

Modular Compilation of a Synchronous Language

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Synchron 2010



Motivation

+ Synchronous languages are **model-driven** ⇒ [▶ see](#)

- Efficiency and reusability of system design
- Formal verification of system behavior

- Large size of models

Modular compilation

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 - Formal verification of system behavior
- Large size of models
- \Rightarrow **Modular compilation**

model-driven + modularity \Rightarrow global causality checking

- synchronous hypothesis \Rightarrow responsiveness.
- modularity
- global causality checking

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- Efficiency and reusability of system design
- Formal verification of system behavior

- Large size of models

➔ **Modular compilation**

We introduce :

- a synchronous language **LE**
- an **equational semantic** allowing **modular** compilation
- an efficient way to check causality relying on a **finalization** phase

Outline

- 1 Introduction
- 2 LE Language
 - LE Language Overview
 - LE Equational Semantic
 - Correctness of the Equational Semantic
- 3 LE Modular Compilation
 - Causality Checking
 - Sorting Algorithms
 - Link of Two Partial Orders
 - Overview of the Compilation Process
- 4 Practical Issues
 - Effective Compilation
 - The Clem Toolkit
- 5 Conclusion and Future Work
 - Conclusion
 - Future Work
- 6 Appendix

LE Language

LE language allows 3 kinds of design :

- 1 Event driven application design
 - synchronous parallel
 - Run module operator to achieve separated compilation
- 2 Automata (State Chart like) design
- 3 Data flow application design

▶ detail

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Mathematical Context

- $\xi = \{\perp, 1, 0, \top\}$;
- notion of environment (E, \preceq)

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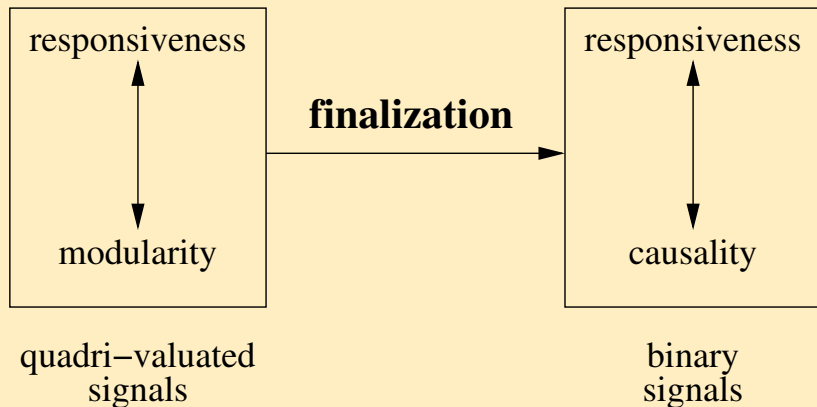
 ξ Rules

\sqcup	1	0	\top	\perp
1	1	\top	\top	1
0	\top	0	\top	0
\top	\top	\top	\top	\top
\perp	1	0	\top	\perp

\sqcap	1	0	\top	\perp
1	1	\perp	1	\perp
0	\perp	0	0	\perp
\top	1	0	\top	\perp
\perp	\perp	\perp	\perp	\perp

x	$\neg x$
1	0
0	1
\top	\perp
\perp	\top

Modularity versus Causality



Notion of Circuit

- W : wires ; R : registers ; S : signals (input, output, locals)
- $\mathcal{C} =_{def} \xi$ equation system
- $p \longrightarrow \mathcal{C}(p)$ with 3 wires :
 - ① Set_p : starts p
 - ② $Reset_p$: stops and reinitiates p
 - ③ RTL_p : p is ready to leave
 - ④ registers (for some instruction only)
- $E \vdash w \leftrightarrow bb$: a constructive propagation law prop-law

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Equational Semantic Definition

- p a LE statement, E : an environment
 $\mathcal{S}_e(p, E) = E'$ iff $E \vdash \mathcal{C}(p) \hookrightarrow E'$. (notation : $\langle p \rangle_E$)
- P :LE program.
 $(P, E) \mapsto E'$ iff $\mathcal{S}_e(\Gamma(P), E) = E'$, where $\Gamma(P)$ is the LE statement body of program P

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Environment Pre Operation

$$\mathcal{P}re(E) = \{S^\perp \mid S^x \in E\} \cup \{S_{pre}^x \mid S^x \in E\}$$

Wait operator Circuit Definition

$$C_{wait\ S} = \left[\begin{array}{ll} R+ & = (Set_{wait\ S} \sqcap \neg Reset_{wait\ S}) \sqcup \\ & (R \sqcap \neg Reset_{wait\ S} \sqcap \neg S) \quad (1) \\ RTL_{wait\ S} & = R \sqcap S \quad (2) \end{array} \right]$$

Wait Semantics

$$\langle P_{wait\ S} \rangle_E = \mathcal{P}re(E') \text{ and } E \vdash C(P_{wait\ S}) \hookrightarrow E'$$

Parallel Operator($P_1 \parallel P_2$) Circuit Definition $C_{P_1 \parallel P_2} =$

$$\left[\begin{array}{l} \text{Set}_{P_1} = \text{Set}_{P_1 \parallel P_2} \\ \text{Set}_{P_2} = \text{Set}_{P_1 \parallel P_2} \\ \text{Reset}_{P_1} = \text{Reset}_{P_1 \parallel P_2} \\ \text{Reset}_{P_2} = \text{Reset}_{P_1 \parallel P_2} \\ R_1^+ = R_1 \sqcap \neg \text{RTL}_{P_2} \sqcap \neg \text{Reset}_{P_1 \parallel P_2} \\ \quad \sqcup \neg R_2 \sqcap \text{RTL}_{P_1} \sqcap \neg \text{RTL}_{P_2} \sqcap \neg \text{Reset}_{P_1 \parallel P_2} \\ R_2^+ = R_2 \sqcap \neg \text{RTL}_{P_1} \sqcap \neg \text{Reset}_{P_1 \parallel P_2} \\ \quad \sqcup \neg R_1 \sqcap \neg \text{RTL}_{P_1} \sqcap \text{RTL}_{P_2} \sqcap \neg \text{Reset}_{P_1 \parallel P_2} \\ \text{RTL}_{P_1 \parallel P_2} = R_1 \sqcap \neg R_2 \sqcap \text{RTL}_{P_2} \sqcup \\ \quad (\neg R_1 \sqcap \text{RTL}_{P_1} \sqcap (R_2 \sqcup \text{RTL}_{P_2})) \end{array} \right]$$

Parallel Semantics

$$\langle P_1 \rangle_E \sqcup \langle P_2 \rangle_E \vdash C(P_1) \cup C(P_2) \cup C_{P_1 \parallel P_2} \leftrightarrow \langle P_1 \parallel P_2 \rangle_E$$

Behavioral Semantic

P program, E input environment, E' output environment :

Rule-based specification : $p \xrightarrow[E]{E', TERM} p'$

$$(P, E) \longmapsto (P', E') \quad \text{iff} \quad \Gamma(P) \xrightarrow[E]{E', TERM} \Gamma(P')$$

Behavioral Semantic

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Theorem

Let P be a LE statement, O its output signal set, and E_C an input environment, the following property holds :

$$P \xrightarrow[E]{E', RTL_P} P' \text{ and } \langle P \rangle_{E_C} \upharpoonright_O = E' \upharpoonright_O$$

where $E \upharpoonright_X = \{S^x \mid S^x \in E, S \in X\}$.

- **Equational semantic** offers a means to compile LE programs.
- **Behavioral semantic** ensures model-checking techniques apply.

New Causality Checking Method

- Problem : the composition of 2 causal systems may introduce causality cycle ▶ causality
- Solution :
 - 1 compute partial orders instead of total orders (thanks to equational semantics)
 - 2 finalization phase : to generate effective output code

Computing Partial Orders

For each equation system, we compute the earliest and latest dates at which each variable can and must be valuated :

- 1 2 dependencies graphs : from system inputs (upstream dependencies graph) and from system outputs (downstream dependencies graph);
- 2 the earliest date of each system variable v is the length of the maximal path from v to system inputs;
- 3 the latest date of each system variable v is the length of the maximal path from v to system outputs;

Example

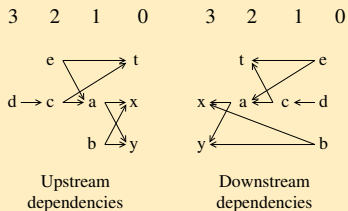
$$a = f_1(x, y)$$

$$b = f_2(x, y)$$

$$c = f_3(a, t)$$

$$d = f_4(a, c)$$

$$e = f_5(a, t)$$



Earliest and Latest Dates

a	b	c	d	e	x	y	t
(1, 1)	(1, 3)	(2, 2)	(3, 3)	(2, 3)	(0, 0)	(0, 0)	(0, 1)

Example

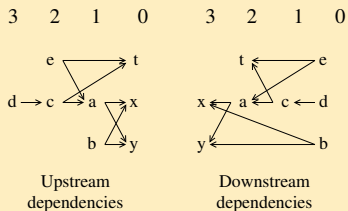
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3 Algorithms

- 1 apply **PERT** method : inputs (resp. outputs) have date 0 and recursively increase of dates for each vertice in the upstream (resp downstream) dependencies graph.
- 2 apply **graph theory** :
 - compute the adjacency matrix \mathcal{U} of upstream (resp. downstream) dependencies graph.
 - the length of the maximal path from a variable v to system inputs is characterized by the maximal k such that $\mathcal{U}^k[v, i] \neq 0$ for all inputs i .
- 3 apply **fix point theory** : the vector of earliest (resp. latest) dates can be computed as the least fix point of a momotonic increasing function.

Partial Orders Composition

To compose two already sorted systems A and B :

- only interface variables may be common ; thus we memorize the upstream dependencies of output variables and the downstream dependencies of input for each equation systems.
- two algorithms :
 - 1 propagation of common variables dates ajustement
 - 2 fix point characterisation starting with the vectors of already computed dates and considering only the variables in the dependencies (upstream and downstream) of common variables

Partial Orders Link

A $a = f_1(x, y)$ $b = f_2(x, y)$ $c = f_3(a, t)$ $d = f_4(a, c)$ $e = f_5(a, t)$		B $y = g_1(m)$ $z = g_2(d)$ $v = g_3(w)$
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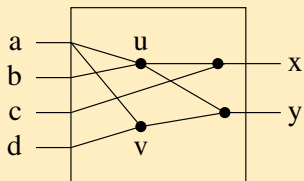
A :	a	b	c	d	e	x	y	t
	(1, 1)	(1, 3)	(2, 2)	(3, 3)	(2, 3)	(0, 0)	(0, 0)	(0, 1)

B :	d	m	v	w	y	z
	(0, 0)	(0, 0)	(1, 1)	(0, 0)	(1, 1)	(1, 1)

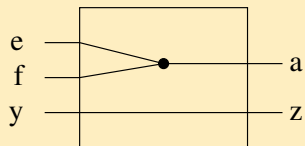
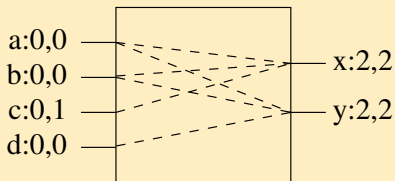
Common variables : d y

	<i>Equations1</i>	<i>Equations2</i>	<i>upstreamdep</i>	<i>downstreamdep</i>	d	y
<i>a</i>	(1, 2)	—	{ <i>c, e, d</i> }	{ <i>x, y</i> }	(1, 3)	(2, 3)
<i>b</i>	(1, 0)	—	\emptyset	{ <i>x, y</i> }	(1, 0)	(2, 0)
<i>c</i>	(2, 1)	—	{ <i>d</i> }	{ <i>a, t</i> }	(2, 2)	(3, 2)
<i>d</i>	(3, 0)	(0, 1)	{ <i>z</i> }	{ <i>a, c</i> }	(3, 1)	(4, 1)
<i>e</i>	(2, 0)	—	\emptyset	{ <i>a, t</i> }	(2, 0)	(3, 0)
<i>x</i>	(0, 3)	—	{ <i>a, b</i> }	\emptyset	(0, 4)	(0, 4)
<i>y</i>	(0, 3)	(1, 0)	{ <i>a, b</i> }	{ <i>m</i> }	(0, 4)	(1, 4)
<i>t</i>	(0, 2)	—	{ <i>c, e</i> }	\emptyset	(0, 3)	(0, 3)
<i>m</i>	—	(0, 1)	{ <i>y</i> }	\emptyset	(0, 1)	(0, 5)
<i>v</i>	—	(1, 0)	\emptyset	{ <i>w</i> }	(1, 0)	(1, 0)
<i>w</i>	—	(0, 1)	{ <i>v</i> }	\emptyset	(0, 1)	(0, 1)
<i>z</i>	—	(1, 0)	\emptyset	{ <i>d</i> }	(4, 0)	(5, 0)

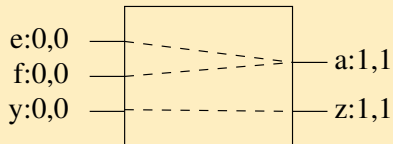
First Compilation Level



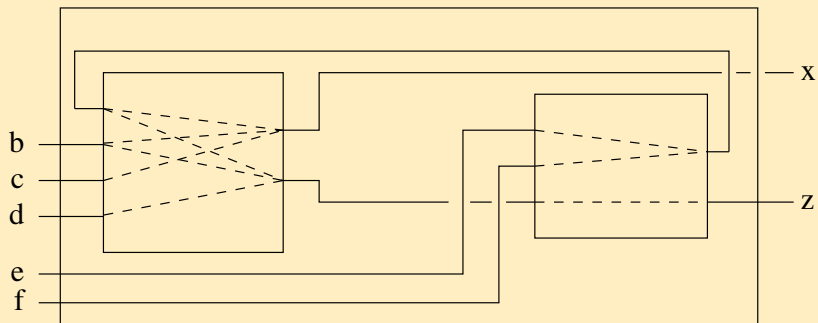
↓ abstraction



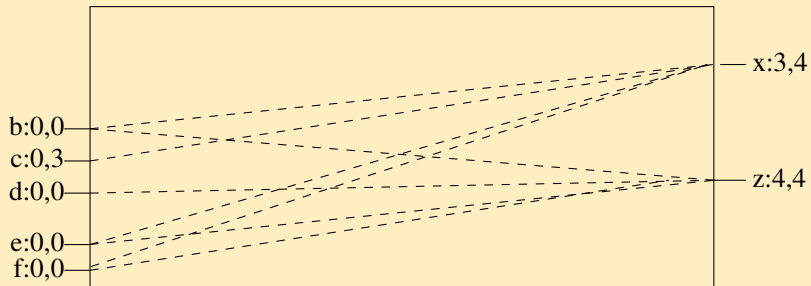
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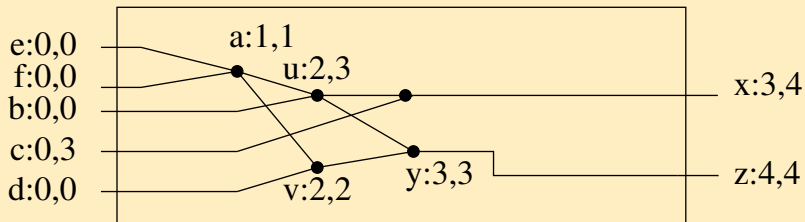
Second Compilation Level



Second Compilation Level



Finalization



Effective Compilation

- 1 P is associated with a ξ equation system ($\mathcal{C}(P)$)
- 2 $\xi \longrightarrow \mathcal{B}$ (BDD implementation)
- 3 compilation = \hookrightarrow propagation law implementation
- 4 separated compilation relies on
 - 1 LEC internal format
 - 2 Finalization operation

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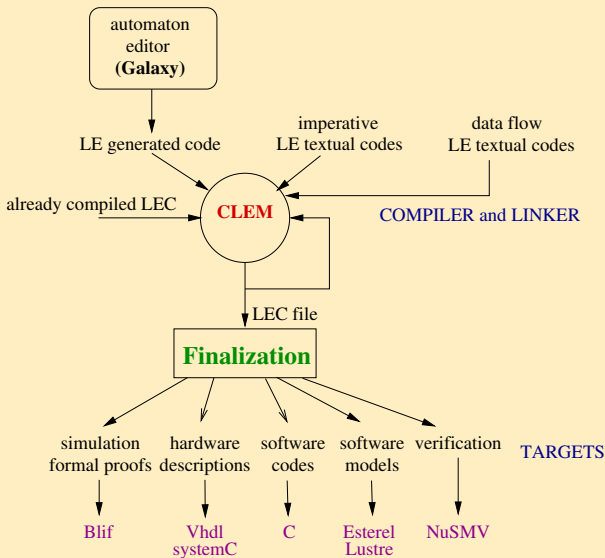
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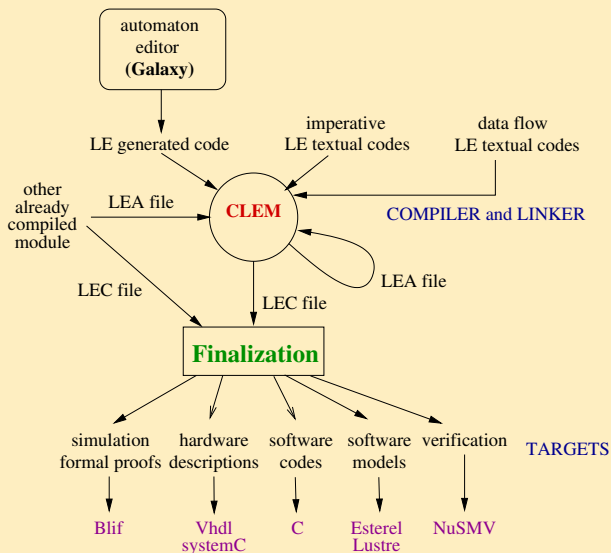
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 - 1 **LEC** internal format
 - 2 **Finilization** operation

CLEM Toolkit : <http://www.inria.fr/sophia/pulsar/projects/Clem>



The Future CLEM Toolkit



Conclusion

- 1 LE language with 2 semantics :
 - the equational semantic offers separated compilation means
 - the behavioral semantic allows NuSMV model-checker usage
- 2 We define the **CLEM** toolkit around LE language modular compilation

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Work in Progress

- ① large industrial application development
- ② extension of LE to deal with data :
 - language improvement
 - semantics extension
 - rely on **Abstract Interpretation** methods (like polyhedron intersection) to still apply model-checking techniques
- ③ improve LE verification :
 - provide facilities to define safety properties as observers.
 - prove that modular and “assume-guarantee” model-checking techniques apply

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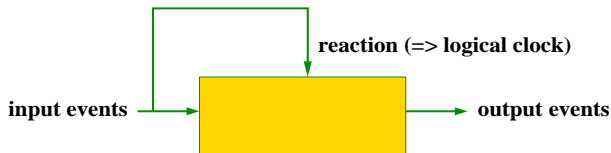
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Synchronous languages rely on the **Synchronous hypothesis**



Synchronous Hypothesis

Model of event driven systems

- **Broadcasting** of events (non blocking communication)
- Reaction is **atomic** : input and resulting output events are **simultaneous**
- Succession of reactions \Rightarrow **logical time**
- Synchronous systems are **deterministic**

Event driven Application Design

Event driven Application Design

LE Operators

- *emit speed*
- *present* $S \{ P1 \}$ *else* $\{ P2 \}$
- $P_1 \gg P_2$: perform P_1 then P_2
- $P_1 \parallel P_2$: **synchronous parallel** : start P_1 and P_2 simultaneously and stop when both have terminated
- *abort* P *when* S : perform P until S presence
- *loop* $\{ P \}$
- *local* $S \{ P \}$: encapsulation, the scope of S is restricted to P
- **Run** M : call of module M
- *pause* : stop until the next reaction
- *wait* S : stop until the next reaction in which S is present

LE Program Example

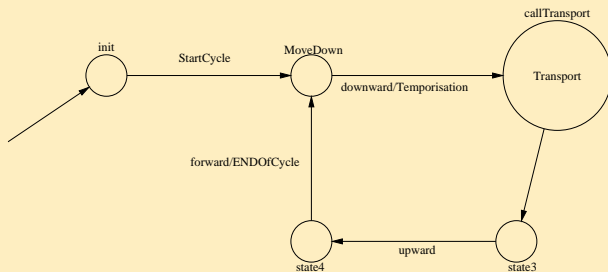
```
module R2WIE0 :  
Input: IO,I1;  
Output: O0,O1;  
Run:"/home/ar/GnuStr1/CLEM_SRC/TEST/" : WIE0;  
{  
  run WIE0[IO \ i, O0 \ o] || run WIE0[I1 \ i, O1 \ o]  
}  
end  
  
module WIE0 :  
Input: i;  
Output: o;  
wait i >> emit o  
end
```

State Chart like Design

State Chart like Design

Automata Design

- $\mathcal{A}(\mathcal{M}, \mathcal{T}, \text{Cond}, M_f, \mathcal{O}, \lambda)$: automata specification



Data flow application Design

▶ return

Data flow application Design

Equation Design

- $\mathcal{E}(\mathcal{I}, \mathcal{O}, \mathcal{R}, \mathcal{D})$: equation system definition

```
module ADDMM:
```

```
Input: Xi, Yi, Rin;
```

```
Output: Si, Rout;
```

Mealy Machine

```
Si = (Xi xor Yi) xor Rin;
```

```
Rout = (Xi and Yi) or (Xi and Rin) or (Yi and Rin);
```

```
end
```

▶ return

$$E \vdash v \hookrightarrow v$$

$$\frac{E(w) = v}{E \vdash w \hookrightarrow v}$$

$$\frac{E \vdash e \hookrightarrow \neg v}{E \vdash \neg e \hookrightarrow v}$$

$$\frac{E \vdash e \hookrightarrow \top \text{ or } E \vdash e' \hookrightarrow \top}{E \vdash e \sqcup e' \hookrightarrow \top}$$

$$\frac{E \vdash e \hookrightarrow \perp \text{ or } E \vdash e' \hookrightarrow \perp}{E \vdash e \sqcap e' \hookrightarrow \perp}$$

$$\frac{(E \vdash e \hookrightarrow 1 \text{ and } E \vdash e' \hookrightarrow 0) \text{ or } (E \vdash e \hookrightarrow 0 \text{ and } E \vdash e' \hookrightarrow 1)}{E \vdash e \sqcup e' \hookrightarrow \top \text{ and } E \vdash e \sqcap e' \hookrightarrow \perp}$$

$$\frac{(E \vdash e \hookrightarrow 1 \text{ and } E \vdash e' \hookrightarrow \perp) \text{ or } (E \vdash e \hookrightarrow \perp \text{ and } E \vdash e' \hookrightarrow 1)}{E \vdash e \sqcup e' \hookrightarrow 1 \text{ and } E \vdash e \sqcap e' \hookrightarrow \perp}$$

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$$\frac{(E \vdash e \hookrightarrow 0 \text{ and } E \vdash e' \hookrightarrow \top) \text{ or } (E \vdash e \hookrightarrow \top \text{ and } E \vdash e' \hookrightarrow 0)}{E \vdash e \sqcap e' \hookrightarrow 0}$$

$$\frac{E \vdash e \hookrightarrow v \text{ and } E \vdash e' \hookrightarrow v}{E \vdash e \sqcup e' \hookrightarrow v \text{ and } E \vdash e \sqcap e' \hookrightarrow v}$$

$$\frac{(E \vdash e \hookrightarrow \top \text{ and } E \vdash e' \hookrightarrow 1) \text{ or } (E \vdash e \hookrightarrow 1 \text{ and } E \vdash e' \hookrightarrow \top)}{E \vdash e \sqcap e' \hookrightarrow 1}$$

Causality Problem Illustration

```

module first:
Input: I1,I2;
Output: O1,O2;
loop {
  pause >>
  {
    present I1 {emit O1}
    ||
    present I2 {emit O2}
  }
}
end

```

O1 = I1
O2 = I2

O = L2
L1 = I

```

module second:
Input: I3;
Output: O3;
loop {
  pause >> present I3 {emit O3}
}
end

```

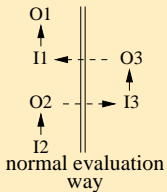
O3 = I3

```

module final:
Input: I;
Output: O;
local L1,L2 {
  run first[ L2\I1,O\O1,I\I2,L1\O2]
  ||
  run second[ L1\I3,L2\O3]
}
end

```

L2 = L1



L1 = I
L2 = L1
O = L2

Causality Problem Illustration

```

module first:
Input: I1,I2;
Output: O1,O2;
loop {
  pause >>
  {
    present I1 {emit O1}
    ||
    present I2 {emit O2}
  }
}
end

```

O1 = I1
O2 = I2

O = L2
L1 = I

```

module second:
Input: I3;
Output: O3;
loop {
  pause >> present I3 {emit O3}
}
end

```

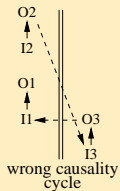
O3 = I3

```

module final:
Input: I;
Output: O;
local L1,L2 {
  run first[ L2\I1,O\O1,I\I2,L1\O2]
  ||
  run second[ L1\I3,L2\O3]
}
end

```

L2 = L1



L2 = L1
O = L2
L1 = I

$$E \vdash bb \hookrightarrow bb$$

$$\frac{E(w) = bb}{E \vdash w \hookrightarrow bb}$$

$$\frac{E \vdash e \hookrightarrow bb}{E \vdash (w = e) \hookrightarrow bb}$$

$$\frac{E \vdash e \hookrightarrow \neg bb}{E \vdash \neg e \hookrightarrow bb}$$