Statistical-Based WCET Estimation and Validation

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Outline

• Measurement-Based WCET Estimation
• About the Data Set
• Extreme Value Theory
• WCET Prediction Algorithm
• Experimental Methodology and Results
Measurement-Based WCET Estimation

Modern processors are optimized for throughput with features such as

- Predictive branching
- Caching

Due to these CPU features

- Execution time can be difficult to model
- True worst case may be unreasonably long

Our Measurement-Based Approach

- Estimate WCET based on execution time measurements
- Use Extreme Value Theory (EVT) to produce estimate
- WCET estimate based on user requirement for probability of being exceeded
- Additional measurement data used to validate predictions
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Data Set Overview

• Trace data taken from an embedded system
• Total of 154 tasks
• For each task
  – 25 runs of 5 min. each
  – 15 min. of data from each trace used for estimation
  – 110 min. of data from each trace used for validation
  – No data-dependant loops
Sample Sizes of Trace Data

- 75,000 Samples
- 122 tasks
Execution Time Distribution for Typical Task
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Extreme Value Theory

Branch of statistics used to reason about rare events
Models tail of distribution
Used to model
  • Insurance claims (e.g., floods)
  • Finance (e.g., market risks)
Central Limit Theorem

The sum of a set of independent identically distributed random variables converges to the normal distribution.

\[ S_n = X_1 + X_2 + \cdots + X_n \]
The maximum of a set of independent identically distributed random variables $X_i$ converges to one of:

- **Gumbel** – if $X_i$ has exponential tail
- **Fréchet** – if $X_i$ has heavy tail
- **Weibull** – if $X_i$ has bounded tail

$$M_n = \max(X_1, X_2, \ldots, X_n)$$
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WCET Estimation Algorithm

1) Raw Execution Time Samples

2) Create Blocks

3) Block Maximums

4) Create QQ Plot

5) Estimate Gumbel

6) Goodness-of-Fit Test

7) Generate Prediction

Test Failed?
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Quantile-Quantile Plot Example

![Quantile-Quantile Plot Example](chart.png)
Chi-Squared Test for Block Size 100

Bins: 100
Collapsed Bins: 62

\[ \chi^2 = 236 \]

Critical value for 59 degrees of freedom at \( p=0.05 \) is 77.93

Reject
Chi-Squared Test for Block Size 400

Bins: 25
Collapsed Bins: 19
\[ \chi^2 = 19.67 \]

Critical value for 16 degrees of freedom at p=0.05 is 26.3

Do Not Reject
Single Task Results

![Graph showing Single Task Results]

Expected

Measured

Trace Data

WCET Exceedance Probability

$10^{-1}$

$10^{-2}$

$10^{-3}$

$10^{-4}$

$10^{-5}$

$10^{-6}$

$10^{-7}$

$50$ $60$ $70$ $80$ $90$ $100$ $110$ $120$ $130$ $140$ $150$

WCET (μs)
Validation Results

Fraction of Samples above Prediction

\( \text{Max Obs.} \quad p_e=10^{-4} \quad p_e=10^{-5} \quad p_e=10^{-6} \)

WCET Prediction

WCET Exceedance Probability

- Measured
- Expected
Conclusion

Applied algorithm to estimate WCET from execution time samples
  • Uses extreme value theory method of block maximum
  • Execution time data grouped into blocks
  • Block size increased until goodness-of-fit test passes

Validated predictions using additional execution time samples
  • Measured WCET exceedances agreed with predicted exceedences
  • Variation in WCET exceedences using EVT algorithm reduced over using maximum observed execution time as WCET.