Techniques for Dimensioning Aggregated Event Streams to Sustain Global Application's Demands

Lorenzo Di Gregorio
lorenzo.digregorio@lantiq.com

Lantiq Deutschland GmbH, Neubiberg, Germany

3rd Workshop on Mapping Applications on MPSoCs, 2010

Legal Information

Disclaimer

All opinions expressed here are those of the author individually and are not reflective or indicative of the opinions and positions of the author's employer.

Outline

1. **Overview and Motivation**
   - The problem of sustaining the performance demand
   - An introductory case study
2. **The achievable performance of event streams**
   - What are event streams
   - Modeling event streams using linear inequalities
   - Time and variability in event streams
3. **Empirical and analytical techniques**
   - Empirical determination of network calculus bounds
   - Analytical determination of sustained properties

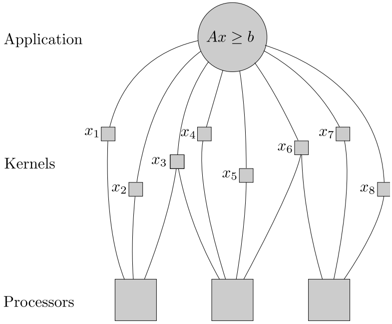
Outline

Overview

Problem

What level of performance is demanded on every processing element, so that they jointly satisfy the global demand?

- Application = interworking of multiple kernels.
- Demand on kernels for aggregated streams of computation events.
    - aggregated = many contributors to one performer.
- Demand on processing elements for performance in executing kernels.

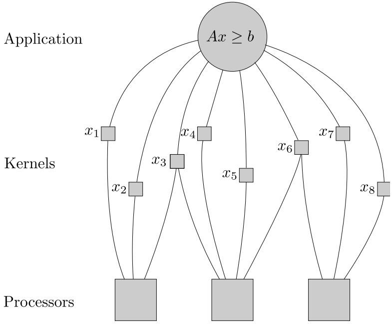


Overview

Problem

What level of performance is demanded on every processing element, so that they jointly satisfy the global demand?

- Application $=$ interworking of multiple kernels.
- Demand on kernels for aggregated streams of computation events.
  - aggregated = many contributors to one performer.
- Demand on processing elements for performance in executing kernels.



Motivation

It is a standard industry practice to solve such problems

- by incremental steps and gut feeling,
- by tabling scenarios on spreadsheets,
- by profiling and tweaking the design,
- sometimes, by developing abstract architecture models.

using mathematics is usually considered not feasible for this complexity and too cumbersome.

How complex is "too complex"?

Some quite generic problems are not as complex as they appear.

- Model many problems by parametric linear inequalities.
- Solve the "architectural part" of the problem only once.
- Apply the "parametric linear solution" to any variable data.

Motivation

It is a standard industry practice to solve such problems

- by incremental steps and gut feeling,
- by tabling scenarios on spreadsheets,
- by profiling and tweaking the design,
- sometimes, by developing abstract architecture models.

using mathematics is usually considered not feasible for this complexity and too cumbersome.

How complex is "too complex"?

Some quite generic problems are not as complex as they appear.

- Model many problems by parametric linear inequalities.
- Solve the "architectural part" of the problem only once.
- Apply the "parametric linear solution" to any variable data.

Motivation

It is a standard industry practice to solve such problems

- by incremental steps and gut feeling,
- by tabling scenarios on spreadsheets,
- by profiling and tweaking the design,
- sometimes, by developing abstract architecture models.

using mathematics is usually considered not feasible for this complexity and too cumbersome.

How complex is "too complex"?

Some quite generic problems are not as complex as they appear.

- Model many problems by parametric linear inequalities.
- Solve the "architectural part" of the problem only once.
- Apply the "parametric linear solution" to any variable data.

Motivation

It is a standard industry practice to solve such problems

- by incremental steps and gut feeling,
- by tabling scenarios on spreadsheets,
- by profiling and tweaking the design,
- sometimes, by developing abstract architecture models.

using mathematics is usually considered not feasible for this complexity and too cumbersome.

How complex is "too complex"?

Some quite generic problems are not as complex as they appear.

- Model many problems by parametric linear inequalities.
- Solve the "architectural part" of the problem only once.
- Apply the "parametric linear solution" to any variable data.

MOTIVATION

It is a standard industry practice to solve such problems

- by incremental steps and gut feeling,
- by tabling scenarios on spreadsheets,
- by profiling and tweaking the design,
- sometimes, by developing abstract architecture models.

using mathematics is usually considered not feasible for this complexity and too cumbersome.

HOW COMPLEX IS "TOO COMPLEX"?

Some quite generic problems are not as complex as they appear.

- Model many problems by parametric linear inequalities.
- Solve the "architectural part" of the problem only once.
- Apply the "parametric linear solution" to any variable data.

MOTIVATION

It is a standard industry practice to solve such problems

- by incremental steps and gut feeling,
- by tabling scenarios on spreadsheets,
- by profiling and tweaking the design,
- sometimes, by developing abstract architecture models.

using mathematics is usually considered not feasible for this complexity and too cumbersome.

HOW COMPLEX IS "TOO COMPLEX"?

Some quite generic problems are not as complex as they appear.

- Model many problems by parametric linear inequalities.
- Solve the "architectural part" of the problem only once.
- Apply the "parametric linear solution" to any variable data.

Motivation

It is a standard industry practice to solve such problems

- by incremental steps and gut feeling,
- by tabling scenarios on spreadsheets,
- by profiling and tweaking the design,
- sometimes, by developing abstract architecture models.

using mathematics is usually considered not feasible for this complexity and too cumbersome.

How complex is "too complex"?

Some quite generic problems are not as complex as they appear.

- Model many problems by parametric linear inequalities.
- Solve the "architectural part" of the problem only once.
- Apply the "parametric linear solution" to any variable data.

MOTIVATION

It is a standard industry practice to solve such problems

- by incremental steps and gut feeling,
- by tabling scenarios on spreadsheets,
- by profiling and tweaking the design,
- sometimes, by developing abstract architecture models.

using mathematics is usually considered not feasible for this complexity and too cumbersome.

HOW COMPLEX IS "TOO COMPLEX"?

Some quite generic problems are not as complex as they appear.

- Model many problems by parametric linear inequalities.
- Solve the "architectural part" of the problem only once.
- Apply the "parametric linear solution" to any variable data.

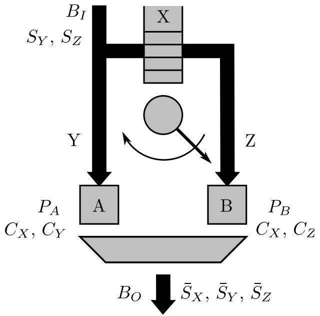
Outline

An introductory case study

Question

Under sufficient load, how many instructions per unit time, P_A and P_B, are demanded from A and B to guarantee that the bandwidth B_O of the output channel is exhausted?

An input channel delivers frame streams Y and Z to two cores A and B, which sustain a fairly distributed traffic flow X from a queue of packets. All the processed streams X, Y and Z are multiplexed into a common output channel.

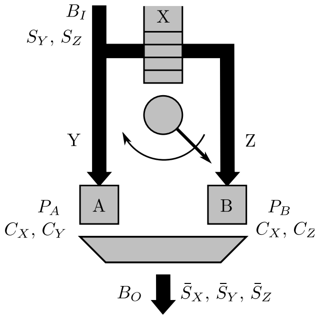


An introductory case study

Question

Under sufficient load, how many instructions per unit time, P_A and P_B, are demanded from A and B to guarantee that the bandwidth B_O of the output channel is exhausted?

An input channel delivers frame streams Y and Z to two cores A and B, which sustain a fairly distributed traffic flow X from a queue of packets. All the processed streams X, Y and Z are multiplexed into a common output channel.

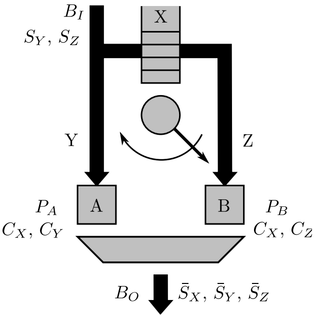


AN INTRODUCTORY CASE STUDY

QUESTION

Under sufficient load, how many instructions per unit time, P_A and P_B, are demanded from A and B to guarantee that the bandwidth B_O of the output channel is exhausted?

- Processing a frame of Y costs up to C_Y instructions and changes its maximum size from S_Y to $\bar{S}_Y$.
- Similarly a frame of Z costs C_Z and changes from S_Z to $\bar{S}_Z$, a packet of X costs C_X and changes from S_X to $\bar{S}_X$
- The input channel guarantees a bandwidth B_I.



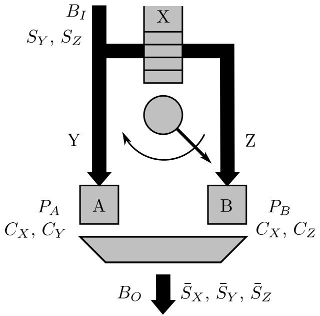
An introductory case study

Question

Under sufficient load, how many instructions per unit time, P_A and P_B, are demanded from A and B to guarantee that the bandwidth B_O of the output channel is exhausted?

$$\begin{aligned} S_Y x_Y + S_Z x_Z &\geq B_I \\ C_X x_X + C_Y x_Y &\geq P_A \\ C_X x_X + C_Z x_Z &\geq P_B \\ \bar{S}_X x_X + \bar{S}_Y x_Y + \bar{S}_Z x_Z &\geq B_O \end{aligned}$$

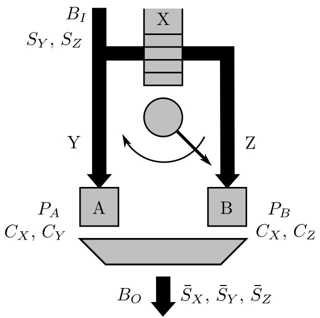
x_Y and x_Z represent respectively the total frames per unit time of the streams Y and Z. Similairly, x_X represents the total packets per unit time of the flow X.



Question

Under sufficient load, how many instructions per unit time, P_A and P_B, are demanded from A and B to guarantee that the bandwidth B_O of the output channel is exhausted?

$$\begin{aligned} S_Y x_Y + S_Z x_Z &\geq B_I \\ C_X x_X + C_Y x_Y &\geq P_A \\ C_X x_X + C_Z x_Z &\geq P_B \\ \bar{S}_X x_X + \bar{S}_Y x_Y + \bar{S}_Z x_Z &\geq B_O \end{aligned}$$



Answer

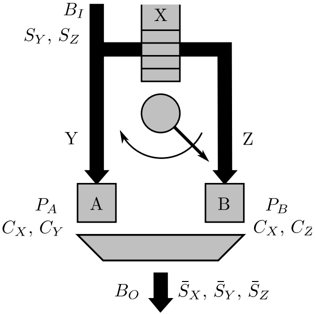
we must determine P_A and P_B such that the last inequality is satisfied for all x_X, x_Y and x_Z.

AN INTRODUCTORY CASE STUDY

QUESTION

Under sufficient load, how many instructions per unit time, P_A and P_B, are demanded from A and B to guarantee that the bandwidth B_O of the output channel is exhausted?

$$
\begin{aligned}
S_Y x_Y + S_Z x_Z &\geq B_I \\
C_X x_X + C_Y x_Y &\geq P_A \\
C_X x_X + C_Z x_Z &\geq P_B \\
\bar{S}_X x_X + \bar{S}_Y x_Y + \bar{S}_Z x_Z &\geq B_O
\end{aligned}
$$



ANSWER

we must determine P_A and P_B such that the last inequality is satisfied for all x_X, x_Y and x_Z.

Solution

The inequality $\bar{S}_X x_X + \bar{S}_Y x_Y + \bar{S}_Z x_Z \geq B_O$ is redundant if:

$$[-\frac{C_Z}{S_Z}K - \frac{\bar{S}_Z}{S_Z}, -K - \frac{\bar{S}_X}{C_X}, K, 1] \begin{bmatrix} B_I \\ P_A \\ P_B \\ B_O \end{bmatrix} \leq 0$$

where

$$K \doteq \frac{\bar{S}_Y - \bar{S}_X \frac{C_Y}{C_X} - S_Y \frac{\bar{S}_Z}{S_Z}}{C_Y + S_Y \frac{C_Z}{S_Z}}$$

Before proceeding with the math ...

What can be modeled by linear inequalities?

Outline

1. Overview and Motivation
   - The problem of sustaining the performance demand
   - An introductory case study
2. The achievable performance of event streams
   - What are event streams
   - Modeling event streams using linear inequalities
   - Time and variability in event streams
3. Empirical and analytical techniques
   - Empirical determination of network calculus bounds
   - Analytical determination of sustained properties

WHAT ARE EVENT STREAMS

- An event is the crossing of one control flow through one selected device and instruction. Example: one function call.
- The type of an event is its equivalence class with respect to multiple objectives. Example: all lookups in a table.
- A stream is a counter over time of events of the same type.

A stream does not need to be localized, events can be anywhere.

QUESTION

What aggregations can be modeled by linear inequalities?

$$\alpha_1 x_1 + \cdots + \alpha_n x_n \geq \beta$$

WHAT ARE EVENT STREAMS

- An event is the crossing of one control flow through one selected device and instruction. Example: one function call.
- The type of an event is its equivalence class with respect to multiple objectives. Example: all lookups in a table.
- A stream is a counter over time of events of the same type.

A stream does not need to be localized, events can be anywhere.

QUESTION

What aggregations can be modeled by linear inequalities?

$$\alpha_1 x_1 + \cdots + \alpha_n x_n \geq \beta$$

Outline

THE ACHIEVABLE PERFORMANCE REGION

AGGREGATED EVENT STREAMS AND THEIR MODELING USING LINEAR INEQUALITIES

QUESTION ...

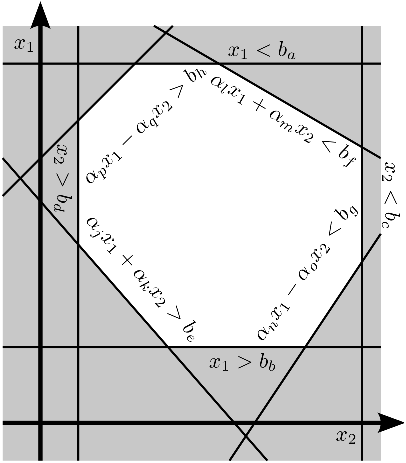
What aggregations can be modeled by linear inequalities?

$$Ax \geq b$$

- It is always an n-dimensional polytope.
- It must be convex, it can be open, it can imply equalities.

... TURNS INTO QUESTION

What do facets model?



THE ACHIEVABLE PERFORMANCE REGION

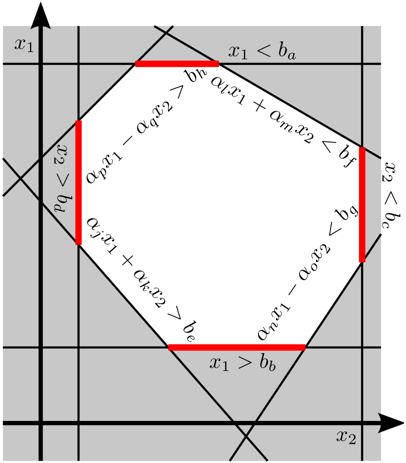
AGGREGATED EVENT STREAMS AND THEIR MODELING USING LINEAR INEQUALITIES

INDIVIDUAL WORST & BEST

$$b_b < x_1 < b_a$$

$$b_d < x_2 < b_c$$

- Absolute best and worst cases for the event streams.
- Worst cases can be caused by bottlenecks like channel capacity.
- Best cases can be intended "under available workload".



The achievable performance region

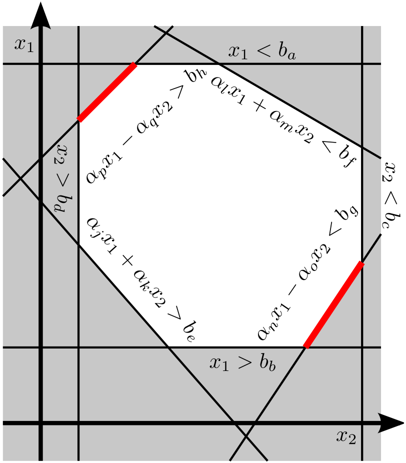
Aggregated event streams and their modeling using linear inequalities

Max & Min Shift

$$\alpha_n x_1 - \alpha_o x_2 < b_g$$

$$\alpha_p x_1 - \alpha_q x_2 > b_h$$

- One stream cannot be increased above a given limit, without increasing also the other one.
- Reasons can be data hazards, but also workload characteristics like frame types in an MPEG stream.



The achievable performance region

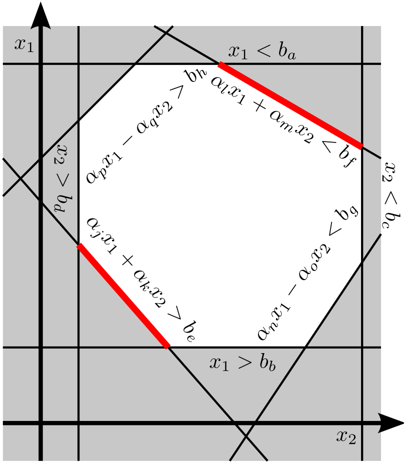
Aggregated event streams and their modeling using linear inequalities

Common Worst & Best

$$\alpha_j x_1 + \alpha_k x_2 > b_e$$

$$\alpha_l x_1 + \alpha_m x_2 < b_f$$

- Upper limit of the system's performance under peak load and improvements over the individual worst cases.
- The system will process more events of one type if it less loaded from events of the other type.



THE ACHIEVABLE PERFORMANCE REGION

AGGREGATED EVENT STREAMS AND THEIR MODELING USING LINEAR INEQUALITIES

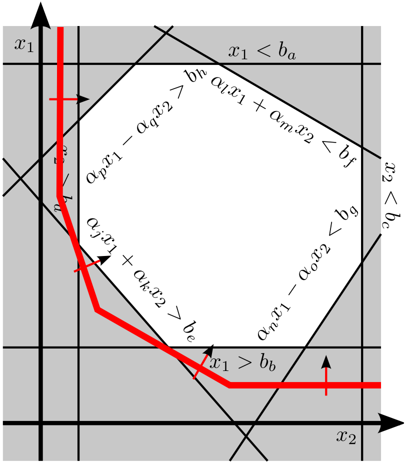
GUARANTEE & LIMITATION

$$Ax \geq b \rightsquigarrow D^{+}x \leq r^{+}$$

$$Ax \geq b \rightsquigarrow D^{\pm}x \geq r^{+}$$

$\rightsquigarrow$ means "is a dependency set for the infeasibility of"

- The demand is guaranteed: $D^{+}x \geq r^{+}$ is always satisfied.
- The limitation is never violated: $D^{\pm}x \leq r^{+}$ is always satisfied.



The achievable performance region

Aggregated event streams and their modeling using linear inequalities

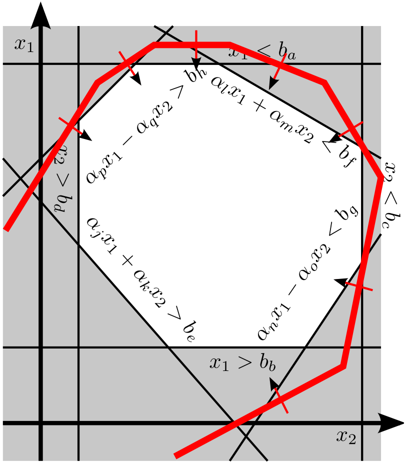
Guarantee & Limitation

$$Ax \geq b \rightsquigarrow D^{+}x \leq r^{+}$$

$$Ax \geq b \rightsquigarrow D^{\pm}x \geq r^{+}$$

$\rightsquigarrow$ means "is a dependency set for the infeasibility of"

- The demand is guaranteed: $D^{+}x \geq r^{+}$ is always satisfied.
- The limitation is never violated: $D^{\pm}x \leq r^{+}$ is always satisfied.



Outline

Performance over time

Average performance at module level is not indicative.

- "phase behaviors" play a major role in programs.
- Applications settle down on these phases.

Disregarding such effects leads to grossly wrong estimations: the bounds for the event counters must be time dependent.

Performance under variability

Effects difficult to model in simulations:

- Same code paths run at different performance levels.
- Same initial states lead to different code paths.

Do not use averages, use confidence intervals.

Performance over time

Average performance at module level is not indicative.

- “phase behaviors” play a major role in programs.
- Applications settle down on these phases.

Disregarding such effects leads to grossly wrong estimations: the bounds for the event counters must be time dependent.

Performance under variability

Effects difficult to model in simulations:

- Same code paths run at different performance levels.
- Same initial states lead to different code paths.

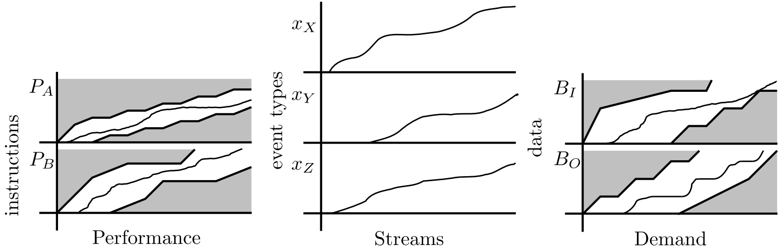
Do not use averages, use confidence intervals.

Performance over time under variability

Defining the performance region using network calculus bounds

Fundamental question

Which performance bounds satisfy the demand bounds?



Network calculus bounds

Performance is characterized within a time dependent confidence interval by network calculus bounds.

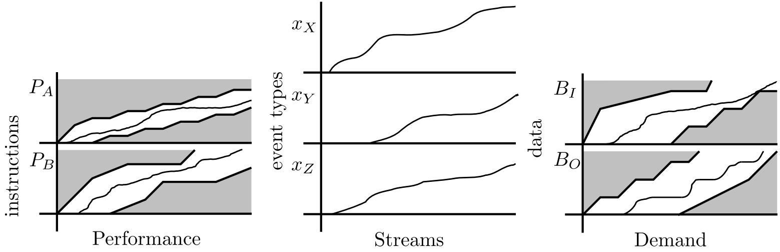
- generalization to generic events: real-time calculus.

Performance over time under variability

Defining the performance region using network calculus bounds

Fundamental question

Which performance bounds satisfy the demand bounds?



Network calculus bounds

Performance is characterized within a time dependent confidence interval by network calculus bounds.

- generalization to generic events: real-time calculus.

Dimensioning Event Streams

Di Gregorio

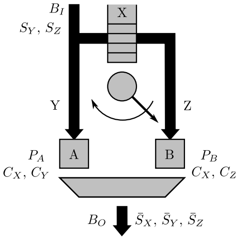
Motivation
- Problem
- Example

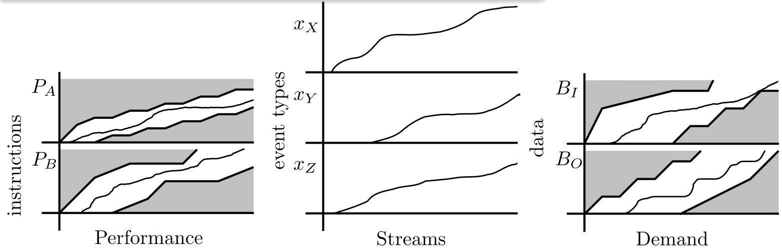
Event Streams
- Definition
- Modeling
- Bounds over time

Techniques
- Empirical
- Analytical

Analytical model

$$\left[-\frac{C_Z}{S_Z}K - \frac{\bar{S}_Z}{S_Z}, -K - \frac{\bar{S}_X}{C_X}, K, 1\right] \begin{bmatrix} B_I \\ P_A \\ P_B \\ B_O \end{bmatrix} \leq 0$$





Outline

Empirical determination

Sliding window under load

For every time window, determine

- the highest increment: arrival curve
- the lowest increment under load: service curve

Complex determination

- must update all sliding windows at every simulation step
- must identify and skip idle conditions

Piecewise linear profiles

Rather than cycle by cycle, characterize event by event and assume linear growth between events.

- calculate and round intersections between linear pieces
- avoid accumulating rounding errors during incremental steps

Outline

1. Overview and Motivation
   - The problem of sustaining the performance demand
   - An introductory case study
2. The achievable performance of event streams
   - What are event streams
   - Modeling event streams using linear inequalities
   - Time and variability in event streams
3. Empirical and analytical techniques
   - Empirical determination of network calculus bounds
   - Analytical determination of sustained properties

Analytical determination

Problem

$$\begin{array}{rcl} S_Y x_Y + S_Z x_Z & \geq & B_I \\ C_X x_X + C_Y x_Y & \geq & P_A \\ C_X x_X + C_Z x_Z & \geq & P_B \\ \bar{S}_X x_X + \bar{S}_Y x_Y + \bar{S}_Z x_Z & \geq & B_O \end{array}$$

Solution

$$[-\frac{C_Z}{S_Z}K - \frac{\bar{S}_Z}{S_Z}, -K - \frac{\bar{S}_X}{C_X}, K, 1] \begin{bmatrix} B_I \\ P_A \\ P_B \\ B_O \end{bmatrix} \leq 0$$

Polar cone of the dual space of the inconsistent problem

$$\begin{bmatrix} 0 & C_X & C_X & \bar{S}_X \\ S_Y & C_Y & 0 & \bar{S}_Y \\ S_Z & 0 & C_Z & \bar{S}_Z \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = 0, \forall \pi_1 \geq 0, \pi_2 \geq 0, \pi_3 \geq 0, \pi_4 < 0$$

- the dual space is a line: 3 equations in four variables
- the polar cone is a halfspace, the parameters vector must remain out of that halfspace: in this case just sign inversion.

Analytical determination

A convex hull problem

Determining the halfspace representation of the prohibited regions for the parameters is a convex hull problem.

- simplified by the null space, but still complex.
- solved by double description method (CDD by Fukuda).
- performance strictly depending on the problem formulation.
- as a rule of thumb, in general no more than 20 variables.

Inverse problem: which demand can be sustained?

Determine the space of redundant inequalities.

- employ the subsumption cone by Lassez
- solve by Fourier-Motzkin elimination (complex!)

Example on the next slide.

Analytical determination

Two threads with overhead

Thread 1 guarantees a service b_1, thread 2 guarantees a service b_2. Either thread 1 or 2 is active, so $b_3 > b_1 + b_2$. Which demand $\alpha_1 x_1 + \alpha_2 x_2 \geq \beta$ can be sustained?

As inequality system $Ax \geq b$

$$\begin{array}{rrcl} & +x_1 & \geq & b_1 \\ x_2 & & \geq & b_2 \\ x_2 & +x_1 & \geq & b_3 \end{array}$$

Subsumption cone:
$\{\pi A - \alpha = 0, \pi b - \beta \geq 0, \pi \geq 0\}$

Subsumption cone

$$\begin{array}{rrrrcl} & \pi_2 & +\pi_3 & -\alpha_1 & = & 0 \\ \pi_1 & & +\pi_3 & -\alpha_2 & = & 0 \\ b_1\pi_1 & +b_2\pi_2 & +b_3\pi_3 & -\beta & \geq & 0 \\ \pi_1 & & & & \geq & 0 \\ & \pi_2 & & & \geq & 0 \\ & & \pi_3 & & \geq & 0 \end{array}$$

Fourier-Motzkin elimination

$$\begin{array}{rcl} \pi_3(b_3 - b_2 - b_1) + \alpha_1 b_2 + \alpha_2 b_1 - \beta & \geq & 0 \\ -\pi_3 & \geq & \alpha_1 \\ -\pi_3 & \geq & -\alpha_2 \\ \pi_3 & \geq & 0 \end{array}$$

The coefficient of π_3 is $b_3 - b_2 - b_1 \geq 0$ per construction.

Analytical determination

Two threads with overhead

Thread 1 guarantees a service b_1, thread 2 guarantees a service b_2. Either thread 1 or 2 is active, so $b_3 > b_1 + b_2$. Which demand $\alpha_1 x_1 + \alpha_2 x_2 \geq \beta$ can be sustained?

As inequality system $Ax \geq b$

$$\begin{array}{ccccc} & +x_1 & \geq & b_1 \\ x_2 & & \geq & b_2 \\ x_2 & +x_1 & \geq & b_3 \end{array}$$

Subsumption cone:
$\{\pi A - \alpha = 0, \pi b - \beta \geq 0, \pi \geq 0\}$

Subsumption cone

$$\begin{array}{cccccc} & \pi_2 & +\pi_3 & -\alpha_1 & = & 0 \\ \pi_1 & & +\pi_3 & -\alpha_2 & = & 0 \\ b_1\pi_1 & +b_2\pi_2 & +b_3\pi_3 & -\beta & \geq & 0 \\ \pi_1 & & & & \geq & 0 \\ & \pi_2 & & & \geq & 0 \\ & & \pi_3 & & \geq & 0 \end{array}$$

Fourier-Motzkin elimination

$$\begin{array}{rcl} \pi_3(b_3 - b_2 - b_1) + \alpha_1 b_2 + \alpha_2 b_1 - \beta & \geq & 0 \\ -\pi_3 & \geq & \alpha_1 \\ -\pi_3 & \geq & -\alpha_2 \\ \pi_3 & \geq & 0 \end{array}$$

The coefficient of π_3 is $b_3 - b_2 - b_1 \geq 0$ per construction.

Analytical determination

Two threads with overhead

Thread 1 guarantees a service b_1, thread 2 guarantees a service b_2. Either thread 1 or 2 is active, so $b_3 > b_1 + b_2$. Which demand $\alpha_1 x_1 + \alpha_2 x_2 \geq \beta$ can be sustained?

As inequality system $Ax \geq b$

$$\begin{array}{rrcl} & +x_1 & \geq & b_1 \\ x_2 & & \geq & b_2 \\ x_2 & +x_1 & \geq & b_3 \end{array}$$

Subsumption cone:
$\{\pi A - \alpha = 0, \pi b - \beta \geq 0, \pi \geq 0\}$

Sustainable demand space

$$\begin{array}{rcl} -\alpha_2 b_1 - \alpha_1 (b_3 - b_1) + \beta & \leq & 0 \\ -\alpha_2 (b_3 - b_2) - \alpha_1 b_2 + \beta & \leq & 0 \\ \alpha_1 & \geq & 0 \\ \alpha_2 & \geq & 0 \end{array}$$

This presentation terminates here

Thank you!

Lorenzo Di Gregorio
lorenzo.digregorio@lantiq.com

For Further Reading I

Komei Fukuda and Alain Prodon.
The double description method revisited.

Harvey J. Greenberg.
Consistency, redundancy, and implied equalities in linear systems.
Annals of Mathematics and Artificial Intelligence, vol. 17, pp. 37–83, 1996

Alaa R. Alameldeen and David A. Wood.
Variability in Architectural Simulations of Multithreaded Workloads
Proceedings of the Ninth IEEE Symposium on High-Performance Computer Architecture, 2003