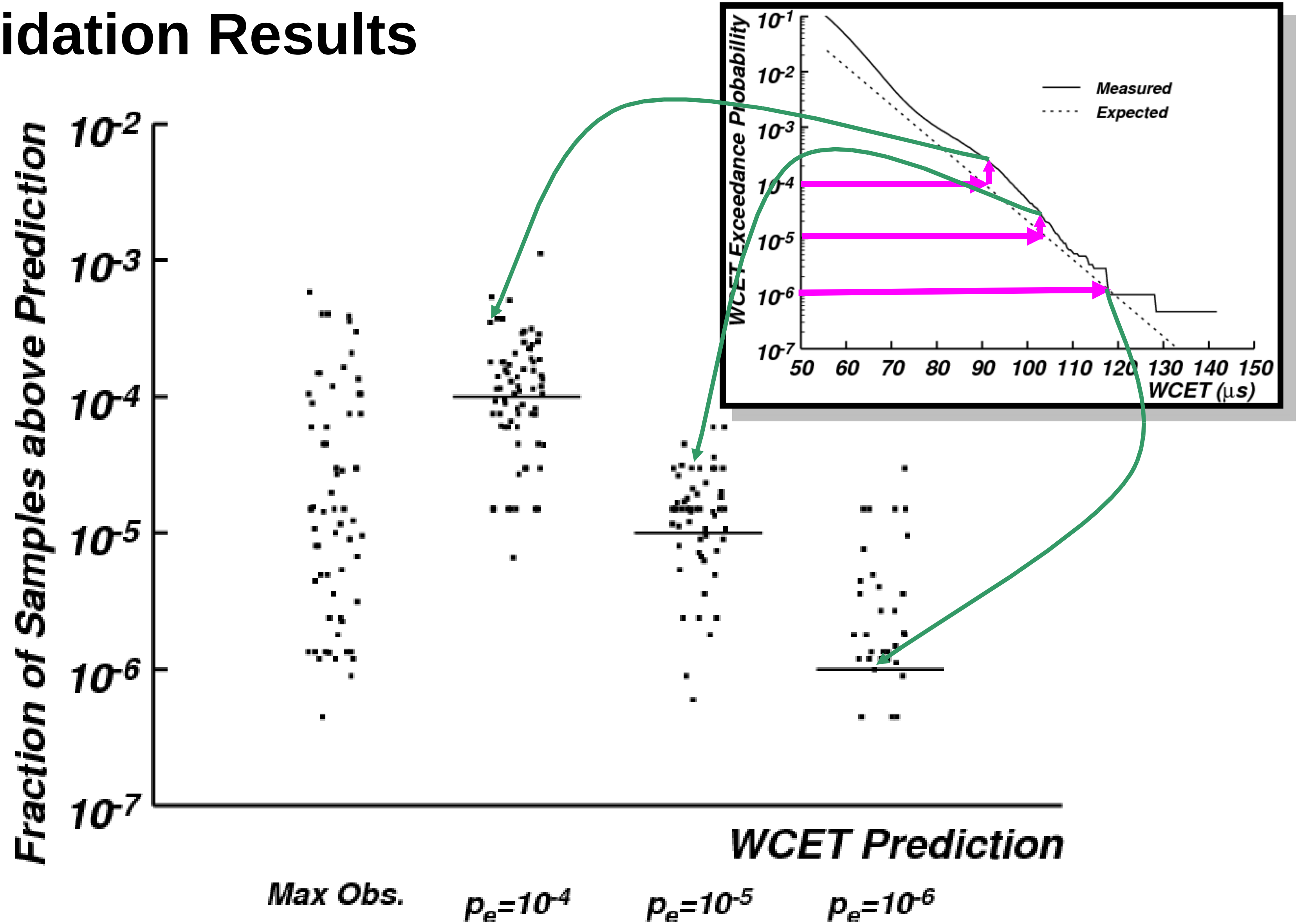
Critical value for 16 degrees of freedom at p=0.05 is 26.3

Do Not Reject

Single Task Results



Validation Results



Conclusion

Applied algorithm to estimate WCET from execution time samples

- Uses extreme value theory method of block maximum
- Execution time data grouped into blocks
- Block size increased until goodness-of-fit test passes

Validated predictions using additional execution time samples

- Measured WCET exceedances agreed with predicted exceedences
- Variation in WCET exceedences using EVT algorithm reduced over using maximum observed execution time as WCET.