

# Statistical-Based WCET Estimation and Validation

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# Outline

- Measurement-Based WCET Estimation
- About the Data Set
- Extreme Value Theory
- WCET Prediction Algorithm
- Experimental Methodology and Results



# Measurement-Based WCET Estimation

Modern processors are optimized for throughput with features such as

- Predictive branching
- Caching

Due to these CPU features

- Execution time can be difficult to model
- True worst case may be unreasonably long

Our Measurement-Based Approach

- Estimate WCET based on execution time measurements
- Use Extreme Value Theory (EVT) to produce estimate
- WCET estimate based on user requirement for probability of being exceeded
- Additional measurement data used to validate predictions



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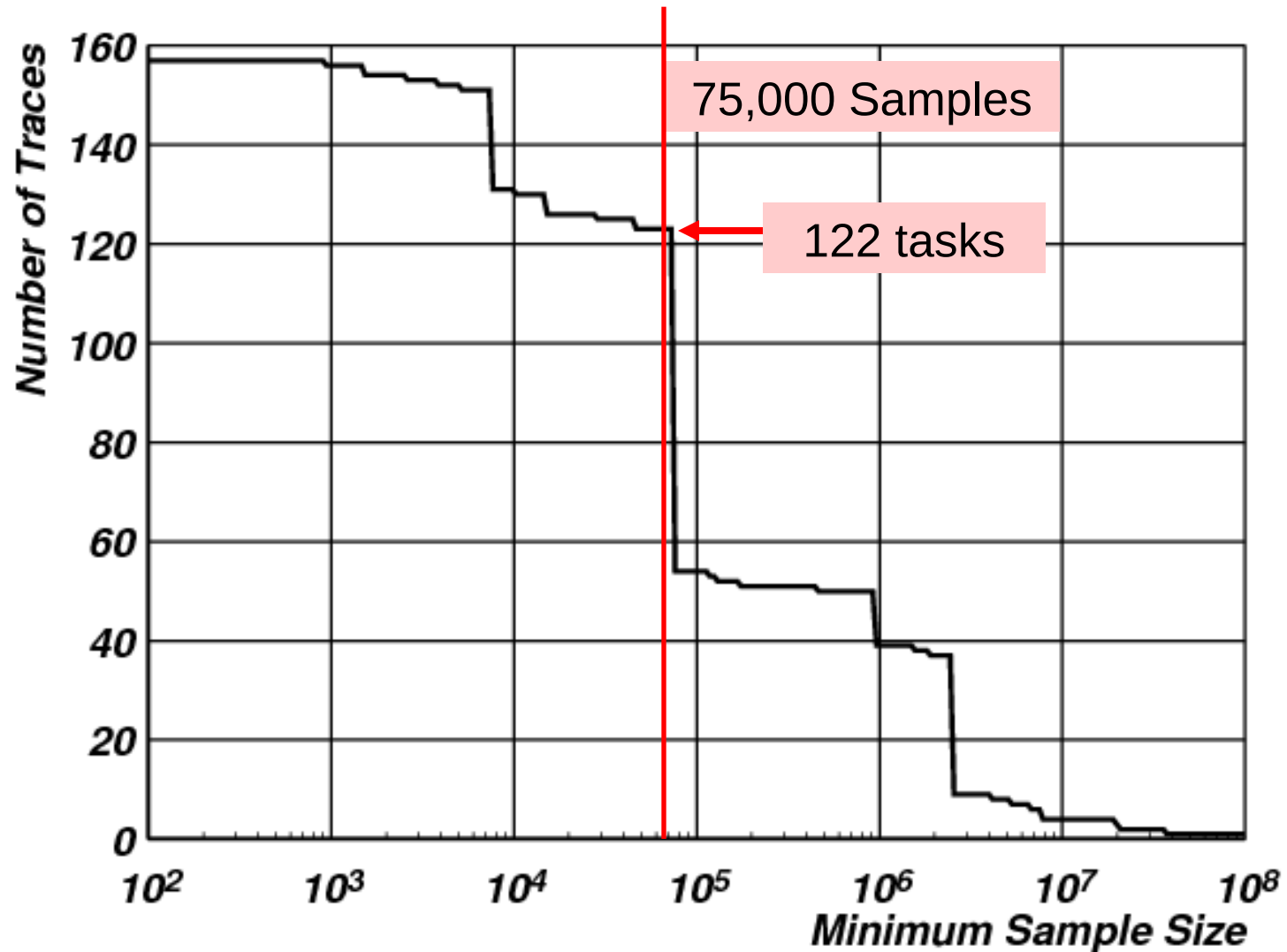


# Data Set Overview

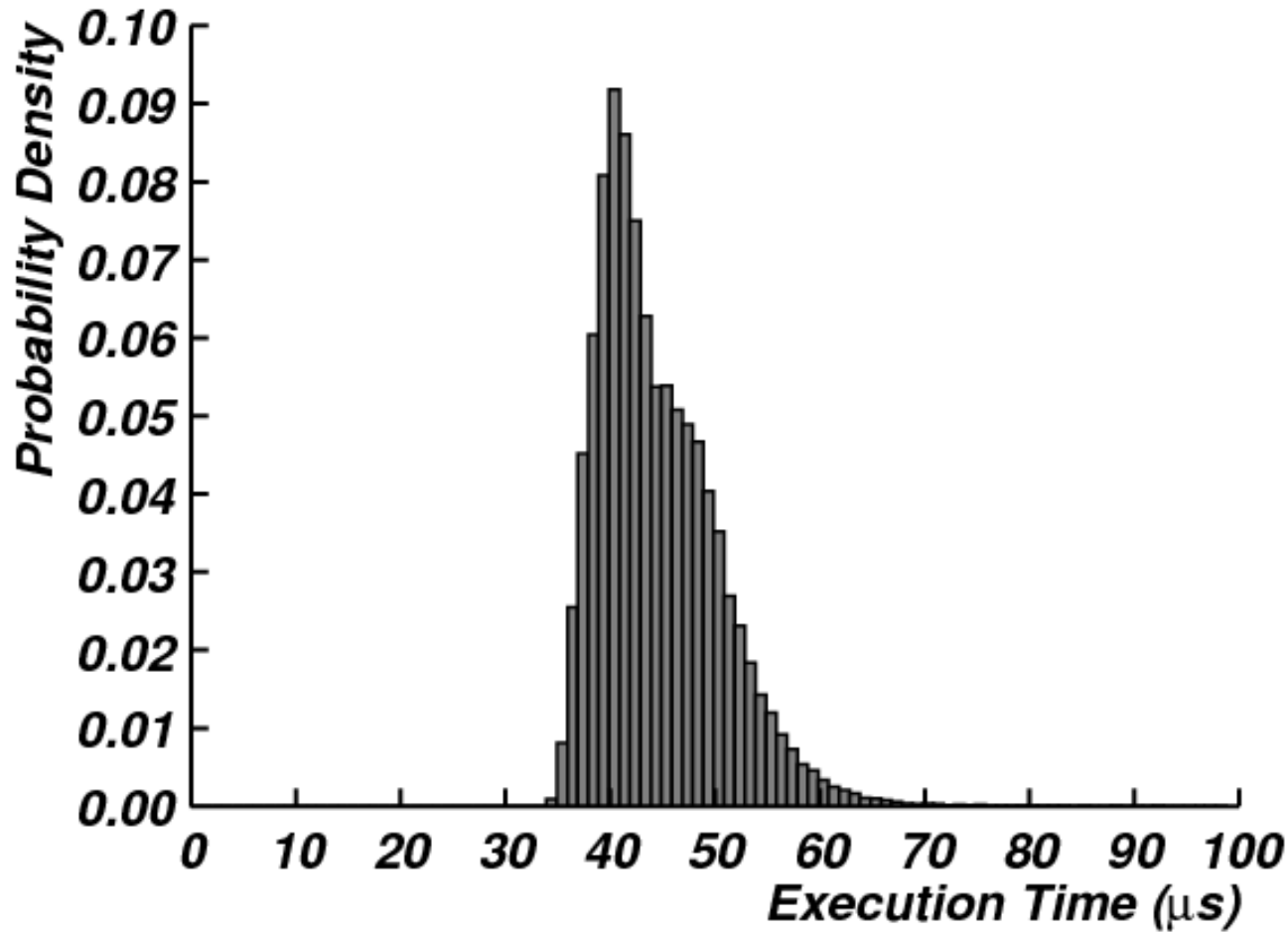
- Trace data taken from an embedded system
- Total of 154 tasks
- For each task
  - 25 runs of 5 min. each
  - 15 min. of data from each trace used for estimation
  - 110 min. of data from each trace used for validation
  - No data-dependant loops



# Sample Sizes of Trace Data



# Execution Time Distribution for Typical Task





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# Extreme Value Theory

Branch of statistics used to reason about rare events

Models tail of distribution

Used to model

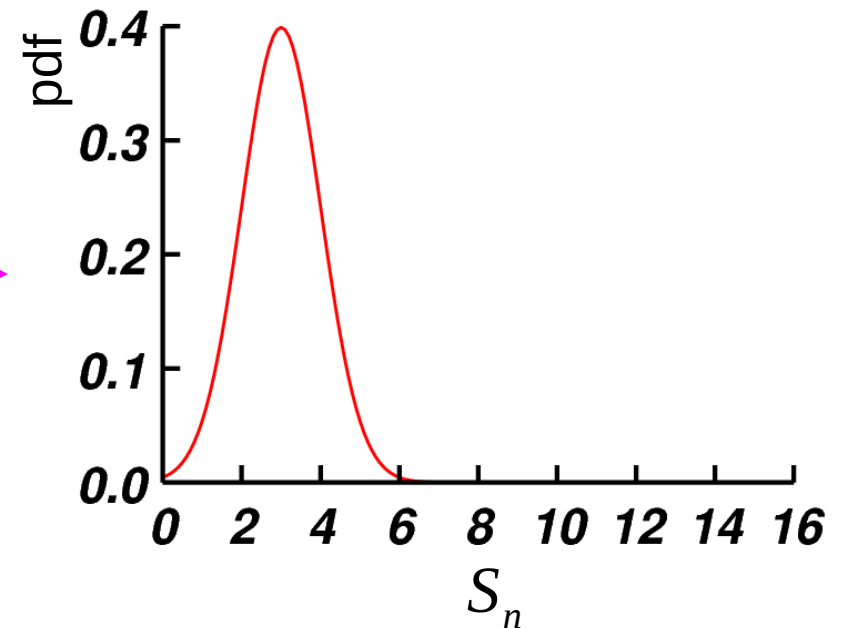
- Insurance claims (e.g., floods)
- Finance (e.g., market risks)



# Central Limit Theorem

The sum of a set of independent identically distributed random variables converges to the normal distribution.

$$S_n = X_1 + X_2 + \cdots + X_n$$

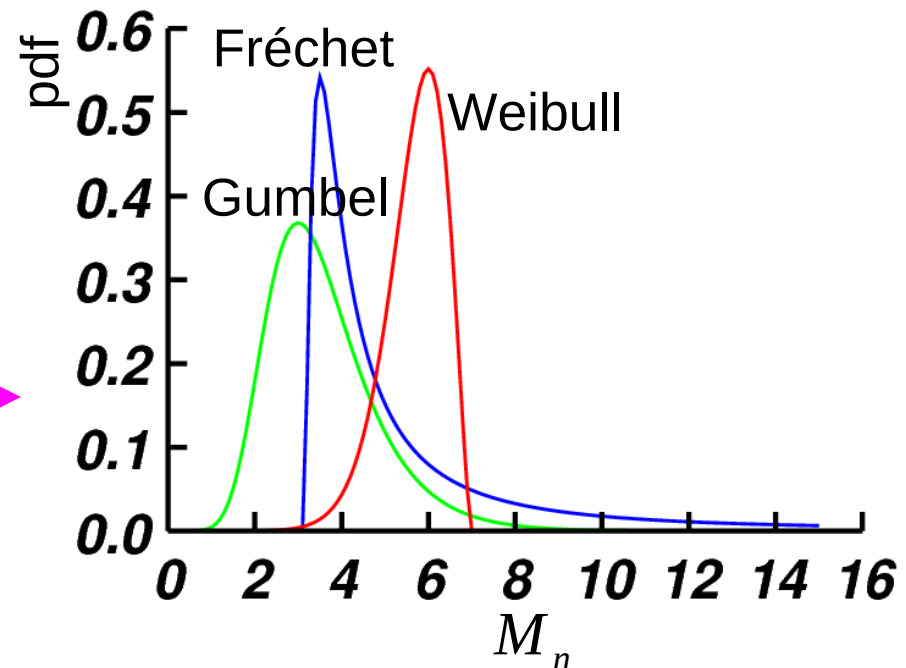


# Fisher-Tippett Theorem

The maximum of a set of independent identically distributed random variables  $X_i$  converges to one of:

- Gumbel – if  $X_i$  has exponential tail
- Fréchet – if  $X_i$  has heavy tail
- Weibull – if  $X_i$  has bounded tail

$$M_n = \max(X_1, X_2, \dots, X_n) \longrightarrow$$



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# WCET Estimation Algorithm

1) Raw Execution Time Samples    2) Create Blocks

3.2 2.8 3.1 2.7 2.6 3.4 3.0 3.1 2.9 2.8 2.7 2.6 3.1 2.9 3.2 3.5

3) Block Maximums

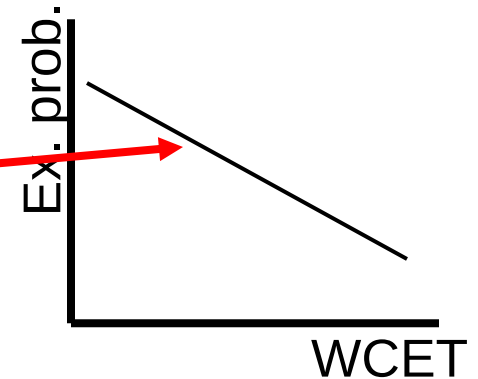
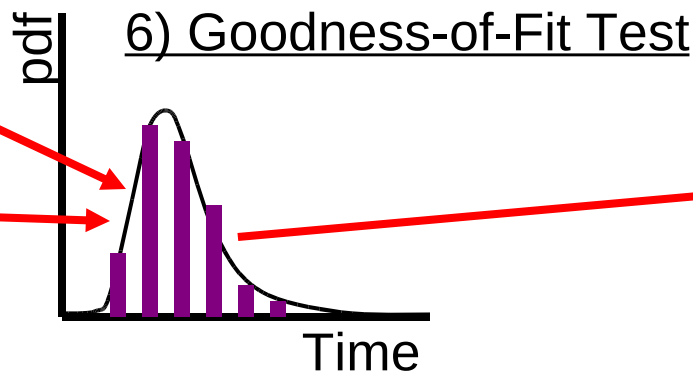
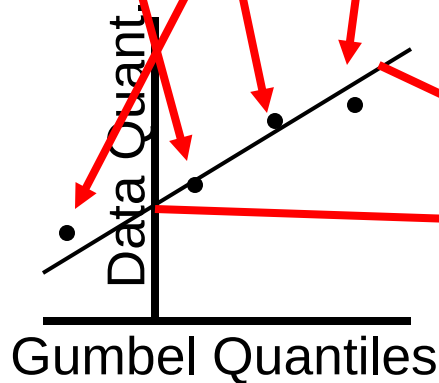
3.2 3.4 2.9 3.5

4) Create QQ Plot

5) Estimate Gumbel

Test Failed?

7) Generate Prediction

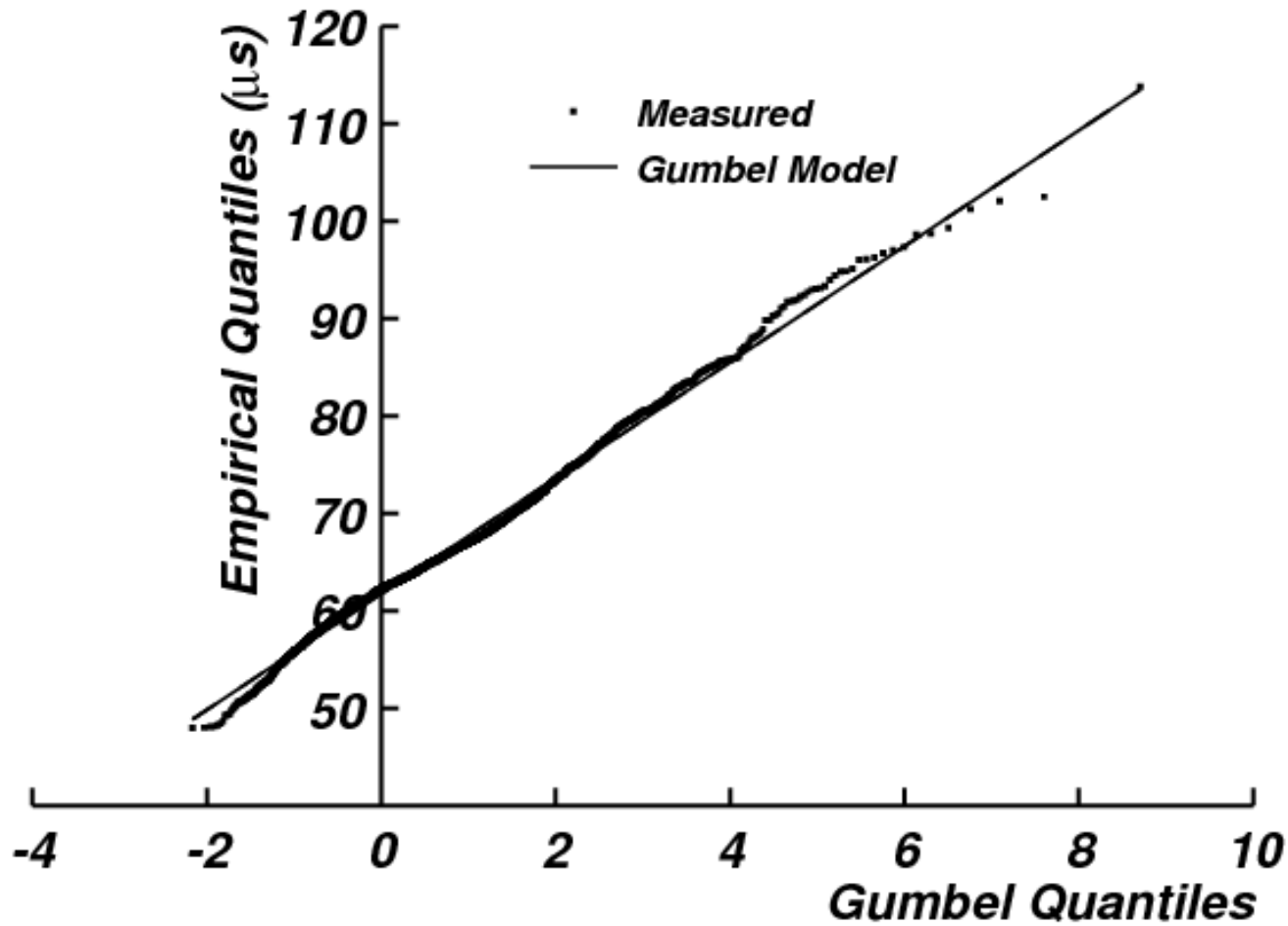


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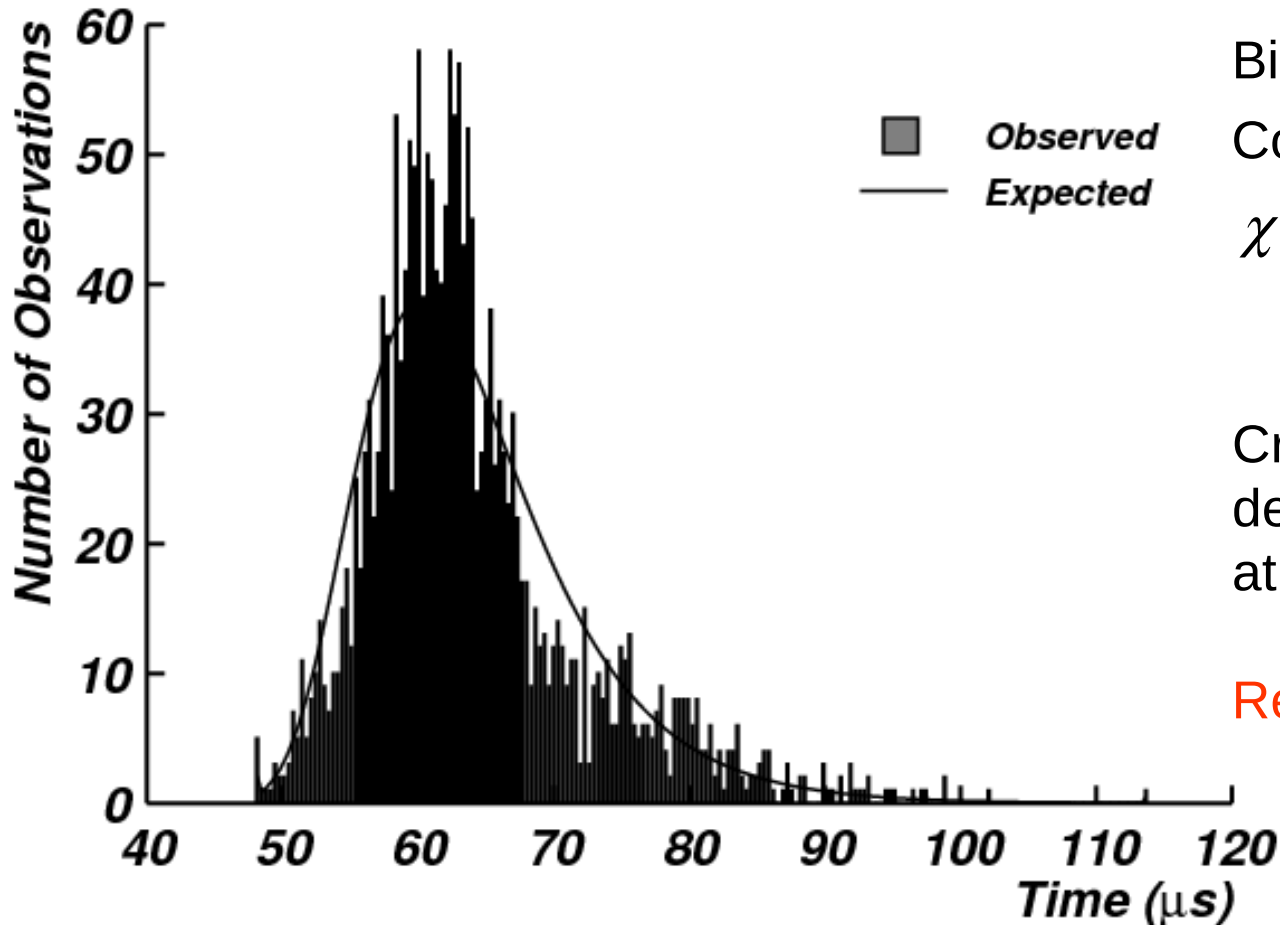


# Quantile-Quantile Plot Example





# Chi-Squared Test for Block Size 100



Bins: 100

Collapsed Bins: 62

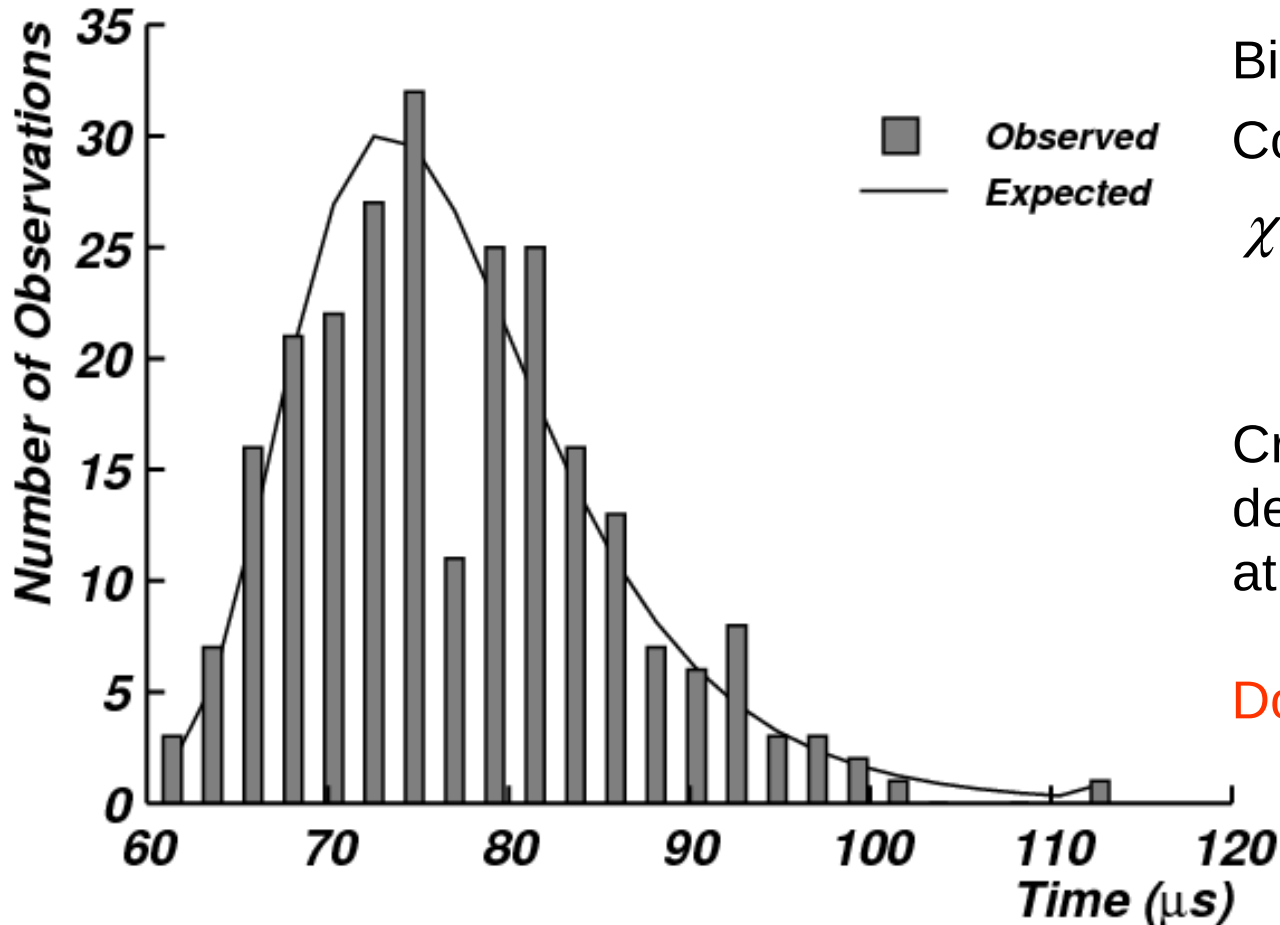
$$\chi^2 = 236$$

Critical value for 59 degrees of freedom at  $p=0.05$  is 77.93

Reject



# Chi-Squared Test for Block Size 400



Bins: 25

Collapsed Bins: 19

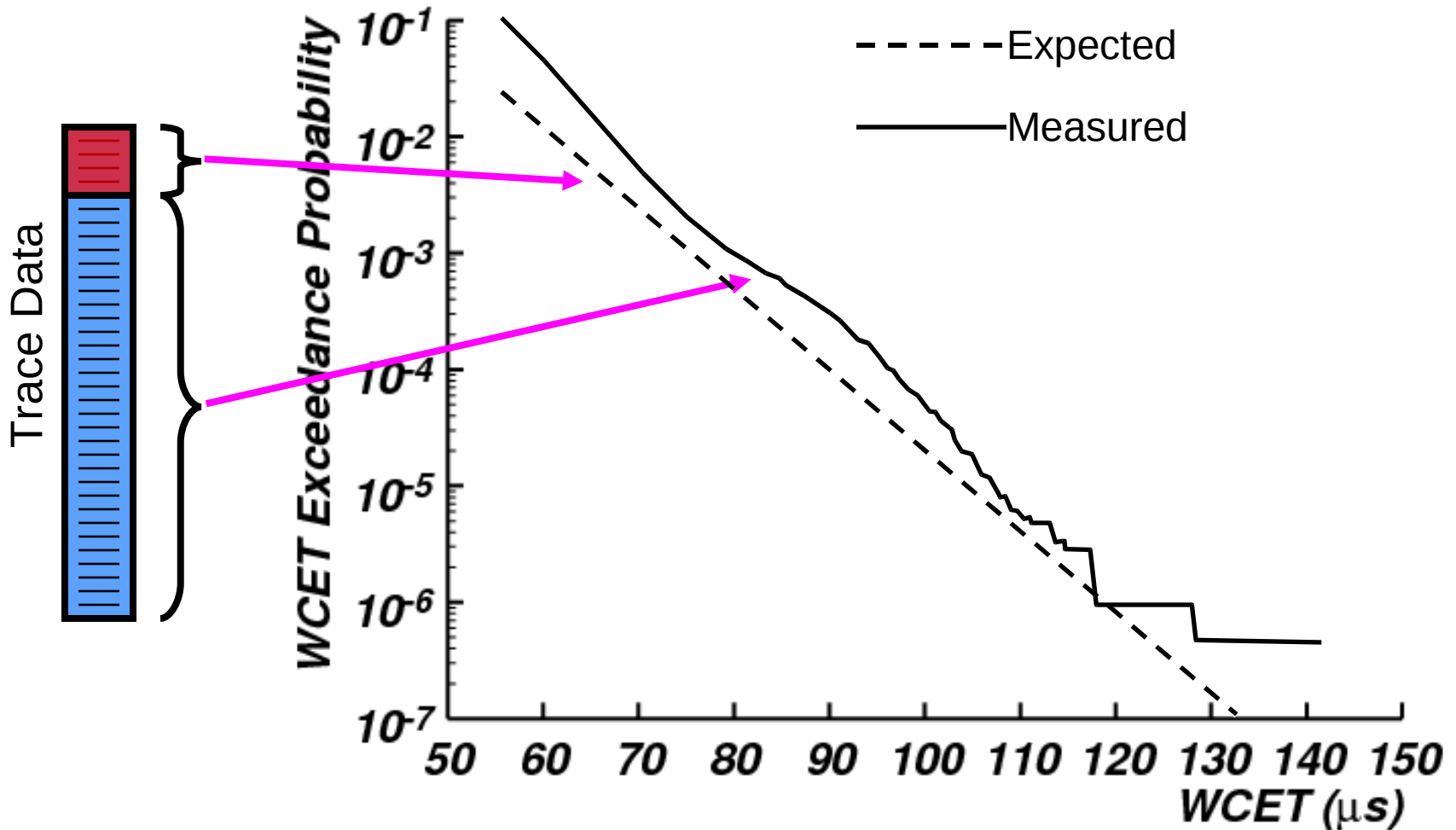
$$\chi^2 = 19.67$$

Critical value for 16 degrees of freedom at  $p=0.05$  is 26.3

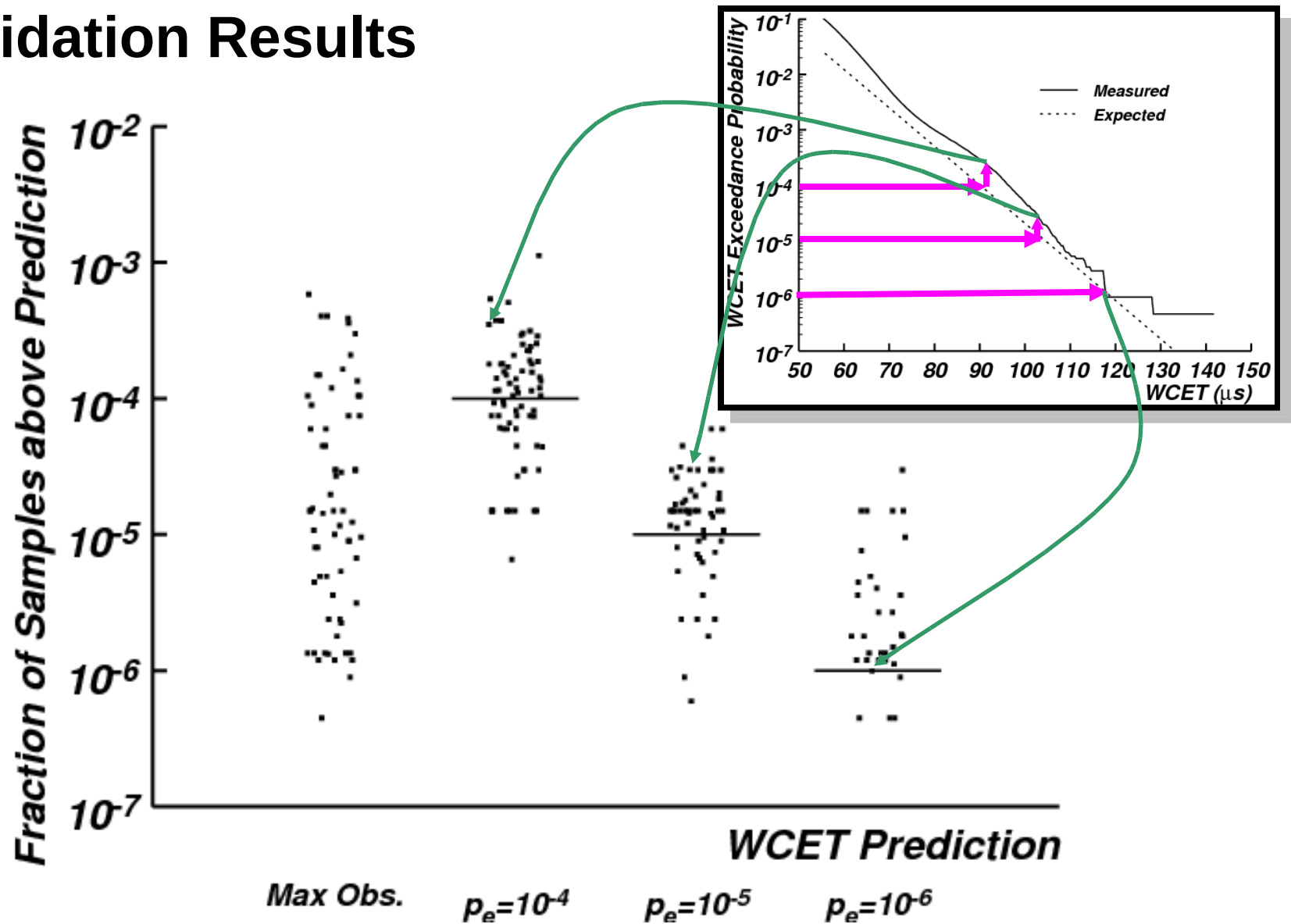
**Do Not Reject**



# Single Task Results



# Validation Results



# Conclusion

Applied algorithm to estimate WCET from execution time samples

- Uses extreme value theory method of block maximum
- Execution time data grouped into blocks
- Block size increased until goodness-of-fit test passes

Validated predictions using additional execution time samples

- Measured WCET exceedances agreed with predicted exceedances
- Variation in WCET exceedances using EVT algorithm reduced over using maximum observed execution time as WCET.

