Lightweight Verification with Stochastic Abstraction

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Introduction

Our goal:

- Specification and analysis of large and complex heterogeneous systems

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- Verifying applications working within a subset of components of the system

Problem:

- *A component may use global ressources*
- *Behaviors of other components have to be considered*

Solution

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- Advantages
  - Stochastic abstraction considerably reduces the size of the heterogeneous system
  - Probabilities to quantify the level of failures
  - Statistical Model Checking allows to go beyond classical reasonings

The rest of this talk

- An introduction to statistical model checking;
- An application of stochastic abstraction (EADS, COMBEST);
- A discussion regarding possible future work.

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Learning from a Simple Problem

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A solution

- Do several flips and deduce the answer from them;
- If the number of flips is infinite, our answer will be correct up to some type error.

This is the statistical model checking approach!

Hypothesis Testing

Test $H_0 : P(\text{having a head}) \geq \theta$ against $H_1 : P(\text{having a head}) < \theta$

With (Type error):

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With (Type error):

1. α : the probability to accept H_1 while H_0 is true;
2. β : the probability to accept H_0 while H_1 is true.

The approach can also be used to compute the probability (PESTIMATION, Monte Carlo)

Indifference Region

We want to test :

$$H_0 : p \geq p_0 \text{ against } H_1 : p \leq p_1, \text{ where}$$
$$p_0 = \theta + \delta \text{ and } p_1 = \theta - \delta.$$

With:

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With:

- Type erros α and β, and
- Indifference region 2δ (needed to terminate in finite time).
- Parameters influence the number of simulations.

Bernouili Variables for experiments

- Bernouili variable X_i of parameter p
  - Takes two values : $X_i = 0$ or $X_i = 1$;
  - $P[X_i = 1] = p$ and $P[X_i = 0] = 1 - p$;
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  - Realization is denoted x_i.
- Experiments:
  - We assume independent trials;
  - We can generate as much trials as we want;
  - p is the probability to get a head ;
  - Associate a bernouili variable X_i to each trial;
  - $X_i = 1$ iff the trial is a tail.

Two Algorithms

- Algorithm 1 : Single Sampling plan (SSP):
  - Pre-compute a number *n* of experiments;
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- Algorithm 2: Basically a on-the-fly version of the Single Sampling Plan

Single Sampling plan : Principles

- Choose n and c with $c \leq n$;

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Difficulty : Find n and c such that α and β are satisfied

Single Sampling plan : Disadvantages

- Computing c and n is equivalent to solve an optimal problem on a sequence of binomial equations;
- This is difficult : No unique solution;
- Difficult to minimize n;
- Approximation algorithms exist (Haakan Youness).
- Better for black box systems

Sequential Hypothesis Testing

- Check hypothesis after each sample and stop as soon as possible
- We can find an acceptance line and a rejection line given $\alpha, \beta, \theta, \delta$.

Wald's Testing (SPRT)

Compute

$$W = \prod_{i=1}^{m} \frac{Pr(X_i = x_i \mid p = \theta - \delta)}{Pr(X_i = x_i \mid p = \theta + \delta)} = \frac{(\theta - \delta)^{d_m}(1 - \theta + \delta)^{m - d_m}}{(\theta + \delta)^{d_m}(1 - \theta - \delta)^{m - d_m}}, \qquad (1)$$

where $d_m = \sum_{i=1}^{m} x_i$.

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where $d_m = \sum_{i=1}^{m} x_i$.

Stop when :

- $W \geq (1 - \beta)/\alpha$: H_1 is accepted;
- $W \leq \beta/(1 - \alpha)$: H_0 is accepted.

More Mathematics

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- New parameters α' and β' such that
  - $\alpha' \leq \frac{\alpha}{1-\beta}$ and $\beta' \leq \frac{\beta}{1-\alpha}$
  - $\alpha' + \beta' \leq \alpha + \beta$;
- In practice : one observes that α and β are almost often guarantee, and it may even be better!

Example

Let $p_0 = 0.5$, $p_1 = 0.3$, $\alpha = 0.2$, $\beta = 0.1$:

- In theory : $\alpha' \leq \frac{0.2}{0.9} = 0.222...$ and $\beta' \leq \frac{0.1}{0.8} = 0.125$;
- Computer simulation : $\alpha' = 0.175$ and $\beta' = 0.082$.

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- Flipping a coin is nothing more than testing whether a finite execution satisfies a property.
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Properties

- Natural : those that can be checked on finite executions: next, bounded until;
- Better than classical logics : Clock drift, Fourier Transform, Systems Biology.

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- Easy to parallelize (independent sampling, unbiased distributed sampling);
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- Easy to trade accuracy for speed;
- Uniform approach;
- Easy to implement :

- In most cases, one only need to implement a "trace checker" that tests whether an execution satisfies a given property;
- No need for complex data structures.

Case Study: Accuracy of clock Synchronization (EADS)

Chalenges:

- Heterogeneous System over an Ethernet backbone
  - Distributed application
  - 280 communicating components
- Local clocks synchronized using the Precision Time Protocol
- Requirement: Verify that the difference between any 2 clocks is lower than a given bound

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Chalenges:

- Heterogeneous System over an Ethernet backbone
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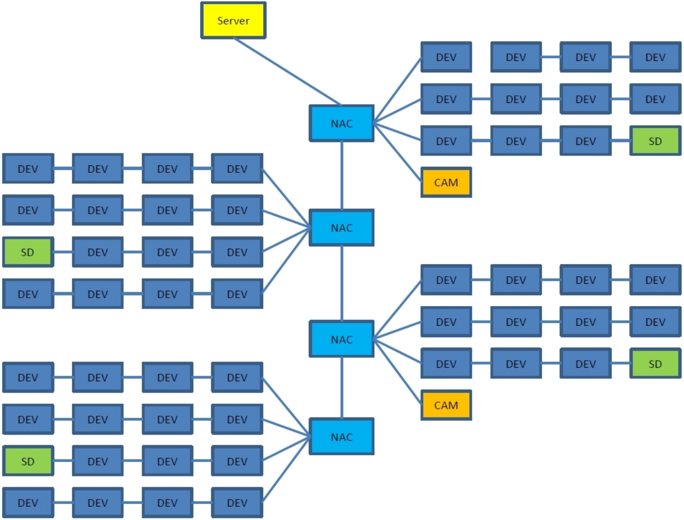
- Local clocks synchronized using the Precision Time Protocol
- Requirement: Verify that the difference between any 2 clocks is lower than a given bound
- Our goals: (1) Compute the best bound to satisfy this requirement without analyzing the whole architecture in a step, (2) compute the probability for a bound fixed by EADS ($50\mu s$).

Solution

- Apply stochastic abstraction between any device an the system;
- Compute the probability to synchronize for several values of the bound;
- Proceed similarly for all the devices;
- Keep the minimal bound for which synchonization is guarantee with probability 1

How to Compute Stochastic Abstraction?



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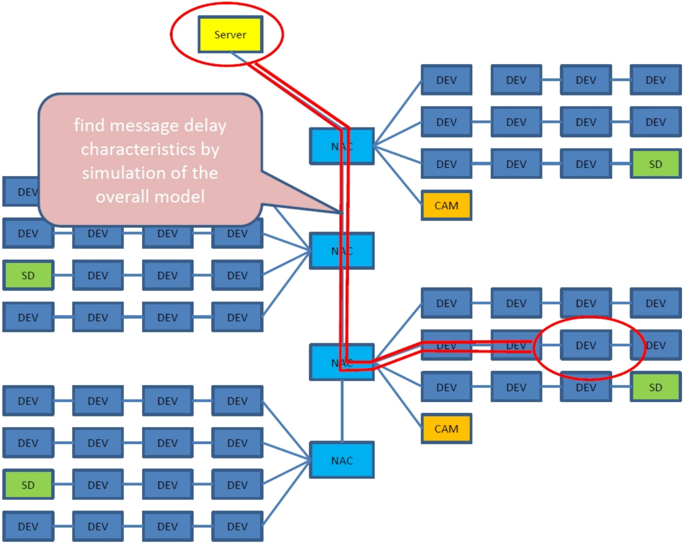

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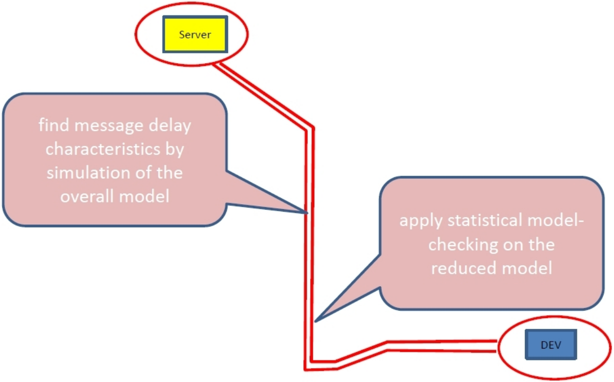

What do we need?

A tool

- whose input language is powerful enough to describe the EADS case study;
- in where stochastic aspects can easily be described;
- in where statistical model checking algorithms can easily be implemented;
- with an engine capable to generate executions in an efficient manner.

Our Solution: BIP!

First step

1. Learn the Probability distributions (1)

Run several simulations on the big model and extract the delays and the number of time they occurs;

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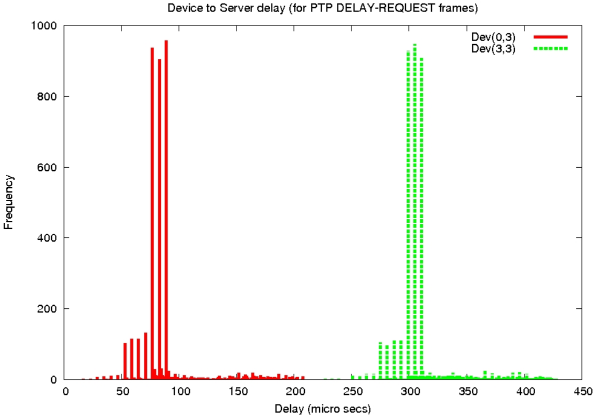

2. Use the distributions to study PTP

Let assume that we measured the delays 33 times. The result will be a series of delay values and, for each value, the number of times it has been observed. As an example, delay 5 has been observed 3 times, delay 19 has been observed 30 times. The probability distribution is represented with a table of 33 cells. In our case, 3 cells of the table will contains the value 5 and 30 will contain the value 19. The BIP engine will simply select a value in the table following a uniform probability distribution.

Second Step

3. Producing Stochastic Abstraction

Stochastic choices are directly integrated in the BIP engine!

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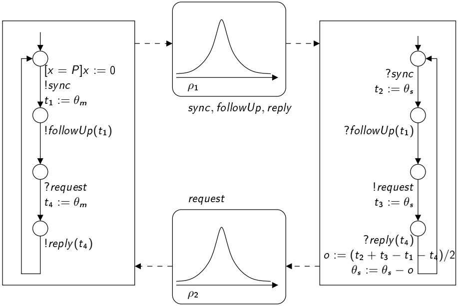

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Statistical model checking (1)

What are the questions?

- Qualitative Question : Does $\mathcal{S} \models P_{\geq\theta}(\varphi)$?
- Quantitative Question : What is the probability for $\mathcal{S}$ to satisfy φ?

Principle

- Reason on a finite set of executions and answer the question;
- We may make mistakes, but we should be as precise as we want!

Statistical model checking (2)

Qualitative question :

- Two algorithsm for the qualitative question : SPRT and SSP;
- They say yes or no, but can make a mistake (confidence).

Quantitative question :

- PESTIMATION computes an estimation p' of the probability for $\mathcal{S}$ to satisfy φ;
- The estimation can be bound : $|p - p'| < q$;
- The algorithm can make a mistake.

Statistical model checking (3)

- PESTIMATION is much slower that SSP or SPRT;
- A good strategy for answering the quantitative question :
    - Computer and estimation with a low confidence;
    - Validate this estimation with SSP or SPRT, but with a high confidence.

In Practice

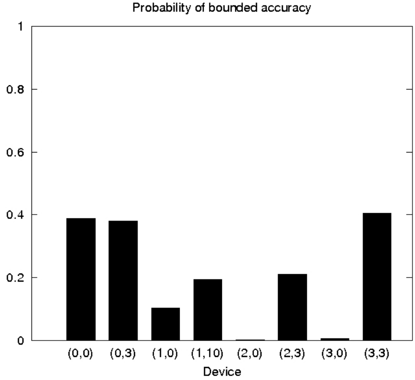
Model/Abstraction:

- PTP and HCS modeled using BIP
- Distributions of delays: 2000 measures

Statistical Model Checking:

- Quantitative question: precision 10^{-2}, confidence 10^{-2}: 100000 simulations
- Qualitative question: precision 10^{-3}, confidence 10^{-10}: 300000 simulations

Some Results 1/2



Probability of satisfying Bounded Accuracy for a bound of $50\mu s$

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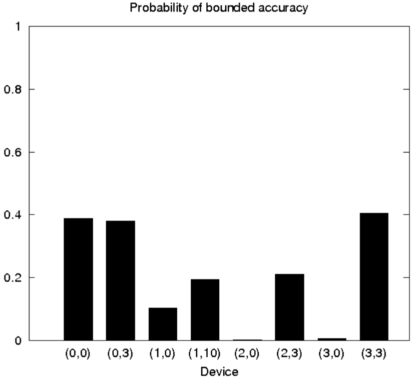
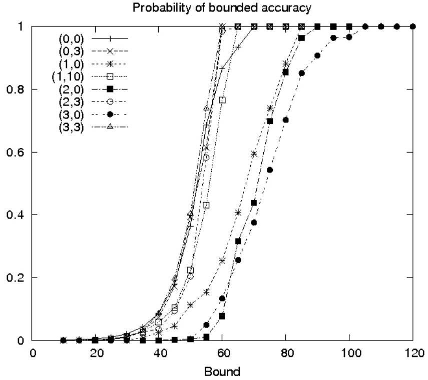


Probability of satisfying Bounded Accuracy for a bound of $50\mu s$

- The property is not satisfied for the given bound !



Probability of satisfying Bounded Accuracy as a function of the bound

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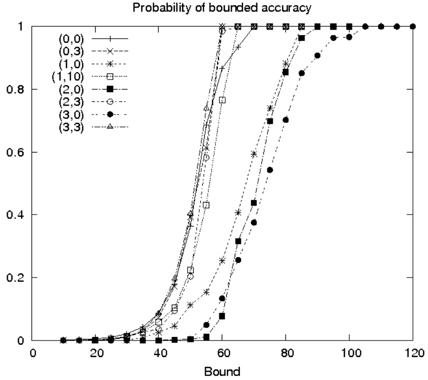


Probability of satisfying Bounded Accuracy as a function of the bound

- The best bound for which B.A. is satisfied with probability 1 is $105\mu s$

SPRT VS. SSP VS. PESTIMATION

Precision	10^{-1}		10^{-2}		10^{-3}	
Confidence	10^{-5}	10^{-10}	10^{-5}	10^{-10}	10^{-5}	10^{-10}
PESTIMATION	4883 $17s$	9488 $34s$	488243 $29m$	948760 $56m$	48824291 $> 3h$	94875993 $> 3h$
SSP	1604 $10s$	3579 $22s$	161986 $13m$	368633 $36m$	16949867 $> 3h$	32792577 $> 3h$
SPRT	316 $2s$	1176 $7s$	12211 $53s$	22870 $1m38s$	148264 $11m$	311368 $31m$

Precision	10^{-1}		10^{-2}		10^{-3}	
Confidence	10^{-5}	10^{-10}	10^{-5}	10^{-10}	10^{-5}	10^{-10}
SSP / SPRT	110 $1s$	219 $1s$	1146 $6s$	2292 $13s$	11508 $51s$	23015 $1m44s$

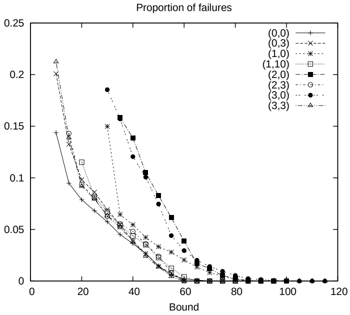
Proportion of Failures (1)

- Proportion of failure = number of failure divided by number of observations on a simulation.
- What is the average proportion of failure?
- What is the worst proportion of failure?

Solution : compute it on 2000 simulations with $\Delta = 50\mu s$:

- Device $(2, 0)$ gives $0, 24$;
- Device $(0, 0)$ gives $0, 076$.

Proportion of Failures (2)



Average proportion of failures as function of the bound Δ for the asymmetric version

Influence of Drift?

- Drift is used to model the fact that, due to the influence of the material, clocks of the master and the device may not progress as the same rate;
- Each time the clock of the server is increased by 1 time unit, the clock of the device is increased by $1 + 10^{-3}$ time units

Observations :

- Experiments : almost the same probabilities; Reason : the value of the drift is much smaller than the on of the jitter;
- With a drift of one time unit : Strong influence : Device $(0, 0)$ goes from a probability of $0,387$ to a probability of $0,007$.

Drift + Simulation = EASY;
Drift + exhaustive Verification = ALMOST IMPOSSIBLE

Results for stochastic abstraction

- Abstraction and verification method
- Applied to 2 case studies:
  - HCS case study [FORTE'10]
  - AFDX network [RV'10]

Future work

- Static analysis of the code for automatic extraction of the abstraction;
- Mixing confidence on the distribution with confidence on the statistical model checking algorithm;
- Handling non determinism;
- Using our knowledge of the system to improve the statistical model checking procedure;
- Various components described in various formalisms.