Resource management through type inference

Dan R. Ghica

Synchron 2010

Joint work with Alex Smith
(to appear at POPL 2011)

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Geometry of Interaction

JY Girard

GoI
1989

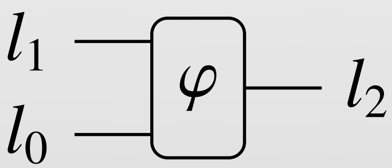
S Abramsky

New foundations
1992

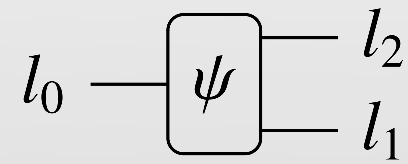
I Mackie

Token machines
1995

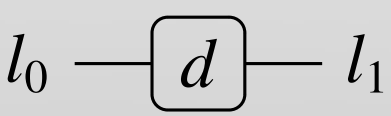
GoI Components



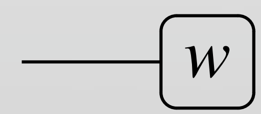
$$\begin{aligned}
\varphi(l_0, n) &= (l_2, pn)\\
\varphi(l_1, n) &= (l_2, qn)\\
\varphi(l_2, pn) &= (l_0, n)\\
\varphi(l_2, qn) &= (l_1, n)
\end{aligned}$$



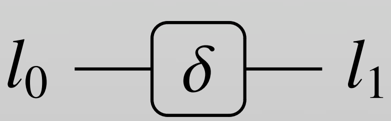
$$\begin{aligned}
\psi(l_0, pn) &= (l_1, n)\\
\psi(l_0, qn) &= (l_2, n)\\
\psi(l_1, n) &= (l_0, pn)\\
\psi(l_2, n) &= (l_0, qn)
\end{aligned}$$



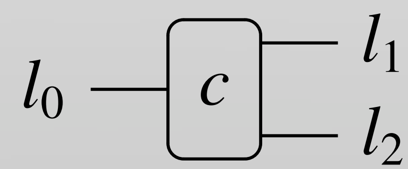
$$\begin{aligned}
d(l_0, \langle 0, n\rangle) &= (l_1, n)\\
d(l_0, \langle i+1, n\rangle) &= \text{undefined}\\
d(l_1, n) &= (l_0, \langle 0, n\rangle)
\end{aligned}$$



$$w = \emptyset$$



$$\begin{aligned}
\delta(l_0, \langle\langle i, j\rangle, n\rangle) &= (l_1, \langle i, \langle j, n\rangle\rangle)\\
\delta(l_1, \langle i, \langle j, n\rangle\rangle) &= (l_0, \langle\langle i, j\rangle, n\rangle)
\end{aligned}$$



$$\begin{aligned}
c(l_0, \langle pi, n\rangle) &= (l_1, \langle i, n\rangle)\\
c(l_0, \langle qi, n\rangle) &= (l_2, \langle i, n\rangle)\\
c(l_1, \langle i, n\rangle) &= (l_0, \langle pi, n\rangle)\\
c(l_2, \langle i, n\rangle) &= (l_0, \langle qi, n\rangle)
\end{aligned}$$

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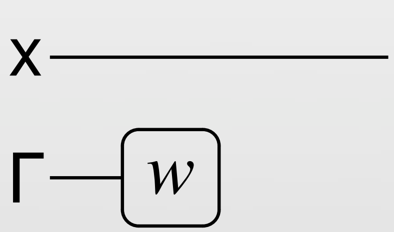
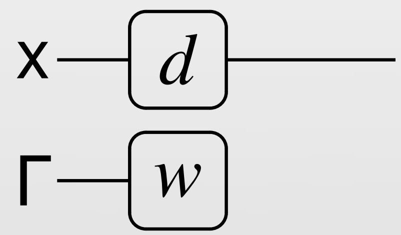
Standard GoI Interpretation

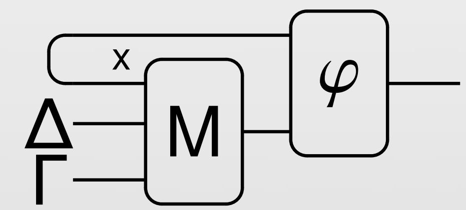
$\Theta \mid \Gamma; x : A \vdash x : A$



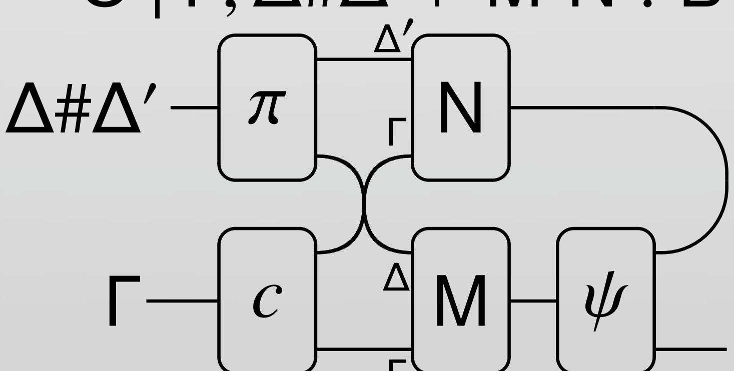
$\Theta \mid \Gamma, x : A; - \vdash x : A$



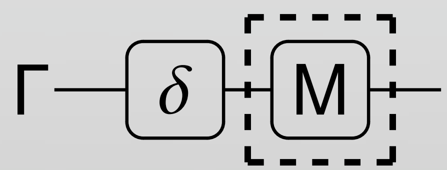
$\Theta \mid \Gamma; \Delta \vdash \lambda x^A.M : A \multimap B$



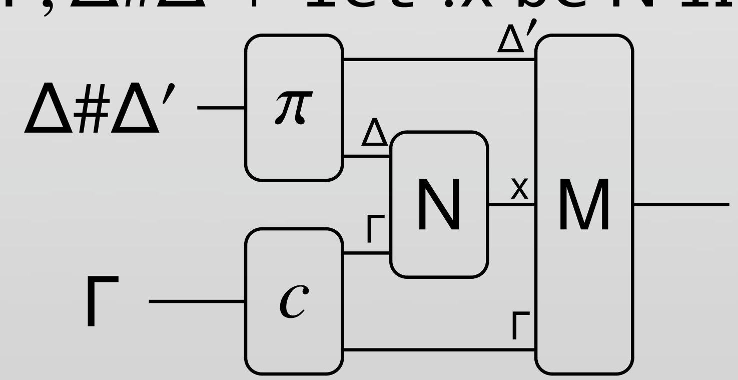
$\Theta \mid \Gamma; \Delta\#\Delta' \vdash M\ N : B$



$\Theta \mid \Gamma; - \vdash !M : !A$



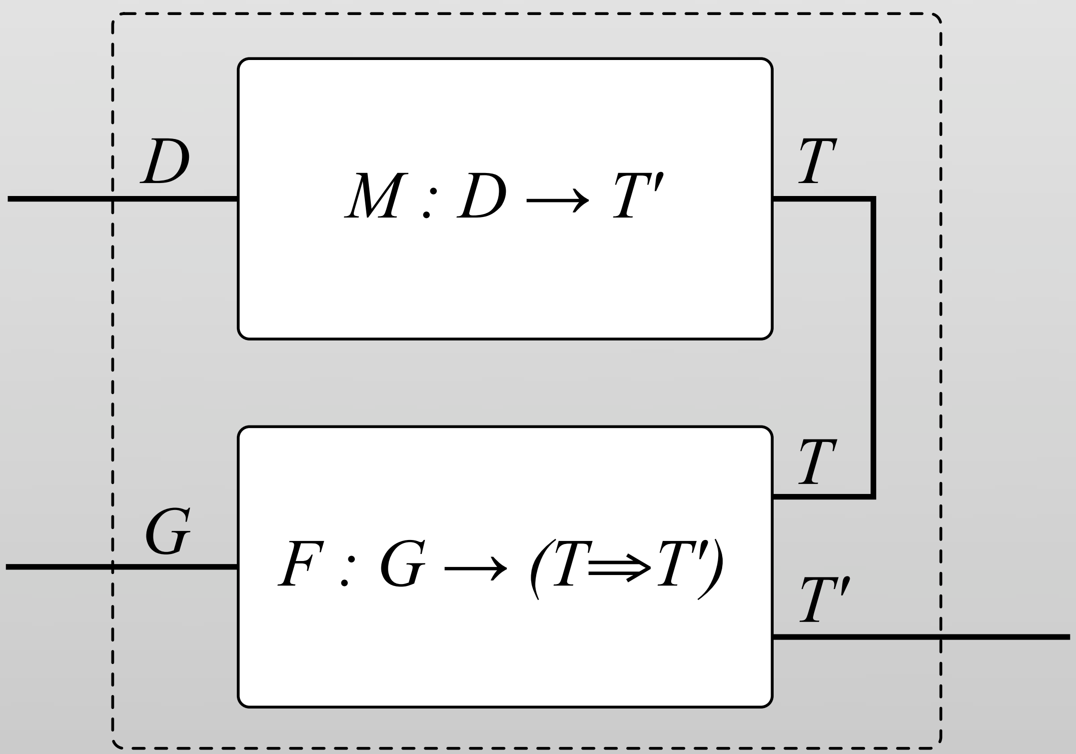
$\Theta \mid \Gamma; \Delta\#\Delta' \vdash \mathtt{let}\ !x\ \mathtt{be}\ N\ \mathtt{in}\ M : A$



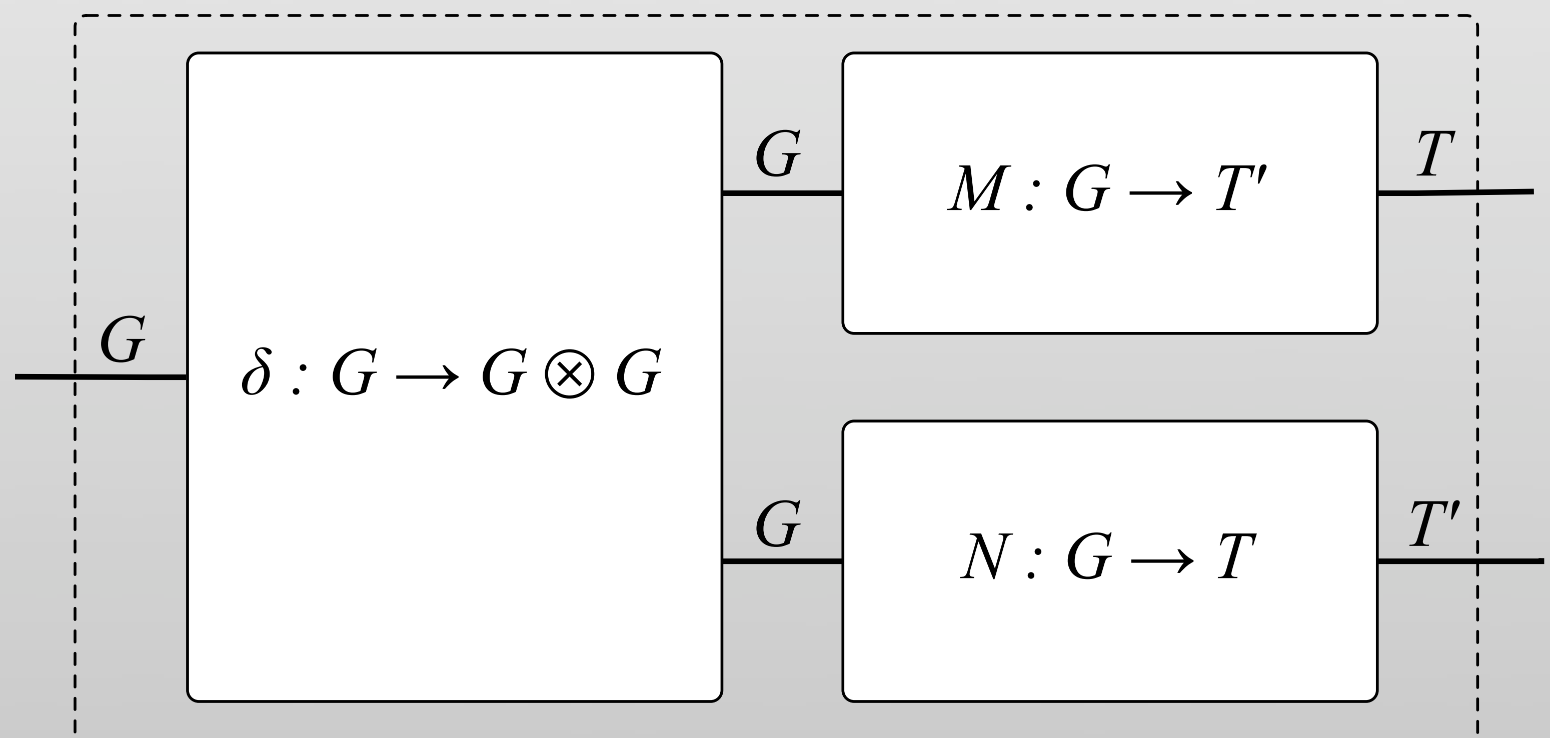
Geometry of Synthesis

$$\frac{\Gamma \vdash F : \theta' \to \theta \qquad \Delta \vdash M : \theta'}{\Gamma, \Delta \vdash FM : \theta} \to \text{Elimination}$$

$$\frac{\Gamma \vdash M : \theta' \qquad \Gamma \vdash N : \theta}{\Gamma \vdash \langle M, N \rangle : \theta' \times \theta} \times \text{Introduction}$$



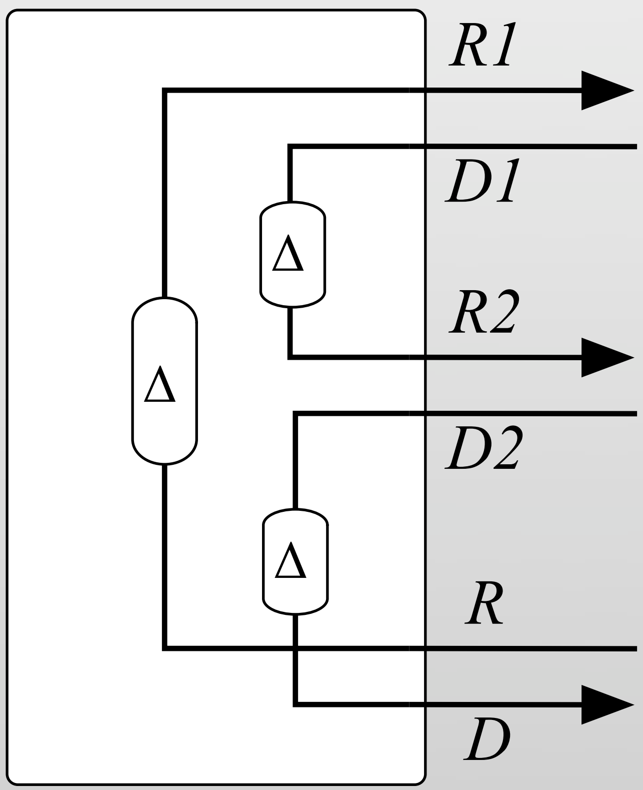
$FM : G \otimes D \to T'$



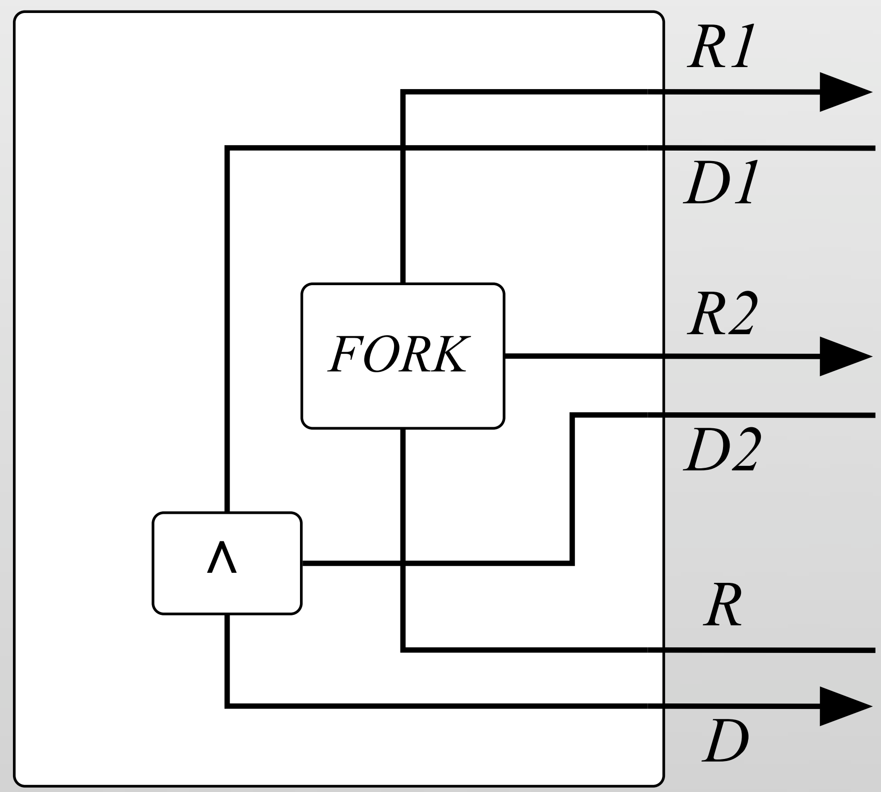
$<M,N> : G \to T \otimes T'$

Ghica, POPL 2007

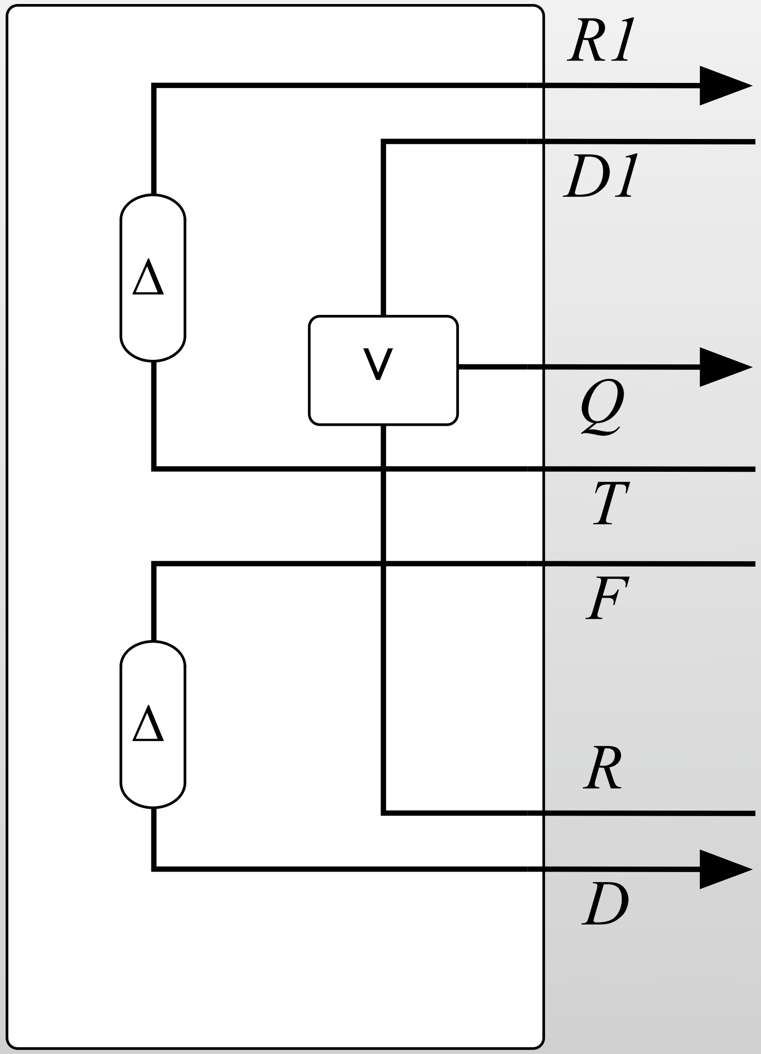
Imperative constants in GoS



$seq : I \to com \otimes com \Rightarrow com$



$par : I \to com \Rightarrow com \Rightarrow com$



$while : I \to bool \otimes com \Rightarrow com$

Features of GoS

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- finite state IAMs (cf GoI)
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- timing independent
  - compatible with asynchronous (GoS II @ MFPS '10)
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- timing independent
  - compatible with asynchronous (GoS II @ MFPS '10)
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- full functional interface
  - FFI, ABI

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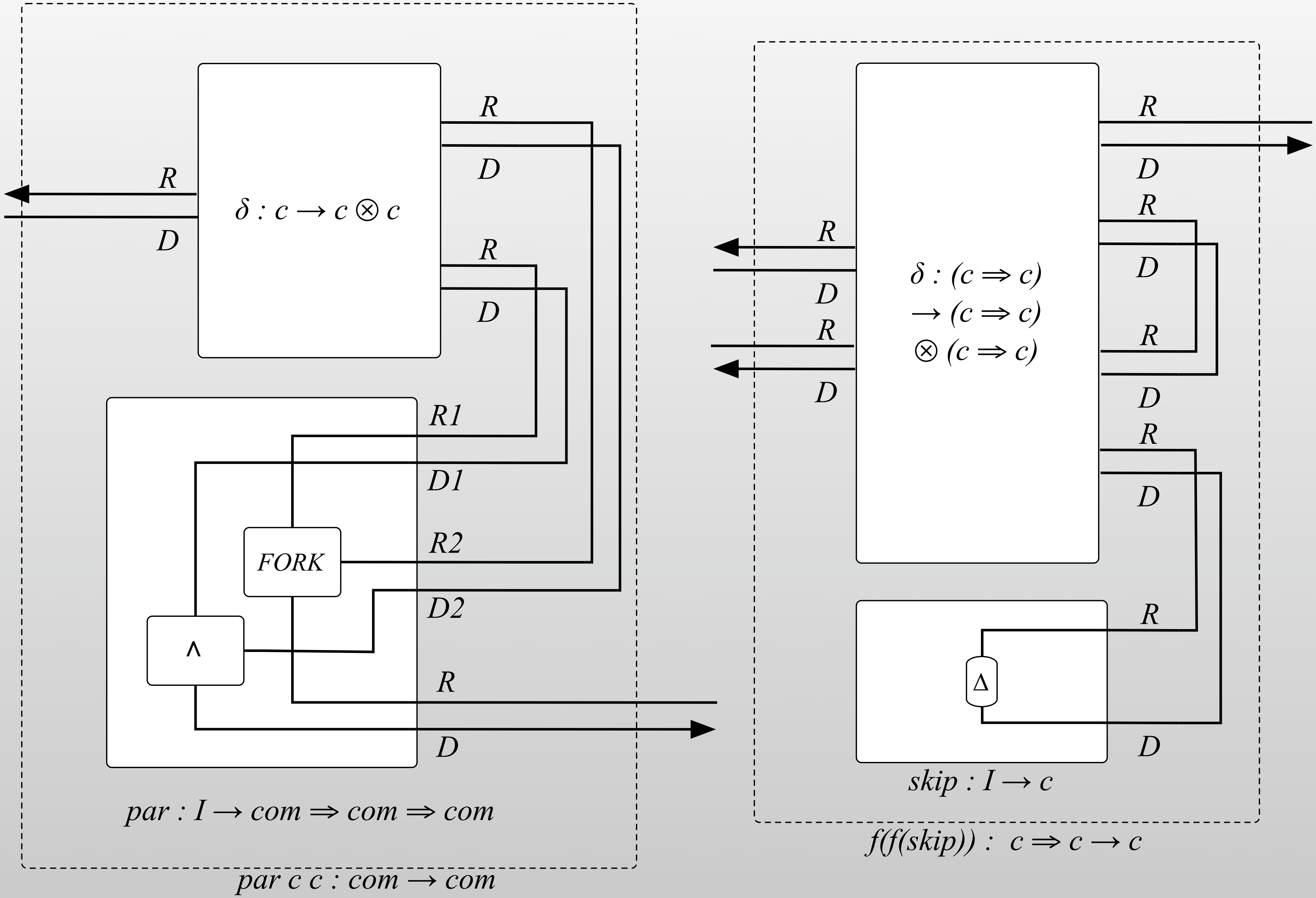
Limitations of GoS



Can't be implemented finite-state.

GMO @ APAL'08

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Solution: systematic replication

$$(\lambda f \lambda x. f x || f x)(\lambda c. c || c)$$

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$$\Downarrow$$

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$$\Downarrow$$

$$(\lambda f_1 f_2 \lambda x_1 x_2 x_3 x_4 . f_1 x_1 x_2 || f_2 x_3 x_4)(\lambda c_1 c_2 . c_1 || c_2)$$

Type-directed transformation

$$\frac{}{x : \theta \vdash x : \theta}\ \text{Identity}$$

$$\frac{\Gamma \vdash M : \theta}{\Gamma, x : \theta' \vdash M : \theta}\ \text{Weakening}$$

$$\frac{\Gamma, x : \theta, y : \theta \vdash M : \theta'}{\Gamma, x : \theta \vdash M[x/y] : \theta'}\ \text{Contraction}$$

$$\frac{\Gamma, x : \theta' \vdash M : \theta}{\Gamma \vdash \lambda x.M : \theta' \to \theta}\ \text{Abstraction}$$

$$\frac{\Gamma \vdash M : \theta \to \theta' \qquad \Gamma \vdash N : \theta}{\Gamma \vdash MN : \theta'}\ \text{Application}$$

$$\frac{\Gamma \vdash M_i : \theta_i}{\Gamma \vdash \langle M_1, M_2 \rangle : \theta_1 \times \theta_2}\ \text{Product}$$

$$\frac{\Gamma \vdash M : \theta \to \theta' \qquad \Gamma' \vdash N : \theta}{\Gamma, \Gamma' \vdash MN : \theta'}\ \text{Application}$$

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$$\frac{\Gamma, x : \theta^m, y : \theta^n \vdash M : \theta'}{\Gamma, x : \theta^{m+n} \vdash M[x/y] : \theta'}\ \text{Contraction (new)}$$

$$\frac{\Gamma \vdash M : \theta^n \to \theta' \qquad \Gamma' \vdash N : \theta}{\Gamma, n \cdot \Gamma' \vdash MN : \theta'}\ \text{Application (new)}$$

$$\frac{\Gamma \vdash M : \theta \qquad \theta \leq \theta'}{\Gamma \vdash M : \theta'}\ \text{Subtyping}$$

$$\frac{\Gamma \vdash M : \theta \to \theta' \qquad \Gamma' \vdash N : \theta}{\Gamma, \Gamma' \vdash MN : \theta'}\ \text{Application}$$

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Step 1: Inferring bounds

The key rules are for contraction

$$\frac{\Gamma', x : \theta^{n_1}, y : \theta^{n_2} \vdash M : \theta' \blacktriangleright C}{\Gamma, x : \theta^{n} \vdash M[x/y] : \theta' \blacktriangleright |\Gamma = \Gamma'| \wedge n \geq n_1 + n_2}$$

and application

$$\frac{\Gamma \vdash M : \theta_1^k \to \theta_2 \blacktriangleright C \qquad \Delta \vdash N : (\theta_1)' \blacktriangleright C'}{\Gamma', \Delta' \vdash MN : \theta_2' \blacktriangleright |\Delta \leq \Delta'| \wedge |\Gamma = \Gamma'| \wedge \left(\bigwedge_{\substack{x:\theta^n \in \Delta \\ x:\theta^{n'} \in \Delta'}} n' \geq k \cdot n \right)}$$

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Example

$$\lambda f x. f(f(x))$$

$$\dfrac{\dfrac{\dfrac{}{f : (\mathsf{com}^{n_1} \to \mathsf{com})^{n_4} \vdash f : (\mathsf{com}^{n_1} \to \mathsf{com}) \blacktriangleright n_4 \geq 1} \qquad \dfrac{\dfrac{}{g : (\mathsf{com}^{n_6} \to \mathsf{com})^{n_7} \vdash g : \mathsf{com}^{n_6} \to \mathsf{com} \blacktriangleright n_7 \geq 1} \qquad \dfrac{}{x : \mathsf{com}^{n_9} \vdash x : \mathsf{com} \blacktriangleright n_9 \geq 1}}{g : (\mathsf{com}^{n_6} \to \mathsf{com})^{n_7}, x : \mathsf{com}^{n_8} \vdash g(x) : \mathsf{com} \blacktriangleright n_8 \geq n_6 \cdot n_9}}{\dfrac{f : (\mathsf{com}^{n_1} \to \mathsf{com})^{n_4}, g : (\mathsf{com}^{n_1} \to \mathsf{com})^{n_5}, x : \mathsf{com}^{n_3} \vdash f(g(x)) : \mathsf{com} \blacktriangleright n_5 \geq n_1 \cdot n_7 \wedge n_6 \geq n_1 \wedge n_3 \geq n_1 \cdot n_8}{f : (\mathsf{com}^{n_1} \to \mathsf{com})^{n_2}, x : \mathsf{com}^{n_3} \vdash f(f(x)) : \mathsf{com} \blacktriangleright n_2 \geq n_4 + n_5}}}{\vdash \lambda f \lambda x. f(f(x)) : (\mathsf{com}^{n_1} \to \mathsf{com})^{n_2} \to \mathsf{com}^{n_3} \to \mathsf{com}) \blacktriangleright \mathit{true}}$$

$$n_2 \geq n_4 + n_5 \wedge n_5 \geq n_1 \cdot n_7 \wedge n_6 \geq n_1 \wedge n_3 \geq n_1 \cdot n_8$$
$$\wedge\, n_8 \geq n_6 \cdot n_9 \wedge n_9 \geq 1 \wedge n_7 \geq 1 \wedge n_4 \geq 1$$

$$(n_1 = 2)$$

Solving the system

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- system is decidable and effectively solveable
    - not all programs can type

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$$\lambda q.(\lambda g.g(\lambda x.g(qx)))(\lambda b.(\lambda k.((k(\lambda u.u))(\lambda l.((kb)(\lambda t.(l(t\,\mathsf{skip})))))))(\lambda v.\lambda w.wv))$$

$$: (\mathsf{com}^{\mathsf{m}} \to \mathsf{com}^{\mathsf{n}} \to \mathsf{com})^{\mathsf{n}^2} \to \mathsf{com},$$

which types only if $m \leq 1$.

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Step 2: Serialization

$$\frac{\Gamma \vdash M : \theta_1 \Longmapsto \Gamma' \vdash M' : \overline{\theta_1} \qquad \theta_1 \leq \theta_2}{\Gamma \vdash M : \theta_2 \Longmapsto \Gamma \vdash \mathrm{subtype}_{\theta_1 \leq \theta_2}(M') : \overline{\theta_2}}$$

$$\frac{\begin{array}{l}\Gamma, x : \theta_1^{n_1}, y : \theta_1^{n_2} \vdash M : \theta_2 \\ \Longmapsto \Gamma', x_1 : \overline{\theta_1}, \ldots, x_{n_1} : \overline{\theta_1}, y_1 : \overline{\theta_1}, \ldots, y_{n_2} : \overline{\theta_1} \vdash M' : \overline{\theta_2}\end{array}}{\begin{array}{l}\Gamma, x : \theta_1^{n_1+n_2} \vdash M[x/y] : \theta_2 \\ \qquad \Longmapsto \Gamma', x_1 : \overline{\theta_1}, \ldots, x_{n_1+n_2} : \overline{\theta_1} \vdash \\ \qquad\qquad M[x_{n_1+1}/y_1] \cdots [x_{n_1+n_2}/y_{n_2}] : \overline{\theta_2}\end{array}}$$

$$\frac{\begin{array}{c}\Gamma \vdash M : \theta_1^n \to \theta_2 \Longmapsto \Gamma' \vdash M' : \overline{\theta_1^n \to \theta_2} \\ \Delta \vdash N : \theta_1 \Longmapsto \Delta' \vdash N' : \overline{\theta_1}\end{array}}{\begin{array}{l}\Gamma, n \cdot \Delta \vdash MN : \theta_2 \\ \Longmapsto \Gamma', \Delta'_1, \ldots, \Delta'_n \vdash M'(N'[\Delta'_1/\Delta']) \cdots (N'[\Delta'_n/\Delta']) : \overline{\theta_2}\end{array}}$$

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Serializing constants

$$\overline{\mathsf{newvar} : (\mathsf{var}^{\mathsf{n}} \longrightarrow \mathsf{com}) \longrightarrow \mathsf{com}}$$

$$= \mathsf{newvar}_{\mathsf{n}} : \overline{(\mathsf{var}^{\mathsf{n}} \longrightarrow \mathsf{com}) \longrightarrow \mathsf{com}}$$

$$= \mathsf{newvar}_{\mathsf{n}} : (\underbrace{\mathsf{var} \longrightarrow \cdots \longrightarrow \mathsf{var}}_{\text{n times}} \longrightarrow \mathsf{com}) \longrightarrow \mathsf{com}.$$

new *x* in *C* ⇔ newvar (λ*x*.*C*)

new3 *x1*, *x2*, *x3* in *C* ⇔ newvar3. (λ*x1*.*x2*.*x3*.*C*)

Example

$$g : (\mathsf{com}^2 \to \mathsf{com}) \vdash g \Longmapsto g : \mathsf{com} \to \mathsf{com} \to \mathsf{com} \vdash g \qquad x : \mathsf{com}^1 \vdash x \Longmapsto x : \mathsf{com} \vdash x$$

$$f : (\mathsf{com}^2 \to \mathsf{com})^1 \vdash f \Longmapsto f_3 : \mathsf{com} \to \mathsf{com} \to \mathsf{com} \vdash f_3 \qquad g : (\mathsf{com}^2 \to \mathsf{com}), x : \mathsf{com}^2 \vdash g(x) \Longmapsto g : \mathsf{com} \to \mathsf{com} \to \mathsf{com}, x_1, x_2 : \mathsf{com} \vdash g(x_1 x_2)$$

$$f : (\mathsf{com}^2 \to \mathsf{com})^1, g : (\mathsf{com}^2 \to \mathsf{com})^2, x : \mathsf{com}^4 \vdash f(fx) \Longmapsto f_3, g, g' : \mathsf{com} \to \mathsf{com} \to \mathsf{com}, x_1, x_2, x_1', x_2' : \mathsf{com} \vdash f_3(g x_1 x_2)(g' x_1' x_2')$$

$$f : (\mathsf{com}^2 \to \mathsf{com})^3, x : \mathsf{com}^4 \vdash f(fx) \Longmapsto f_3, f_1, f_2 : (\mathsf{com} \to \mathsf{com} \to \mathsf{com}) \to \mathsf{com}, x_1, x_2, x_1', x_2' : \mathsf{com} \vdash f_3(f_1 xx)(f_2 xx)$$

$$\vdash \lambda f x. f(f(x)) \Longmapsto \ \vdash \lambda f_3 f_2 f_1 x_1 x_2 x_1' x_2'. f_3(f_1 x_1 x_2)(f_2 x_1' x_2')$$

Comparisons

$f : \mathsf{com}^1 \longrightarrow \mathsf{com}.$

$$\lambda f x.f(fx){:}(\mathsf{com}^1{\to}\mathsf{com})^2{\to}\mathsf{com}^1{\to}\mathsf{com} \Longmapsto \lambda f_1 f_2 x.f_1(f_2 x)$$

$$\lambda f x.fx; fx{:}(\mathsf{com}^1{\to}\mathsf{com})^1{\to}\mathsf{com}^1{\to}\mathsf{com} \Longmapsto \lambda f x.fx; fx$$

$$\lambda f x.fx||fx{:}(\mathsf{com}^1{\to}\mathsf{com})^2{\to}\mathsf{com}^2{\to}\mathsf{com}$$

$$\Longmapsto \lambda f_1 f_2 x_1 x_2.f_1 x_1; f_2 x_2$$

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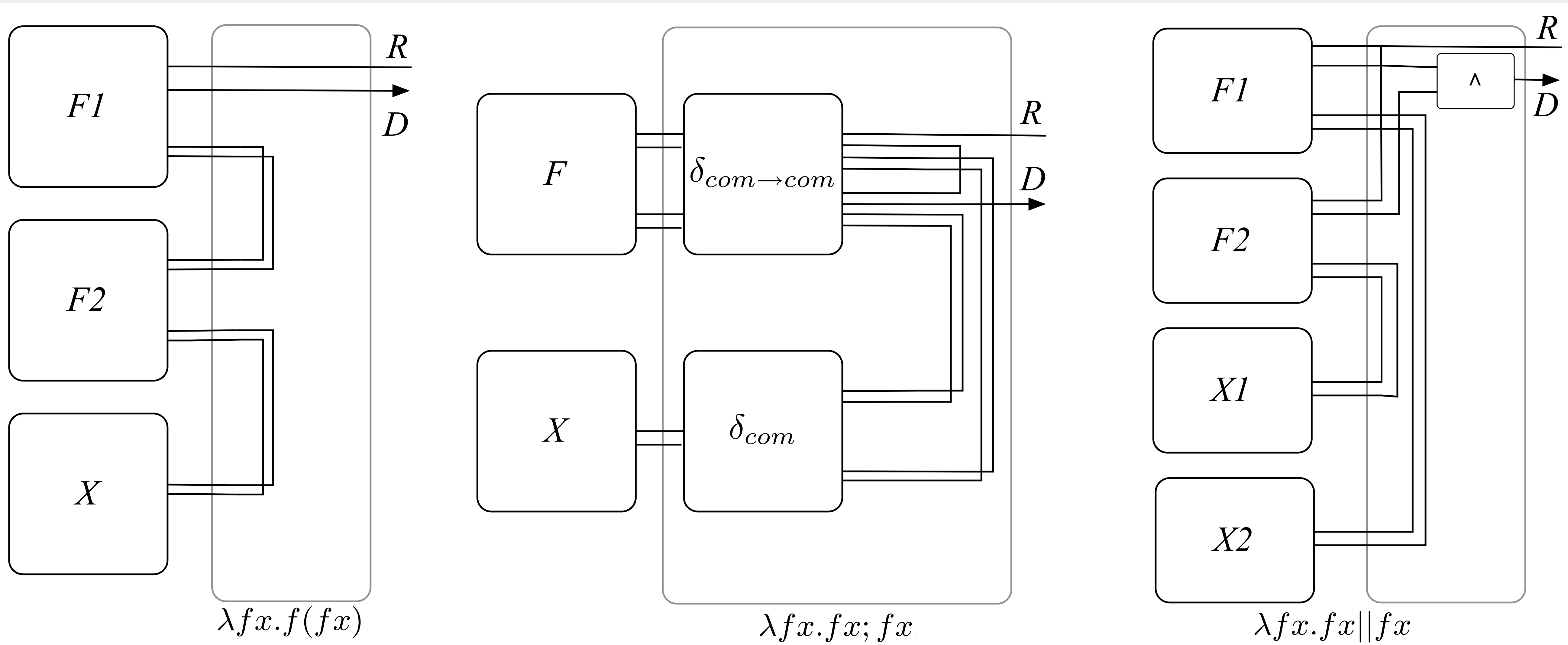

Conclusions

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- pipelining (ongoing, with A Smith)
- CBV, het'geneous computing (ongoing, with H Thielecke)

References

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